2025 *Activity Report*

RESEARCH CENTRE: **Inria Centre at Université Grenoble Alpes**
IN PARTNERSHIP WITH: **CNRS, Université de Grenoble Alpes**

Project-Team

TRIPOP

Modeling, Simulation and Control of Nonsmooth Dynamical Systems

In collaboration with Laboratoire Jean Kuntzmann (LJK)

Project-Team TRIPOP

Creation of the Project-Team: 2019 June 01

Each year, Inria research teams publish an Activity Report presenting their work and results over the reporting period. These reports follow a common structure, with some optional sections depending on the specific team. They typically begin by outlining the overall objectives and research programme, including the main research themes, goals, and methodological approaches. They also describe the application domains targeted by the team, highlighting the scientific or societal contexts in which their work is situated. The reports then present the highlights of the year, covering major scientific achievements, software developments, or teaching contributions. When relevant, they include sections on software, platforms, and open data, detailing the tools developed and how they are shared. A substantial part is dedicated to new results, where scientific contributions are described in detail, often with subsections specifying participants and associated keywords. Finally, the Activity Report addresses funding, contracts, partnerships, and collaborations at various levels, from industrial agreements to international cooperations. It also covers dissemination and teaching activities, such as participation in scientific events, outreach, and supervision. The document concludes with a presentation of scientific production, including major publications and those produced during the year.

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- A6.1.1. – Continuous Modeling (PDE, ODE)
- A6.1.4. – Multiscale modeling
- A6.2.6. – Optimization
- A6.4.1. – Deterministic control
- A6.4.3. – Observability and Controlability
- A6.4.4. – Stability and Stabilization
- A6.4.5. – Control of distributed parameter systems
- A6.4.6. – Optimal control
- A6.5.1. – Solid mechanics
- A6.5.4. – Waves

Other research topics and application domains

- B3.3.1. – Earth and subsoil
- B3.4.1. – Natural risks
- B5.2.1. – Road vehicles
- B5.2.3. – Aviation
- B5.2.4. – Aerospace
- B5.4. – Microelectronics
- B5.6. – Robotic systems
- B7.1.2. – Road traffic
- B9.5.2. – Mathematics
- B9.5.5. – Mechanics
- B9.11.1. – Environmental risks

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2 Overall objectives

2.1 Introduction

The joint research team, TRIPOP, between INRIA Grenoble Rhône–Alpes, Grenoble INP and CNRS, part of the Laboratoire Jean Kuntzmann (LJK UMR 5224) is mainly concerned with the modeling, the mathematical analysis, the simulation and the control of nonsmooth dynamical systems, with a strong application to modeling natural environmental risks in mountains.

Nonsmooth dynamics concerns the study of the time evolution of systems that are not smooth in the mathematical sense, *i.e.* systems that are characterized by a lack of differentiability, either of the mappings in their formulations, or of their solutions with respect to time. In mechanics, the main instances of nonsmooth dynamical systems are multibody systems with Signorini unilateral contact, set-valued (Coulomb-like) friction and impacts. In electronics, examples are found in switched electrical circuits with ideal components (diodes, switches, transistors). In control, nonsmooth systems arise in the sliding mode control theory and in optimal control. Many examples can also be found in cyber-physical systems (hybrid systems), in transportation sciences, in mathematical biology or in finance.

The team is organized along two research axes:

1) nonsmooth simulation and numerical modeling for natural gravitational risks in mountains and

2) modeling, simulation and control of nonsmooth dynamical systems.

The idea is to put forward a strong application axis for which there is a strong academic research dynamic in the Grenoble region and a network of socio-economic actors very interested in an industrial transfer of digital science methods on these subjects. The second axis remains at the heart of the team's activities and expertise.

2.2 General scope and motivations

Nonsmooth dynamics concerns the study of the time evolution of systems that are not smooth in the mathematical sense, *i.e.*, systems that are characterized by a lack of differentiability, either of the mappings in their formulations, or of their solutions with respect to time. The class of nonsmooth dynamical systems recovers a large variety of dynamical systems that arise in many applications. The term "nonsmooth", like the term "nonlinear", does not precisely define the scope of the systems we are interested in but, and most importantly, they are characterized by the mathematical and numerical properties that they share. To give more insight into nonsmooth dynamical systems, we give in the following a very brief introduction of their salient features. For more details, we refer to [59, 26][2][76, 44, 62].

As we have indicated there are many applications to the methods of nonsmooth dynamics. We have chosen a strong particular application for this technique of nonsmooth dynamics which is that of natural gravity risk in the mountains. The choice of this application is particularly motivated by global climate change which has increased the number of rockfall and landslide events very significantly in recent decades.

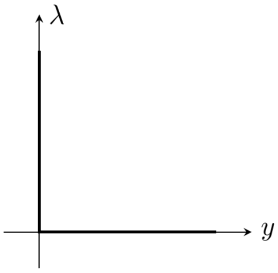


Figure 1: Complementarity condition $0 \leq y \perp \lambda \geq 0$.

Especially, the effects of melting permafrost, increased rainfall and rapid temperature changes means that alpine regions are particularly at risk [89, 74]. The team will conduct research on the mechanical modeling and simulation of natural hazards in mountains (floods and debris flows, block falls, glacial hazards), bringing new software development in a high performance computing (HPC) framework.

2.3 A flavor of nonsmooth dynamical systems

As a first illustration, let us consider a linear finite-dimensional system described by its state $x(t) \in \mathbb{R}^n$ over a time-interval $t \in [0, T]$:

$$\dot{x}(t) = Ax(t) + a, \quad A \in \mathbb{R}^{n \times n},\ a \in \mathbb{R}^n, \tag{1}$$

subjected to a set of m inequality (unilateral) constraints:

$$y(t) = Cx(t) + c \geq 0, \qquad C \in \mathbb{R}^{m \times n},\ c \in \mathbb{R}^m. \tag{2}$$

If the constraints are physical constraints, a standard modeling approach is to augment the dynamics in (1) by an input vector $\lambda(t) \in \mathbb{R}^m$ that plays the role of a Lagrange multiplier vector. The multiplier restricts the trajectory of the system in order to respect the constraints. Furthermore, as in the continuous optimization theory, the multiplier must be signed and must vanish if the constraint is not active. This is usually formulated as a complementarity condition:

$$0 \leq y(t) \perp \lambda(t) \geq 0, \tag{3}$$

which models the one-sided effect of the inequality constraints. The notation $y \geq 0$ holds component–wise and $y \perp \lambda$ means $y^T \lambda = 0$. All together we end up with a Linear Complementarity System (LCS) of the form,

$$\begin{aligned} \dot{x}(t) &= Ax(t) + a + B\lambda(t) \\ y(t) &= Cx(t) + c \\ 0 &\leq y(t) \perp \lambda(t) \geq 0 \end{aligned} \tag{4}$$

where $B \in \mathbb{R}^{n \times m}$ is the matrix that models the input generated by the constraints. In a more general way, the constraints may also involve the Lagrange multiplier,

$$y(t) = Cx(t) + c + D\lambda(t) \geq 0, \qquad D \in \mathbb{R}^{m \times m}, \tag{5}$$

leading to a general definition of LCS as

$$\begin{aligned} \dot{x}(t) &= A\,x(t) + a + B\,\lambda(t) \\ y(t) &= C\,x(t) + c + D\,\lambda(t) \\ 0 &\leq y(t) \perp \lambda(t) \geq 0. \end{aligned} \tag{6}$$

The complementarity condition, illustrated in Figure 1 is the archetype of a nonsmooth graph that we extensively use in nonsmooth dynamics. The mapping $y \mapsto \lambda$ is a multi-valued (set-valued) mapping, that is nonsmooth at the origin. It has many interesting mathematical properties and reformulations that come mainly from convex analysis and variational inequality theory. Let us introduce the indicator function of $\mathbb{R}_+$ as

$$\Psi_{\mathbb{R}_+}(x) = \begin{cases} 0 \text{ if } x \geq 0, \\ +\infty \text{ if } x < 0. \end{cases} \tag{7}$$

This function is convex, proper and can be sub-differentiated [68]. The definition of the subdifferential of a convex function $f : \mathbb{R}^m \longrightarrow \mathbb{R}$ is defined as:

$$\partial f(x) = \left\{ x^\star \in \mathbb{R}^m \mid f(z) \geq f(x) + (z-x)^\top x^\star, \forall z \right\}. \tag{8}$$

A basic result of convex analysis is

$$0 \leq y \perp \lambda \geq 0 \Longleftrightarrow -\lambda \in \partial \Psi_{\mathbb{R}_+}(y) \tag{9}$$

that gives a first functional meaning to the set-valued mapping $y \mapsto \lambda$. Another interpretation of $\partial \Psi_{\mathbb{R}_+}$ is based on the normal cone to a closed and nonempty convex set C:

$$N_C(x) = \left\{ v \in \mathbb{R}^m | v^\top (z-x) \leq 0 \text{ for all } z \in C \right\}. \tag{10}$$

It is easy to check that $\partial \Psi_{\mathbb{R}_+}(x) = N_{\mathbb{R}_+}(x)$ and it follows that

$$0 \leq y \perp \lambda \geq 0 \Longleftrightarrow -\lambda \in N_{\mathbb{R}_+}(y). \tag{11}$$

Finally, the definition of the normal cone yields a variational inequality:

$$0 \leq y \perp \lambda \geq 0 \Longleftrightarrow \lambda^\top (y-z) \leq 0, \forall z \geq 0. \tag{12}$$

The relations (11) and (12) allow one to formulate the complementarity system with $D = 0$ as a differential inclusion based on a normal cone (see (15)) or as a differential variational inequality. By extending the definition to other types of convex functions, possibly nonsmooth, and using more general variational inequalities, the same framework applies to the nonsmooth laws depicted in Figure 2 that includes the case of piecewise smooth systems.

The mathematical concept of solutions depends strongly on the nature of the matrix quadruplet (A, B, C, D) in (6). If D is a positive definite matrix (or a P-matrix), the Linear Complementarity problem

$$0 \leq Cx + c + D\lambda \perp \lambda \geq 0, \tag{13}$$

admits a unique solution $\lambda(x)$ which is a Lipschitz continuous mapping. It follows that the Ordinary Differential Equation (ODE)

$$\dot{x}(t) = Ax(t) + a + B\lambda(x(t)), \tag{14}$$

is a standard ODE with a Lipschitz right-hand side with a C^1 solution for the initial value problem. If $D = 0$, the system can be written as a differential inclusion in a normal cone as

$$-\dot{x}(t) + Ax(t) + a \in BN_{\mathbb{R}_+}(Cx(t)), \tag{15}$$

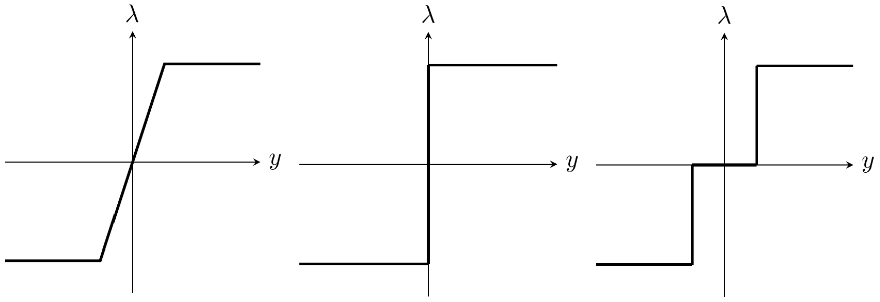
that admits a solution that is absolutely continuous if CB is a definite positive matrix and the initial condition satisfies the constraints. The time derivative $\dot{x}(t)$ and the multiplier $\lambda(t)$ may have jumps and are generally considered as functions of bounded variations. If $CB = 0$, the order of nonsmoothness increases and the Lagrange multiplier may contain Dirac atoms and must be considered as a measure. Higher–order index, or higher relative degree systems yield solutions in terms of distributions and derivatives of distributions [39].

A lot of variants can be derived from the basic form of linear complementarity systems, by changing the form of the dynamics including nonlinear terms or by changing the complementarity relation by other multivalued maps. In particular the nonnegative orthant may be replaced by any convex closed cone $K \subset \mathbb{R}^m$ leading to complementarity over cones

$$K^\star \ni y \perp \lambda \in K, \tag{16}$$

where $K^\star$ its dual cone given by

$$K^\star = \left\{ x \in \mathbb{R}^m \mid x^\top y \geq 0 \text{ for all } y \in K \right\}. \tag{17}$$



(a) Saturation (b) Relay (c) Relay with dead zone

Figure 2: Examples of multivalued piecewise linear models

In Figure 2, we illustrate some other basic maps that can be used for defining the relation between λ and y. The saturation map, depicted in Figure 2(a) is a single valued continuous function which is an archetype of a piece-wise smooth map. In Figure 2(b), the relay multi-function is illustrated. If the upper and the lower limits of λ are respectively equal to 1 and -1, we obtain the multivalued sign function defined as

$$\mathrm{Sgn}(y) = \begin{cases} 1, & y > 0 \\ [-1, 1], & y = 0 \\ -1, & y < 0. \end{cases} \tag{18}$$

Using again convex analysis, the multivalued sign function may be formulated as an inclusion into a normal cone as

$$\lambda \in \mathrm{Sgn}(y) \Longleftrightarrow y \in N_{[-1,1]}(\lambda). \tag{19}$$

More generally, any system of the type,

$$\begin{aligned} \dot{x}(t) &= Ax(t) + a + B\lambda(t) \\ y(t) &= Cx(t) + a \\ -\lambda(t) &\in \mathrm{Sgn}(y(t)), \end{aligned} \tag{20}$$

can reformulated in terms of the following set-valued system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + a + B\lambda(t) \\ y(t) &= Cx(t) + c \\ -y(t) &\in N_{[-1,1]^m}(\lambda(t)). \end{aligned} \tag{21}$$

The system (21) appears in a lot of applications; among them, we can cite the sliding mode control, electrical circuits with relay and Zener diodes [34], or mechanical systems with friction [26].

Though this class of systems seems to be rather specific, it includes as well more general dynamical systems such as piecewise smooth systems and discontinuous ordinary differential equations. Indeed, the system (20) for scalars y and λ can be viewed as a discontinuous differential equation:

$$\dot{x}(t) = \begin{cases} Ax + a + B \text{ if } Cx + c > 0 \\ Ax + a - B \text{ if } Cx + c < 0. \end{cases} \tag{22}$$

One of the most well-known mathematical frameworks to deal with such systems is the Filippov theory [59] that embeds the discontinuous differential equations into a differential inclusion. In the case of a single discontinuity surface given in our example by $S = \{x \mid Cx + c = 0\}$, the Filippov differential inclusion based on the convex hull of the vector fields in the neighborhood of S is equivalent to the use of the multivalued sign function in (20). Conversely, as it has been shown in [40], a piecewise smooth system can be formulated as a nonsmooth system based on products of multivalued sign functions.

2.4 Nonsmooth Dynamical systems in the large

Generally, the nonsmooth dynamical systems we propose to study mainly concern systems that possess the following features:

- A nonsmooth formulation of the constitutive/behavioral laws that define the system. Examples of nonsmooth formulations are piecewise smooth functions, multi–valued functions, inequality constraints, yielding various definitions of dynamical systems such as piecewise smooth systems, discontinuous ordinary differential equations, complementarity systems, projected dynamical systems, evolution or differential variational inequalities and differential inclusions (into normal cones). Fundamental mathematical tools come from convex analysis [90, 67, 68], complementarity theory [55], and variational inequalities theory [58].

- A concept of solutions that does not require continuously differentiable functions of time. For instance, absolutely continuous, Lipschitz continuous functions or functions of local bounded variation are the basis for solution concepts. Measures or distributions are also solutions of interest for differential inclusions or evolution variational inequalities.

2.5 Nonsmooth systems versus hybrid systems

The nonsmooth dynamical systems we are dealing with, have a nonempty intersection with hybrid systems and cyber-physical systems, as is briefly discussed in Sect. 3.3.1. Like in hybrid systems, nonsmooth dynamical systems define continuous-time dynamics that can be identified with modes separated by guards, defined by the constraints. However, the strong mathematical structure of nonsmooth dynamical systems allows us to state results on the following points:

- Mathematical concept of solutions: well-posedness (existence, and possibly, uniqueness properties, (dis)continuous dependence on initial conditions).

- Dynamical systems theoretic properties: existence of invariants (equilibria, limit cycles, periodic solutions,. . .) and their stability, existence of oscillations, periodic and quasi-periodic solutions and propagation of waves.

- Control theoretic properties: passivity, controllability, observability, stabilization, robustness.

These latter properties, that are common for smooth nonlinear dynamical systems, distinguish the nonsmooth dynamical systems from the very general definition of hybrid or cyber-physical systems [42, 66]. Indeed, it is difficult to give a precise mathematical concept of solutions for hybrid systems since the general definition of hybrid automata is usually too loose.

2.6 Numerical methods for nonsmooth dynamical systems

To conclude this brief exposition of nonsmooth dynamical systems, let us recall an important fact related to numerical methods. Beyond their intrinsic mathematical interest, and the fact that they model real physical systems, using nonsmooth dynamical systems as a model is interesting, because there exists a large set of robust and efficient numerical techniques to simulate them. Without entering into the finer details, let us give two examples of these techniques:

- *Numerical time integration methods: convergence, efficiency (order of consistency, stability, symplectic properties).* For the nonsmooth dynamical systems described above, there exist event-capturing time-stepping schemes with strong mathematical results. These schemes have the ability to numerically integrate the initial value problem without performing an event location, but by capturing the event within a time step. We call an event, or a transition, every change into the index set of the active constraints in the complementarity formulation or in the normal cone inclusion. Hence these schemes are able to simulate systems with a huge number of transitions or even with finite accumulation of events (Zeno behavior). Furthermore, the schemes do not suffer from the weaknesses of the standard schemes based on a regularization (smoothing) of the multi-valued mapping resulting in stiff ordinary differential equations. For the time-integration of the initial value problem (IVP), or Cauchy problem, a lot of improvements of the standard time-stepping schemes for nonsmooth dynamics (Moreau–Jean time-stepping scheme) have been proposed in the last decade, in terms of accuracy and dissipation properties [30, 31, 91, 92, 29, 52, 48, 94, 50]. A significant number of these schemes have been developed by members of the BIPOP team and have been implemented in the Siconos software.

- *Numerical solution procedure for the time–discretized problem, mainly through well-identified problems studied in the optimization and mathematical programming community.* Another very interesting feature is the fact that the discretized problem that we have to solve at each time–step is generally a well-known problem in optimization. For instance, for LCSs, we have to solve a linear complementarity problem [55] for which there exist efficient solvers in the literature. Compared to the brute force algorithm with exponential complexity that consists of enumerating all the possible modes, the algorithms for linear complementarity problem have polynomial complexity when the problem is monotone.

3 Research program

3.1 Introduction

In this section, we develop our scientific program. In the framework of nonsmooth dynamical systems, the activities of the project–team will be focused on the following research axes:

- **Axis 1**: Nonsmooth simulation and numerical modeling for natural gravitational risk in mountains. (detailed in Sect. 3.2).
- **Axis 2**: Modeling, simulation and control (detailed in Sect. 3.3).

These research axes will be developed with a strong emphasis on the software development and the industrial transfer.

3.2 Axis 1: Nonsmooth simulation and numerical modeling for natural gravitational risk in mountains.

In this research axis, we propose, on the one hand, to extend existing methods of simulation in mechanics of complex flows in a nonsmooth framework, which allows us to simplify the models by decreasing the physical parameters, and to make more robust the numerical simulations and thus to make possible the construction of reduced models or meta-models. On the other hand, the so-called "data-driven modeling" methods will be explored for gravity flows and prevention structures. The aim is to make the most of laboratory and observational data in order to build and calibrate the models, to evaluate their sensitivity, to improve their predictive character, *i.e.* to control and take into account the uncertainties, thanks to variational, statistical and AI methods.

This work will be conducted in close collaboration with the UR IGE of INRAE as well as other researchers from INRIA (AIRSEA, LEMON). More generally, our collaboration with INRAE opens new long term perspectives on granular flow applications such as debris and mud flows, granular avalanches and the design of structural protections. The numerical methods that go with these new modeling approaches will be implemented in our software Siconos).

This research is also part of the more general context of a digital platform on environmental risk in the mountains, including intensive and cloud computing.

3.2.1 Rockfall trajectory modeling

Trajectory analysis of falling rocks during rockfall events is limited by the currently unrefined modeling of the impact phase [46, 45, 77]. The goal of this axis is to improve reliability of simulation techniques.

- *Rock fracturing:* When a rock falls from a steep cliff, it stores a large amount of kinetic energy that is partly dissipated though the impact with the ground. If the ground is composed of rocks and the kinetic energy is sufficiently high, the probability of the fracture of the rock is high and yields an extra amount of dissipated energy but also an increase of the number of blocks that fall. In this topic, we want to use the capability of the nonsmooth dynamical framework for modeling cohesion and fracture [73, 28] to propose new cohesive zone models with contact and friction.

- *Rock/forest interaction:* To prevent damage and incidents to infrastructure, a smart use of the forest is one of the ways to control trajectories (decrease of the run-out distance, jump heights and the energy) of the rocks that fall under gravity [56, 57]. From the modeling point of view and to be able to improve the protective function of the forest, an accurate modeling of impacts between rocks and trees is required. Due to the aspect ratio of the trees, they must be considered as flexible bodies that may be damaged by the impact. This new aspect offers interesting modeling research perspectives, especially, building rockfall simulation method with mechanical models of trees including damage, fracture and plasticity.

- *Experimental validation:* The participation of INRAE with F. Bourrier makes possible the experimental validation of models and simulations through comparisons with real data. INRAE has extensive experience of lab and in-situ experiments for rockfall trajectory modeling [46, 45]. It is a unique opportunity to strengthen our model and to prove that nonsmooth modeling of impacts is reliable for such experiments and forecasting of natural hazards.

3.2.2 Modeling and simulation of gravity hazards (debris flows, avalanches and large-scale rock flows)

Different modeling approaches are used in the literature depending on the type of hazard.

For rockfalls and dense snow avalanches, methods that explicitly model the particles of granular materials (notably Discrete Element Methods - DEM) are preferred, whereas for flows (debris flows, avalanches and large-scale rockfalls), methods that assimilate the large number of individual constituents to materials with complex rheology are more commonly used (notably Material Point Method - MPM, Smoothed-Particle Hydrodynamics - SPH, Shallow Water models - SWM). It should be noted that these methods are most often explicit and regularize the constraints of inequalities and thresholds.

This research item will develop the following points:

- *Rethinking DEM, MPM, SPH and SWM methods in the nonsmooth framework.* This will allow a simple and efficient modeling of threshold and inequality phenomena (one-sided contact, impact with Coulomb friction, threshold behavior laws such as plasticity, damage or fracture, Bingham-type fluids) in order to develop new, implicit and robust numerical methods, where the most important physical features of frictional cohesive materials are well-modeled neglecting the second order phenomena. In a context of data utilization and prediction, these methods seem particularly well suited as our first experiments on block trajectography and rock flows have already shown.

- *Couple these methods to integrate the "multi-scale (micro/meso/macro)" character* of these problems or, more simply, to spatially couple at the same scale several physical phenomena better taken into account by different methods, for example a debris flow containing a material with complex rheology (MPM or SPH) and large size particles (DEM)

- *Use “data-driven mechanics” approaches* when behavioral models are not reliable and faithful to the observed physical phenomena. These techniques can also be used to model “sub-mesh” phenomena, which are not or only slightly taken into account in large-scale phenomenological models.

3.2.3 Data-driven modelling for prediction and mitigation of gravity hazards

The objective is to develop simplified models that can be used extensively for the development of calibration and uncertainty quantification methods that allow for the joint use of data from various sources to evaluate and improve the predictive capacity of gravity hazard models.
The following points will be developed:

- *Statistical models* integrating various types of data and the hazard models developed in the previous section. The identification of the parameters of these hazard models, in particular using Bayesian approaches, will also allow the calibration and quantification of the uncertainties associated with the hazard models.

- *Model reduction approaches* (POD, PGD,...) or construction of substitution models (Sparse Polynomial Chaos, Gaussian Processes,...) to build simplified models usable in this context.

- *Application of different data assimilation techniques* (particle filters or variational methods) on the models described in the first axis and the reduced order models.

3.3 Axis 2: Modeling, simulation and control of non-smooth dynamical systems.

This axis is dedicated to the modeling and the mathematical analysis of nonsmooth dynamical systems. It consists of two main directions: 1) Modeling, analysis and numerical methods and 2) Automatic control.

3.3.1 Modeling, analysis and numerical methods

Multibody vibro-impact systems

- *Multiple impacts with or without friction (short-term):* there are many different approaches to model collisions, especially simultaneous impacts (so-called multiple impacts) [87]. One of our objectives is on one hand to determine the range of application of the models (for instance, when can one use "simplified" rigid contact models relying on kinematic, kinetic or energetic coefficients of restitution?) on typical benchmark examples (chains of aligned beads, rocking block systems). On the other hand, we will try to take advantage of the new results on nonlinear wave phenomena, to better understand multiple impacts in 2D and 3D granular systems. The study of multiple impacts with (unilateral) nonlinear visco-elastic models (Simon–Hunt–Crossley, Kuwabara–Kono), or visco-elasto-plastic models (assemblies of springs, dashpots and dry friction elements), is also a topic of interest, since these models are widely used.

- *Artificial or manufactured or ordered granular crystals, meta-materials (short-term):* Granular metamaterials (or more general nonlinear mechanical metamaterials) offer many perspectives for the passive control of waves originating from impacts or vibrations. The analysis of waves in such systems is delicate due to spatial discreteness, nonlinearity and non-smoothness of contact laws [88, 71, 72, 78]. We will use a variety of approaches, both theoretical (e.g. bifurcation theory, modulation equations) and numerical, in order to describe nonlinear waves in such systems, with special emphasis on energy localization phenomena (excitation of solitary waves, fronts, breathers).

- *Systems with clearances, modeling of friction (long-term):* joint clearances in kinematic chains deserve specific analysis, especially concerning friction modeling [41]. Indeed contacts in joints are often conformal, which involve large contact surfaces between bodies. Lubrication models should also be investigated.

- *Painlevé paradoxes (long-term):* the goal is to extend the results in [61], which deal with single-contact systems, to multi-contact systems. One central difficulty here is the understanding and the analysis of singularities that may occur in sliding regimes of motion.

As a continuation of the work in the BIPOP team, our software Siconos will be our favored software platform for the integration of these new modeling results.

Systemic risk

- The high consumption of natural resources by our society puts in question its long-term sustainability. The decrease of natural resources results in a deterioration of human welfare with a risk of society instability. Recently, a simple nature-society interrelations model, called the HANDY model (Human And Nature DYnamics), has been proposed by Montesharrei *et al* (2014) to address this concern with a special emphasis on the role of the stratification of the society. The Handy model is a four dimensional nonlinear dynamical system that describes the evolution of population, resources and accumulated wealth. We analyse the dynamics of this model and we explore the influence of two parameters: the nature depletion rate and the inequality factor. We characterize the asymptotic states of the system through a bifurcation analysis and we derive several quantitative results on the trajectories. We show that some collapses are irreversible and, depending on the wealth production factor, a bistability regime between a sustainable equilibrium and cycles of *collapse-and-regeneration* can be obtained. We discuss possible policies to avoid dramatic scenarios.

Cyber-physical systems (hybrid systems) Participants: V. Acary, B. Brogliato, C. Prieur, A. Tonnelier

Nonsmooth systems have a non-empty intersection with hybrid systems and cyber–physical systems. However, nonsmooth systems enjoy strong mathematical properties (concept of solutions, existence and uniqueness) and efficient numerical tools. This is often the result of the fact that nonsmooth dynamical systems are models of physical systems, and so can take advantage of their intrinsic properties (conservation or dissipation of energy, passivity, stability). A standard example is a circuit with n ideal diodes. From the hybrid point of view, this circuit is a piecewise smooth dynamical system with 2^n modes, that can be quite cumbersome to enumerate in order to determinate the current mode. As a nonsmooth system, this circuit can be formulated as a complementarity system for which there exist efficient time-stepping schemes and polynomial time algorithms for the computation of the current mode. The key idea of this research action is to benefit from this observation to improve hybrid system modeling tools.

- *Structural analysis of multimode DAE :* When a hybrid system is described by a Differential Algebraic Equation (DAE) with different differential indices in each continuous mode, the structural analysis has to be completely rethought. In particular, the re-initialization rule, when a switching occurs from one mode to another, has to be consistently designed. We propose in this action to use our knowledge in complementarity and (distribution) differential inclusions [39] to design consistent re-initialization rules for systems with nonuniform relative degree vector $(r_1, r_2, \ldots, r_m)$ and $r_i \neq r_j, i \neq j$.

- *Cyber–physical in hybrid systems modeling languages :* Nowadays, some hybrid modeling languages and tools are widely used to describe and to simulate hybrid systems (MODELICA, SIMULINK, and see [51] for references therein). Nevertheless, the compilers and the simulation engines behind these languages and tools suffer from several serious weaknesses (failure, nonsensical output or extreme sensitivity to simulation parameters), especially when some components, that are standard in nonsmooth dynamics, are introduced (piecewise smooth characteristic, unilateral constraints and complementarity condition, relay characteristic, saturation, dead zone, . . .). One of the main reasons is the fact that most of the compilers reduce the hybrid system to a set of smooth modes modeled by differential algebraic equations and some guards and reinitialization rules between these modes. Sliding mode and Zeno-type behaviour are extremely difficult for hybrid systems and relatively simple for nonsmooth systems. With B. Caillaud (Inria HYCOMES) and M. Pouzet (Inria PARKAS), we propose to improve this situation by implementing a module able to identify/describe nonsmooth elements and to efficiently handle them with SICONOS as the simulation engine. They have already carried out a first implementation [49] in Zelus, a synchronous language for hybrid systems Zelus. Removing the weaknesses related to the nonsmoothness of solutions should improve hybrid systems towards robustness and certification.

- *A general solver for piecewise smooth systems* This direction is the continuation of the promising result on modeling and the simulation of piecewise smooth systems [40]. As for general hybrid automata, the notion or concept of solutions is not rigorously defined from the mathematical point of view. For piecewise smooth systems, multiplicity of solutions can happen and sliding solutions are common. The objective is to recast general piecewise smooth systems in the framework of differential inclusions with Aizerman–Pyatnitskii extension [40, 59]. This operation provides a precise meaning to the concept of solutions. Starting from this point, the goal is to design and study an efficient numerical solver (time-integration scheme and optimization solver) based on an equivalent formulation as mixed complementarity systems of differential variational inequalities. We are currently discussing the issues in the mathematical analysis. The goal is to prove the convergence of the time-stepping scheme to get an existence theorem. With this work, we should also be able to discuss the general Lyapunov stability of stationary points of piecewise smooth systems.

Numerical optimization for discrete nonsmooth problems

- Second Order Cone Complementarity Problems (SOCCP) for discrete frictional systems (short-term): After some extensive comparisons of existing solvers on a large collection of examples [25, 36], the numerical treatment of constraint redundancy by the proximal point technique and the augmented Lagrangian formulation seems to be a promising path for designing new methods. From the comparison results, it appears that the redundancy of constraints prevents the use of second order methods such as semi-smooth Newton methods or interior point methods. With P. Armand (XLIM, U. de Limoges), we propose to adapt recent advances for regularizing constraints for the quadratic problem [60] for the second-order cone complementarity problem.

- The other question is the improvement of the efficiency of the algorithms by using accelerated schemes for the proximal gradient method that come from large-scale machine learning and image processing problems. Learning from the experience in large-scale machine learning and image processing problems, the accelerated version of the classical gradient algorithm [84] and the proximal point algorithm [43], and many of their further extensions, could be of interest for solving discrete frictional contact problems. Following the visit of Y. Kanno (University of Tokyo) and his preliminary experience on frictionless problems, we will extend its use to frictional contact problem. When we face large-scale problems, the main available solvers is based on a Gauss–Seidel strategy that is intrinsically sequential. Accelerated first-order methods could be a good alternative to benefit from distributed scientific computing architectures.

3.3.2 Automatic Control

This last item is dedicated to the automatic control of nonsmooth dynamical systems, or the nonsmooth control of smooth systems. The first research direction concerns the discrete-time sliding mode control and differentiation. The second research direction concerns multibody systems with unilateral constraint, impacts and set-valued friction. The third research direction concerns a class of dynamics which is an extension of linear complementarity systems (or, equivalently, of differential algebraic equations).

Discrete-time Sliding-Mode Controllers (SMC), State Observers (SMSO) and Differentiators (SMD)

- *SMC with output feedback:* Output feedback can take different forms, like the use of observers/differentiators in the loop (specific dynamic output feedback), or the design of static or dynamic output feedback (without state observation). The time-discretization of such feedback systems and its analysis remains largely open.

- *Unifying algorithm for discrete-time SMC and SMD:* All of sliding mode algorithms are built from interactions of maximal monotone mappings. This shared property of monotonicity allows us to set up a common framework for solving the associated selection problem via proximal algorithms. In the cases when the associated proximal mappings are too convoluted, splitting techniques are going to be developed.

Control of nonsmooth discrete Lagrangian systems

- *Linear Complementarity Systems (LCS):* the PhD thesis of Aya Younes is dedicated to the trajectory tracking control in LCS. In particular the cases with uncertainties and with state jumps are carefully analysed. The PhD thesis of Quang-Hung Pham focuses on networks of LCS. In both cases passivity is a central tool for the analysis.

- *Optimal control:* the optimal control of mechanical systems with unilateral constraints and impacts, largely remains an open issue. Through a collaboration with Moritz Diehl (Freiburg University) the problem has been tackled using a suitable dynamics transformation of Lagrangian complementarity systems into a Filippov "classical" differential inclusion with absolutely continuous solutions. The results are restricted to a single unilateral frictionless constraint. The global objective is to enlarge it to multiple unilateral constraints (hence multiple impacts) and friction.

- *Cable-driven systems:* these systems are typically different from the cable-car systems, and are closer in their mechanical structure to so-called tensegrity structures. The objective is to actuate a system *via* cables supposed in the first instance to be flexible (slack mode) but non-extensible in their longitudinal direction. This gives rise to complementarity conditions, one big difference with usual complementarity Lagrangian systems being that the control actions operate directly in one of the complementary variables (and not in the smooth dynamics as in cable-car systems). Therefore both the cable models and the control properties are expected to differ a lot from what we may use for cableway systems (for which guaranteeing a positive cable tension is usually not an issue, hence avoiding slack modes, but the deformation of the cables due to the nacelles and cables weights, is an important factor). Tethered systems are a close topic.
- *Robot-object underactuated dynamical systems:* such systems are made of a controlled part (called the robot) and an uncontrolled part (called the object). Both are linked by Lagrange multipliers which represent the contact forces. The object can be controlled only through the multipliers, which are in turn a function of the system's state. Examples are bipeds which walk, run, jump, juggling, tapping, pushing robots, prehensile and non-prehensile tasks, some cable-driven systems, and some circuits with nonsmooth set-valued components. A global approach consists in a backstepping-like control strategy. The goal is to derive a unifying framework which can be easily adapted to all these various systems and tasks.

Switching LCS and DAEs

- We have gained a strong experience in the field of complementarity systems and distribution differential inclusions [39, 47], that may be seen as some kind of switching DAEs. More recently we have obtained preliminary results on the analysis of so-called differential-algebraic linear complementarity systems (DALCS) and descriptor-variable LCS (DVLCS), as well as on switching DAEs with state-dependent swithing bilateral constraints. These systems can be seen as DAEs with added complementarity constraints, or as LCS with added equality constraints, or as DAEs with nonsmooth equality constraints. Their well-posedness (existence and uniqueness of solutions to the one-step nonsmooth problem of the implicit Euler scheme, existence and uniqueness of solutions to the continuous-time system) is non-trivial. The case of systems with state-jumps also requires careful analysis.
- A closely related subject is that of interconnections of LCSs or extensions of these (like differential inclusions with maximal monotone properties). The stability of the interconnected system is a topic of interest, as well as, the resulting collective behavior.

Dynamics of complex nonlinear networks, set-valued couplings

- The interconnections of uncertain dynamical systems is a topic of broad interest within the control community. For the case of nonlinear agents with set-valued coupling laws, many questions remain open regarding the resulting behavior of the network, as well as, their robustness properties against parametric uncertainties and external disturbances. The PhD thesis of Quang-Hung Pham focuses on such issues within the context of robust synchronization of LCSs.
- Recently, novel extensions of the concept of passivity have been studied for the analysis of systems away from equilibrium [79]. However, their relevance in the context of networks remains largely unexplored.
- Two-dimensional networks of oscillators with set-valued generalized Coulomb friction laws arise challenging questions regarding their dynamics (nonlinear oscillations, localized waves, excitability), with applications in earthquake dynamics and friction control.
- G. James has recently introduced in collaboration with F. Karbou (Centre d'Etudes de la Neige, Grenoble) a nonsmooth dynamical system on a network suitable for segmenting wet snow areas in SAR (synthetic-aperture radar) satellite images. The network corresponds to a large ensemble of pixels of a grayscale image, whose evolutions are coupled or uncoupled depending on their distance and local topography given by a digital elevation model. This yields an excitable dynamical system that tends to

create domain walls surrounding snowy areas. The system provides very good identification results and arises nontrivial questions regarding its theoretical analysis, optimization (parameters, complexity) and generalizations.

4 Application domains

Nonsmooth dynamical systems arise in many application fields. We briefly highlight here some applications that have been treated in the TRIPOP team, as a validation for the research axes and also in terms of transfer.

In mechanics, the main instances of nonsmooth dynamical systems are multibody systems with Signorini's unilateral contact, set-valued (Coulomb-like) friction and impacts, or in continuum mechanics, ideal plasticity, fracture or damage. Some illustrations are given in Figure 3(a-f). Other instances of nonsmooth dynamical systems can also be found in electrical circuits with ideal components (see Figure 3(g)) and in control theory, mainly with sliding mode control and variable structure systems (see Figure 3(h)). More generally, every time a piecewise, possibly set–valued, model of systems is invoked, we end up with a nonsmooth system. This is the case, for instance, for hybrid systems in nonlinear control or for piecewise linear modeling of gene regulatory networks in mathematical biology (see Figure 3(i)). Another common example of nonsmooth dynamics is also found when the vector field of a dynamical system is defined as a solution of an optimization problem under constraints, or a variational inequality. Examples of this kind are found in optimal control theory, in dynamic Nash equilibrium or in the theory of dynamic flows over networks.

5 Social and environmental responsibility

Regarding our environmental footprint, we have already decided to drastically reduce air travel and limit participation in international conferences, where possible. For instance, trips that can be completed in less than ten hours by train should not be made by plane. When international travel is necessary, conference attendance should be combined with visits to collaborators or other scientific events to maximize the value of each trip. Concerning computing equipment, we do not replace devices before five years, and we aim to keep office machines in service for seven to ten years.

Regarding social impact, the development of research axis 1 on natural gravitational hazards in relation to climate change, together with our work on systemic risk, reflects our intention to address major societal challenges. Industrial collaborations are also evaluated in light of partners' commitments and actions in social and environmental responsibility.

6 Latest software developments, platforms, open data

6.1 Latest software developments

6.1.1 SICONOS

Name: Modeling, simulation and control of nonsmooth dynamical systems

Keywords: NSDS, MEMS, DCDC, SD, Collision, Friction, Mechanical multi-body systems

Scientific Description: Siconos is an open-source scientific software primarily targeted at modeling and simulating nonsmooth dynamical systems in C++ and in Python:

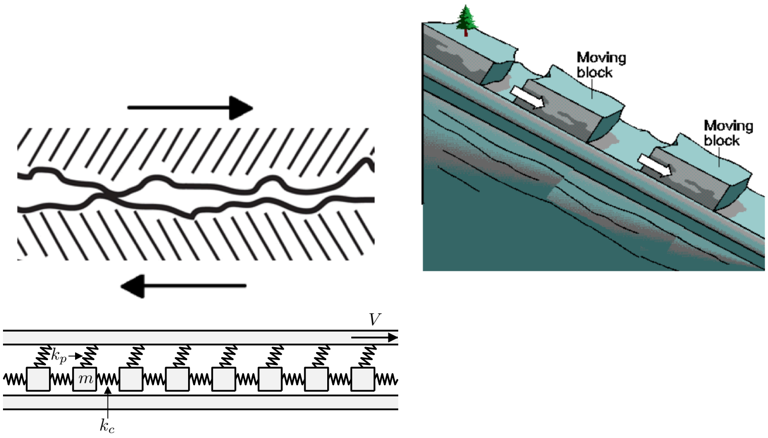
- Mechanical systems (rigid or solid) with unilateral contact and Coulomb friction and impact (nonsmooth mechanics, contact dynamics, multibody systems dynamics or granular materials).

- Switched Electrical Circuit such as electrical circuits with ideal and piecewise linear components: power converter, rectifier, Phase-Locked Loop (PLL) or Analog-to-Digital converter.

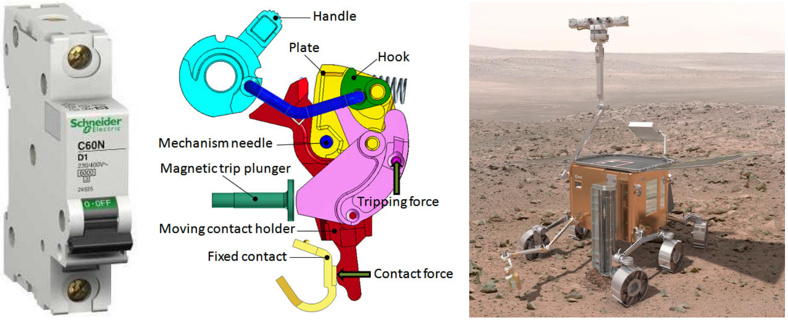
- Sliding mode control systems.

- Biology (Gene regulatory network).

(a) Rockfall [46, 45, 57], granular and debris flows

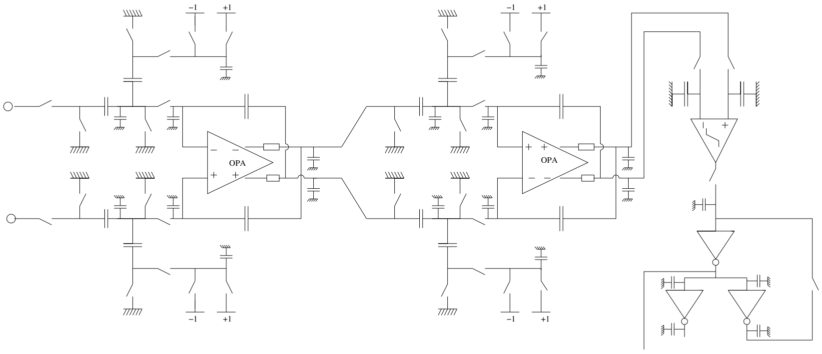


(b) Frictional interface and solitary waves in the Burridge-Knopoff model [81]

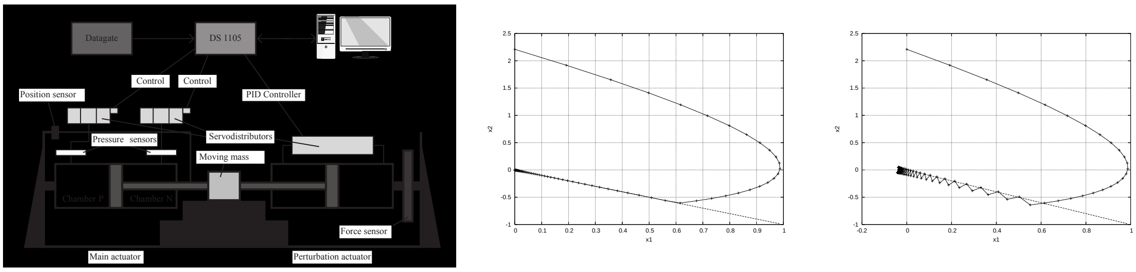


(c) Circuit breakers mechanisms [41] and Robots (ESA ExoMars Rover [24])

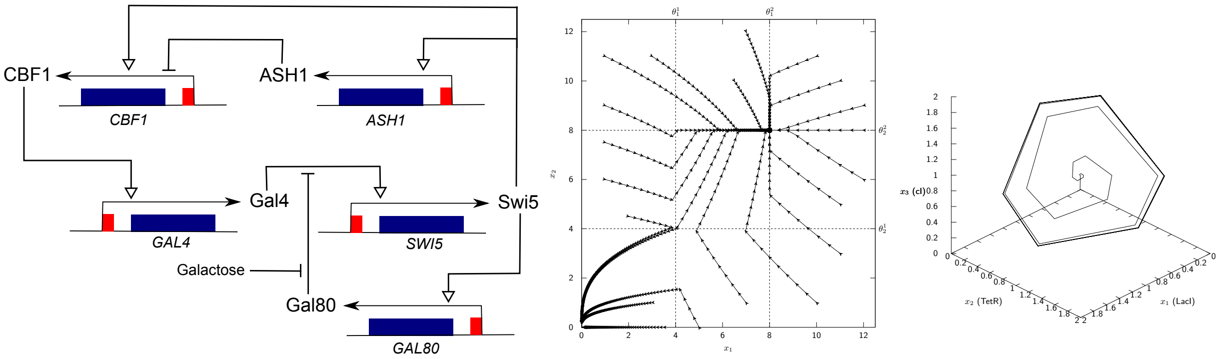
Figure 3: Application fields of nonsmooth dynamics (mechanics)



(d) Switched electrical circuits (delta-sigma converter) [34]



(e) Sliding mode control [38, 27, 70, 69, 80]



(f) Gene regulatory networks [40]

Figure 3: Application fields of nonsmooth dynamics (continued)

Other applications are found in Systems and Control (hybrid systems, differential inclusions, optimal control with state constraints), Optimization (Complementarity systems and Variational inequalities), Fluid Mechanics, and Computer Graphics.

Functional Description: Read more about Siconos at the [Siconos homepage] http://siconos.org

Release Contributions: see of release on https://github.com/siconos/siconos/releases

URL: `http://siconos.org`

Publication: hal-04056972v1

Contact: Vincent Acary

Participants: Franck Pérignon, Maurice Bremond, Vincent Acary

7 New results

7.1 Numerical Modeling for natural risk in mountains

7.1.1 Energy conservation and dissipation properties for elastodynamics with contact impact and friction

Participants: Vincent Acary, Nicholas Collins-Craft.

It has long been known that the standard implementation of impact and Coulomb friction leads to the creation of energy in cases where the sliding direction changes over the impact. The paper [29] proposes a time integration scheme for nonsmooth mechanical systems involving unilateral contact, impact and Coulomb friction, that respects the principles of discrete-time energy balance with positive dissipation. To obtain energetic consistency in the continuous time model, we work with an impact law inspired by the work of M. Frémond, which ensures that dissipation is positive, *i.e.* that the Clausius–Duhem inequality is satisfied. On this basis, we propose in [3] a time integration method based on the Moreau–Jean scheme with a discrete version of the Frémond impact law, and show that this method has the correct dissipation properties, *i.e.* no energy is created.

7.1.2 Numerical modeling of fracture in solids

Participants: Vincent Acary, Franck Bourrier, Nicholas Collins-Craft.

In [54], a new extrinsic cohesive model is developed together with a consistent time–stepping scheme to simulate fracture in quasi-brittle material like rock or concrete. An extrinsic cohesive zone model with a novel unload-reload behaviour is developed in the framework of non-smooth mechanics. The model is extended to include the effects of dynamics with impact, and is discretised in such a way that it can be written as a Linear Complementarity Problem (LCP). This LCP is proved to be well-posed, and to respect the discrete energy balance of the system. Finally, the LCP system is validated numerically, in both statics and dynamics, by simple test cases, and more involved finite element simulations that correspond to standard test geometries in the literature. The results correspond well with those of other authors, while also demonstrating the simulations' ability to resolve with relatively large time steps while respecting the energetic balance. We are now working on the development of a model taking into account the tangential cohesion coupled with the Coulomb friction [53]. The objective is to propose a model coupled with hydro-thermal freezing and thawing phenomena in rock interfaces, which will be used to simulate the stability of cliffs in connection with the thawing of permafrost. This is still on-going work.

7.1.3 Variational approach for nonsmooth elasto-plastic dynamics with contact and impacts

Participants: Louis Guillet, Vincent Acary, Franck Bourrier, Olivier Goury.

The objective of this work [35] was the modelling and the numerical simulation of the response of elastoplastic structures to impacts. To this end, a numerical method is proposed that takes into account one-sided contacts (Signorini condition) and impact phenomena together with plasticity in a monolithic solver, while accounting for the non-smooth character of the dynamics. The formulation of the plasticity and the contact laws are based on inclusions into normal cones of convex sets, or equivalently, variational inequalities following the pioneering work of Moreau (1974) and Halphen and Nguyen (1975), who introduced the assumptions of normal dissipation and of generalised standard materials (GSM) in the framework of associated plasticity with strain hardening. The proposed time-stepping method is an extension of the Jean and Moreau (1987) scheme for nonsmooth dynamics. The discrete energy balance shows that spurious numerical damping can be suppressed and that the scheme is in practice unconditionally stable. Furthermore, the finite-dimensional variational inequality at each time-step is well-posed and can be solved by optimization methods for convex quadratic programs, providing an interesting alternative to the return mapping algorithm coupled with a dedicated frictional contact method.

A contribution [63, 21] presents an implicit solver for non-associative plasticity problems based on the semi-smooth Newton method in the context of the material point method. The method is derived from the Implicit Standard Material and is easily compatible with various space discretization techniques, particularly the Material Point Method and the Finite Element Method. The solver converges quadratically, even for large time steps, although we have only demonstrated theoretical results for restricted cases. The method is demonstrated through a footing simulation.

7.1.4 Application of nonsmooth models of rockfall protection structures for the quantification of energy dissipation, warning and survey

Participants: Vincent Acary, Franck Bourrier, Ritesh Gupta.

This work is based on the nonsmooth model developed in [64] which simulates the response of a novel rockfall protection structure, made of piled-up concrete blocks interconnected via metallic components, subjected to impact of rock blocks.

This model was first used to investigate the key issue of energy dissipation in passive rockfall protection structures when exposed to impact by a rock block [7, 65]. Based on the work done in [3], energy dissipation due to friction between the system bodies and to plastic strain at contacts were quantified. The evolution with time of energy dissipation by each dissipative mechanism provides insights into the global structure response with time in terms of displacement and contact force amplitude. The influence of the model parameters on the contribution of these two dissipative mechanisms is evaluated. The variability in energy dissipation varying the impact conditions is addressed. In the end, this study reveals the benefits derived from a precise quantification of energy dissipation in passive rockfall protection structures.

As the viability of a rockfall protection structure is vital for hazard mitigation [75], the nonsmooth model developed in [64] was also used to assess the potential of inverse analysis applied to data collected on on-site rockfall protection structures exposed to real events. Extensive simulations allowed to investigate the variability in wall mechanical response against different impact conditions. The simulation results served as input data for developing the inverse analysis method. As a first application, it is proposed to use the inverse analysis to aid in remote decision-making shortly after an event, based on real-time measurements. Then, the use of inverse analysis to retrieve the impact condition characteristics (energy, location) from data collected after the event is addressed. The proposed approach appeared efficient for back-analyzing (i.e., output to input) data related to the wall response for being used as a warning based on its displacement and damage to the wall.

The report [22] is a part of the project AEx-GRANIER (Action Exploratoire - GRAvitatioNal hazards in mountaIns in the contExt of Risks prediction — Nonsmooth modeling and simulation with data in

Geomechanics). The Non-smooth contact dymanics (NSCD) method based numerical model, implemented in Siconos software package, is presented for the simulation of gravitational natural hazards in the mountain environments. The fundamental idea is to narrow down the existing gap between the real event data and the numerical models towards hazard prevention and risk prediction. In this work, the NSCD model simulations are combined with the so-called 'data-driven modelling' techniques to access the descriptive statistics of the phenomenon. The characteristics of the rockfall process in a given terrain are highlighted by statistically presenting a correlation and quantification of the uncertainty between input and output parameters.

7.2 Systemic risk

Participants: Antoine Cordoba, Arnaud Tonnelier, Vincent Acary.

The HANDY model, proposed by Motesharrei *et al.* (2014), describes the nonlinear interactions between human society and natural resources. The system can be viewed as a predator–prey model augmented with a variable representing accumulated wealth. In previous work, we analysed the model's dynamics with particular attention to bifurcations with respect to two parameters: the depletion factor and the inequality factor. We have recently extended this study in two directions. First, we consider the coupling of two HANDY-type models and investigate the impact of population mobility on the dynamics of the coupled system. Second, we introduce a general class of HANDY-type models and identify their generic properties (Hopf bifurcations, and cycles of prosperity and collapse).

The HANDY model provides a highly simplified description of human–nature interactions. A more realistic, but more complex, framework is the World3 model, proposed in the context of the *Club of Rome*. This model describes the interactions between five sectors: world population, non-renewable natural resources, the industrial sector, the agricultural sector, and pollution. We have obtained analytical results for the dynamics of the resources–capital subsystem. However, classical dynamical-systems approaches are not sufficient to analyse the behaviour of the full system due to its complexity, and more original methods are required. To this end, we used and further developed loop dominance analysis to study the dynamics of the World3 model, with a particular focus on loop eigenvalue elasticity analysis (LEEA) methods. This work is ongoing; preliminary results have been obtained but are not yet published.

7.3 Numerical solvers for frictional contact problems.

Participants: Vincent Acary, Maurice Brémond, Paul Armand.

In [37], we review several formulations of the discrete frictional contact problem that arises in space and time discretized mechanical systems with unilateral contact and three-dimensional Coulomb's friction. Most of these formulations are well–known concepts in the optimization community, or more generally, in the mathematical programming community. To cite a few, the discrete frictional contact problem can be formulated as variational inequalities, generalized or semi–smooth equations, second–order cone complementarity problems, or as optimization problems such as quadratic programming problems over second-order cones. Thanks to these multiple formulations, various numerical methods emerge naturally for solving the problem. We review the main numerical techniques that are well-known in the literature and we also propose new applications of methods such as the fixed point and extra-gradient methods with self-adaptive step rules for variational inequalities or the proximal point algorithm for generalized equations. All these numerical techniques are compared over a large set of test examples using performance profiles. One of the main conclusion is that there is no universal solver. Nevertheless, we are able to give some hints to choose a solver with respect to the main characteristics of the set of tests.

Recently, new developments have been carried out on applications of well-known numerical methods in optimization. With the visit of Paul Armand, Université de Limoges, we co-supervise a M2 internship, Maksym Shpakovych on the application of interior point methods for quadratic problem with second-order cone constraints. The results are encouraging [93], [85], [86]. A first publication on rolling friction has been published [33] and another publication [32] in Optimization Methods and Software. Following the defense of the PhD these of Hoang Minh Nguyen [18], a new article is in preparation on the asymptotic numerical methods for nonconvex interior point method.

7.4 Modeling of Impact Phenomena

Participants: Bernard Brogliato.

Multiple impacts (i.e., impacts that occur simultaneously in a mechanical system, not to be confused with finite accumulations of impacts—also known as Zeno phenomena) exhibit specific features that are not encountered in single-impact events (for instance, non-uniqueness of the output for a given energetic behaviour, or discontinuities with respect to initial data). As such, their modelling is a non-trivial task that requires dedicated analysis. We provide a survey of multiple-impact models in [17] (see also Chapter 6 of [2] for further developments and additional information), where the various modelling approaches are classified into three main classes and their distinctive features are discussed in detail.

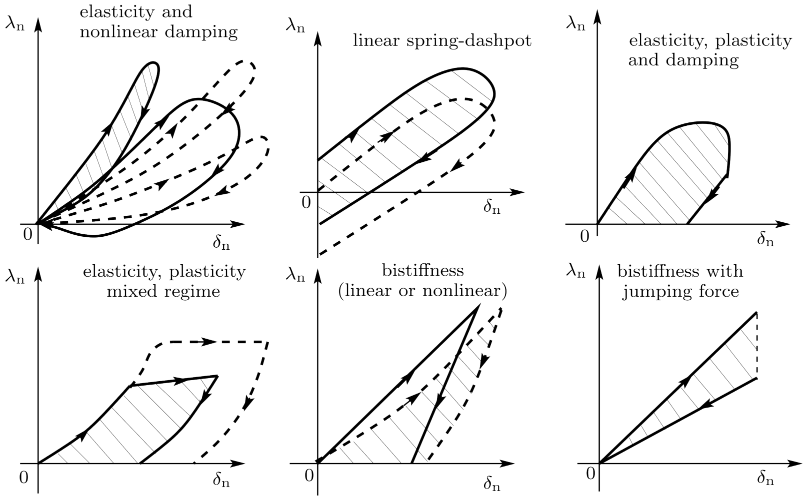


Figure 4: Classification of force–indentation curves and corresponding models (dashed areas represent dissipated energies), [17].

7.5 Analysis and Control of Set-Valued Systems

Participants: Bernard Brogliato, Félix Miranda-Villatoro, Aya Younes, Quang Hung Pham, Van Nam Vo.

Discrete-time implicit Euler sliding-mode control In [12], it is shown that some discrete-time algorithms (such as those referred to as minimum-operator algorithms), which are finite-time stable and written in explicit form, can in fact be interpreted as the implicit (backward) Euler discretisation of appropriate set-valued systems. Equivalently, they can be cast as proximal-point algorithms involving resolvents of maximal monotone operators. This viewpoint paves the way for a unified framework for computing sliding-mode controllers and differentiators based on implicit discretisations.

The work [11] offers a fresh perspective on sliding-mode techniques for control, observation, and differentiation through their discrete-time implementation via backward Euler schemes. In this setting, maximal monotonicity of the backward terms plays a central role in ensuring closed-loop well-posedness and enabling stability analysis. This approach also highlights strong links between sliding-mode control and optimisation through proximal-point iterations. The manuscript further underscores the significance of passivity in the resulting closed-loop system, an aspect that has received comparatively little attention in the sliding-mode community. As a by-product, a novel robust variant of the proximal-point algorithm is introduced and used to study finite-time stability for conventional (first-order) sliding-mode control. The analysis combines tools from optimisation and maximal monotone operator theory, and shows how set-valued control maps and suitable selection schemes can yield control strategies that avoid the common issue of numerical chattering.

In [10] the discrete analysis of multi-variable supertwisting-like algorithms is presented. The approach departs from the emulation of the continuous-time design, yielding improved precision and robustness. In addition, an dynamic splitting approach for the computation of the selection values of the controller and its convergence properties are studied as well as its performance in numerical simulations.

Well-posedness of Time-Varying Linear Complementarity Systems Linear Complementarity Systems (LCS), as in (4), form a well-known class of nonsmooth, nonlinear dynamical systems. Their analysis becomes significantly more challenging when the system data are time-dependent. In [14, 19], we study several settings: the case where $D(t)$ is positive definite, the case $D(t) = 0$, and the case where the quadruple $\big(A(t), B(t), C(t), D(t)\big)$ is passive (in the sense of Willems), so that $D(t)$ is positive semidefinite and possibly nonsymmetric. Depending on the assumptions, solutions are shown to be either absolutely continuous or right-continuous with locally bounded variation. The analysis builds on earlier results by Camlibel *et al.* for time-varying differential inclusions and by Brogliato–Thibault for time-varying LCS studied via first-order sweeping processes, extending these frameworks in a non-trivial manner. Applications include electrical circuits with set-valued or piecewise-linear components, and with time-varying resistances, capacitances, or inductances.

Multivalued Hamiltonian Systems in Discrete-Time In [6], we analyze the stability properties of a class of discrete-time Hamiltonian systems in which both the lossless part and the dissipation function are allowed to be multivalued. We study in particular the backward Euler discretisation and show how its implicit structure supports a rigorous stability analysis in this nonsmooth setting. Examples include feedback control algorithms (twisting and super-twisting), mechanical systems, and optimisation algorithms.

Passive Linear Complementarity Systems interconnections The generic interconnection of two passive Linear Complementarity Systems (LCS) is analyzed in [5]. The main difficulty lies in the fact that the interconnection variables are not, in general, the passivity input–output pairs, in contrast with the classical passivity theorem. Several cases are examined in detail (interconnections of passive, strictly state-passive, strongly passive LCS), relying on a careful analysis of the corresponding passivity linear matrix inequalities (LMIs). Most importantly, state jumps induced by the interconnection are characterised: interconnecting two systems may generate state jumps (even when each subsystem admits absolutely continuous solutions for any initial condition) or, conversely, suppress jumps that would otherwise occur.

Tracking control in frictional oscillators In [15] the tracking control of frictional oscillators, which can be recast into LCS, is considered. A frictional oscillator, as usually considered in the Nonlinear Dynamics literature, consists of a mass sliding/sticking on a moving belt. A blanket assumption is that the belt's velocity can be used as an input. Robustness with respect to uncertain friction coefficients is carefully analyszd. The case of Stribeck friction (which yields a set-valued hypomonotone operator) is studied.

Heterogeneous Networks Synchronization In [23] we study the robust synchronization of heterogeneous networks, using feedback controllers built from complementarity conditions. This yields closed-loop systems which are Linear Complementarity Systems. The advantage is good robustness properties together with finite-time synchronization. The discrete-time implementation is analyzed carefully, where backward Euler algorithms are shown to preserve the major properties of the continuous-time scheme.

7.6 Differential Algebraic Linear Complementarity Systems (DALCS)

Participants: Bernard Brogliato.

DALCS are an extension of DAE and of LCS, where both equality and complementarity constraints are present. As such they involve both algebraic and differential states (familiar to DAE specialists). In [20] we analyse state jumps in DALCS. It happens that algebraic states can undergo discontinuities while differential states are continuous, creating a kind of impulse-free state jumps. Lur'e operators and maximal monotonicity are the main tools to analyse the variational inequalities from which the jumps are characterized.

7.7 Additional new results

From constitutive modelling to the physics of friction in fault gouges

Participants: Filippo Masi, Itai Einav (The University of Sydney).

The development of rate- and state-dependent friction laws offered important insights into the key physical mechanisms of the frictional behavior of fault gouges and their seismic cycle. However, past approaches were specifically tailored to address the problem of fault shearing, leaving questions about their ability to comprehensively represent the gouge material under general loading conditions. The work presented in [8] establishes an alternative approach for developing a physical friction law for fault gouges that is grounded on the rigour of the hydrodynamic procedure with two-scale temperatures through Terracotta, a thoroughly robust constitutive model for clay in triaxial loading conditions. By specifying the model for direct shearing, the approach yields an alternative friction law that readily captures the frictional dynamics of fault gouges, including explicit dependencies on gouge layer thickness, normal stress, and solid fraction. Validated against available laboratory experiments, the friction law retains the original predictive capabilities of Terracotta in triaxial conditions and explains the rate-and-state, dilatational behavior of fault gouges in direct shear conditions. Finally, when the Terracotta friction law is connected to a spring-dashpot representation of the host rock, the combined model predicts an elastic buildup precursor to the onset of and subsequent seismicity, with results closely reflecting experimental evidence and field observations.

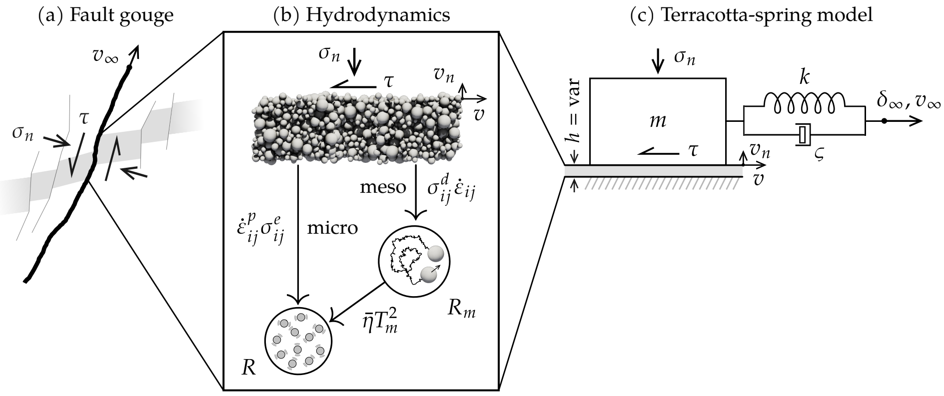


Figure 5: Hydrodynamics of fault gouges: from constitutive to friction law [8]. A fault gouge is modelled as a shearing layer of thickness h under overburden stress σ_n, where the slip and normal rates control the evolution of shear stress τ and the energy is stored elastically and dissipated through coupled thermal micro- and meso-scopic mechanisms. Coupling this gouge model to a spring–dashpot representation of the host rock enables simulations of earthquake dynamics in clay-rich faults.

Towards Real-Time Simulation of Soft Robots with Contacts using a Method of Hybrid Hyper-Reduction

Participants: Olivier Goury, Samuel M Youssef (Istituto Italiano di Tecnologia), Simon Le Berre (CEA), Christian Duriez (DEFROST Inria).

Soft robotics has emerged as an important part of robotics in recent years. Soft robots have an inherent view of contacts that is dramatically different from traditional rigid robots. Indeed, for rigid robots, contacts are either forbidden to avoid damage to the robot, the environment and humans, or precisely controlled for locomotion or interaction with an object. For soft robots, contacts may happen without damage, and when interacting with an object, local deformations allows for smoother interactions and potentially better performance. These prospects make soft robots attractive for tasks such as grasping. Fast finite element simulation is very useful for control and design. However, simulating collision adds a major numerical cost as it requires first a collision detection algorithm to detect collisions, and most importantly, it requires solving a constrained problem to avoid inter-penetrations and compute contact forces. When the number of contact points is large, this computation slows down the simulation dramatically. In the contribution [16], we apply a hybrid hyper-reduction method to alleviate the FEM cost, the collision detection as well as the contact response computation. The deformations are computed in a low-dimensional subspace computed from offline experiments. The mechanical matrices are reduced through a method of hyper-reduction and the collision model is reduced following a hybrid reduction strategy. We show good agreement between original and reduced simulation while speeding up dramatically the computation. We first apply the method in simulation on a soft bouncing ball to explain the method. We then show an example with a soft gripper. The method is generic and can be used for control, design or learning algorithms.

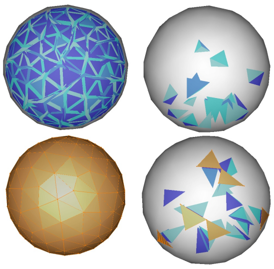


Figure 6: Different models of a soft ball [16]. Top left and right: full order model and sampled model. Bottom left and right: full collision model and reduced collision model, the ball is turned for a better view.

Modeling, Embedded Control, and Design of Soft Robots Using a Learned Condensed FEM Model

Participants: Tanguy Navez (DEFROST Inria), Etienne Ménager (DEFROST Inria), Paul Chaillou (DEFROST Inria), Olivier Goury, Alexandre Kruszewski (DEFROST Inria), Christian Duriez (DEFROST Inria).

The finite element method (FEM) is a powerful modeling tool for predicting soft robots' behavior, but its computation time can limit practical applications. In the contribution [13], a learning-based approach based on condensation of the FEM model is detailed. The proposed method handles several kinds of actuators and contacts with the environment. We demonstrate that this compact model can be learned as a unified model across several designs and remains very efficient in terms of modeling since we can deduce the direct and inverse kinematics of the robot. The learned model is presented as a general framework for modeling, controlling, and designing soft manipulators. First, the method's adaptability and versatility are illustrated through optimization-based control problems involving positioning and manipulation tasks with mechanical contact-based coupling. Second, the low-memory consumption and the high prediction speed of the learned condensed model are leveraged for real-time embedding control without relying on costly online FEM simulation. Finally, the ability of the learned condensed FEM model to capture soft robot design variations and its differentiability are leveraged in calibration and design optimization applications.

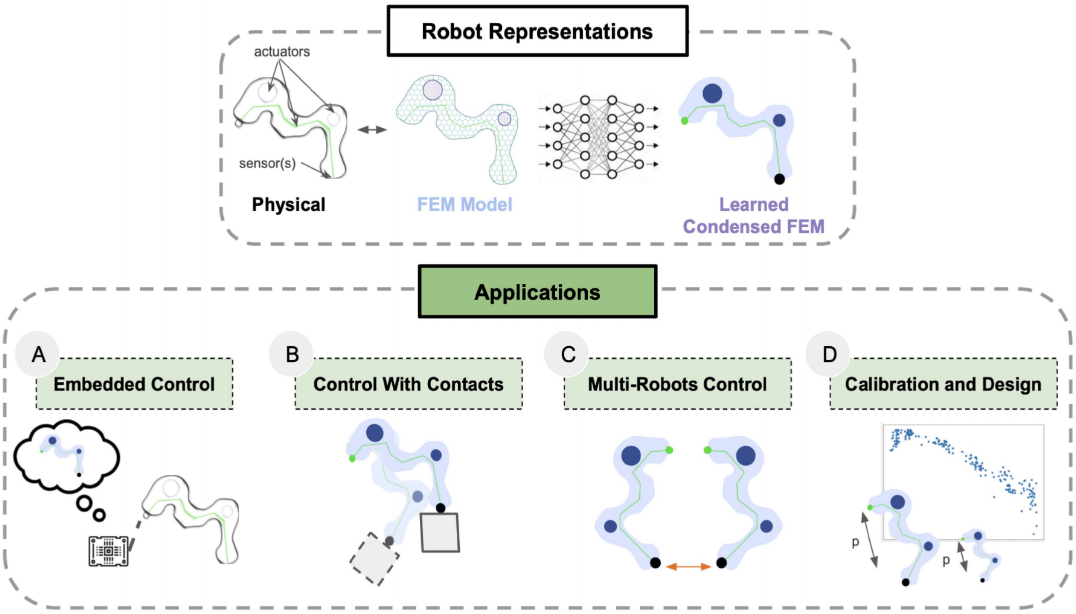


Figure 7: Illustration of the proposed framework and its applications [13]. The FEM model of a robot is projected in the constraint space, and the corresponding matrices are learned using a neural network. The learned matrices can be used in different applications like (a) real-time embedded control, (b) inverse control involving predefined contact points, (c) control of multiple identical robots from a single learned model, and (d) both design optimization and calibration applications.

Reduced-scale laboratory testing of structures and scaling laws for blasts from exploding wires

Participants: Filippo Masi, Ahmad Morsel (École Centrale Nantes), Panagiotis Kotronis (École Centrale Nantes), Ioannis Stefanou (École Centrale Nantes).

In [83, 82], we introduce and validate a reduced-scale experimental facility for laboratory testing of structures subjected to blast loading (miniBLAST) based on the electrical discharge of thin metallic wires. The setup enables safe, systematic and repeatable generation of blast-type shock waves with controlled intensity. The investigastions show that the blast parameters generated by exploding wires follow self-similar scaling with respect to the Hopkinson–Cranz scaled distance, in close analogy with conventional high explosives. This provides a physically consistent framework to interpret reduced-scale tests and to connect them to engineering practice through TNT-equivalence factors. These results support the use of exploding wires as a robust and cost-effective alternative for parametric blast loading campaigns, model validation, and reduced-scale structural dynamics experiments.

Multi-physics modeling for ion homeostasis in multi-compartment plant cells using an energy function

Participants: Guillaume Mestdagh, Alexis de Angeli (IPSiM, Univ. Montpellier), Christophe Godin (Laboratoire Reproduction et Développement des Plantes, Univ. Lyon).

Plant cells control their volume by regulating the osmotic potential of their cytoplasm and vacuole. Water is attracted into the cell as the result of a cascade of solute exchanges between the cell subcompartments and the cell surroundings, which are governed by chemical, electrostatic and mechanical forces. Due to this multi-physics aspect and to couplings between volume changes and chemical effects, modeling these exchanges remains a challenge that has only been partially addressed. As interest for multi-compartment

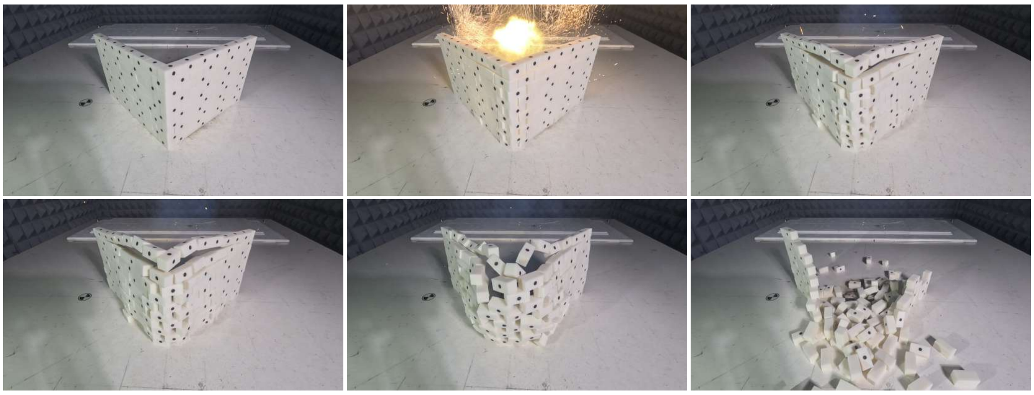


Figure 8: Time evolution of the response of a masonry wall subjected to blast loads from the discharge of 5 kJ [83].

models grows in the plant cell community, this challenge calls for new modeling strategies. In [9], we introduce an energy-based approach to couple chemical, electrical and mechanical processes taking place between several subcompartments of a plant cell. The contributions of all physical effects are gathered in an energy function, which allows us to derive the equations satisfied by each variable in a systematic way. We illustrate the properties of this modular, unified approach on the modeling of ion and water transport in a guard cell during stoma opening. We represent the stoma opening process as a quasi-static evolution driven by hydrogen pumps in the plasma and vacuolar membranes, and we show that the new formalism explains why the system varies in a particular direction in response to perturbations. Additional numerical simulations allow us to investigate the role of each hydrogen pump in this process. Altogether, we show that this energy-based approach highlights a hierarchy between the forces involved in the system, and to dissect the role of each physical effect in the complex behavior of the system.

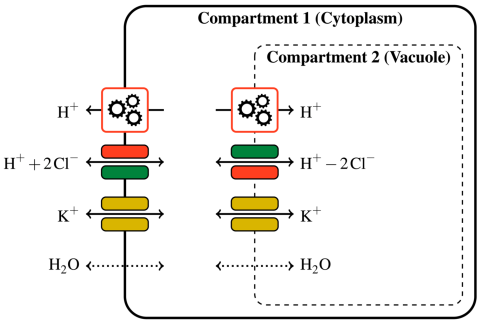


Figure 9: Representation of the multi-physics model involving two membranes and a hand-picked selection of transporters (image adapted from Issi at openclipart.org).

8 Bilateral contracts and grants with industry

Participants: Vincent Acary, Bernard Brogliato.

8.1 Bilateral grants with industry

Schneider Electric. The longest and most established partnership is with Schneider Electric, which has been ongoing since 2001. This collaboration initially began with a post-doctoral position co-funded by

Schneider Electric and CNRS and has continued into the present. Over the years, it has covered the simulation and modeling of multibody systems involving contact, friction, and impacts, with a direct application to the virtual prototyping of electrical circuit breakers. Schneider Electric has funded two PhD theses and engaged in several research contracts with INRIA, and it has also participated as a major partner in the ANR Saladyn project. The collaboration evolved from interactions with the R&D innovation department to working directly with the business unit responsible for designing circuit breakers, and it now includes new challenges such as modeling flexible parts and managing multiple impact laws. Additional work with Schneider Electric also took place in the project ANR-10-AIRT-05 of IRT NanoElec Pulse, which focused on modeling and controlling overhead cranes.

- 2025: master internship (PFE Ensimag/Schneider Electric) – Rémy Roubinet, co-supervised by B. Brogliato with E. Frangin (Schneider Electric): impact mechanics, SICONOS simulations, and vibration tests on Schneider Electric's shaking table.

Service Technique des remontées mécaniques et des transports guidés (STRMTG). A long-term partnership with STRMTG supports a research contract on the modelling, simulation and control of cable-transport systems. The work focuses on cable dynamics and interactions with supports under unilateral contact and friction, with the objective of delivering an open-source simulation tool to support operations and certification.

- 2025: postdoctoral researcher – Guillaume Mestdagh (with V. Acary).

Safran Tech. In 2024, TRIPOP initiated a collaboration with Safran Tech through the recruitment of a postdoctoral researcher (S. Le Berre). This project targets the modelling and simulation of flexible multibody systems with impacts and frictional contact, extending the team's industrial partnerships to the aerospace sector.

NGE Fondations. Since 2018, the team has maintained regular exchanges with NGE Fondations, a European leader in the design of protection solutions against gravity-driven natural hazards. These exchanges were formalized in 2024 through a collaboration within the FEREC project PROTECMO, in which TRIPOP initiated the development of numerical models for temporary rockfall protection solutions.

Natural risk management. The team works with several socio-economic actors through the OCIRN project (and SMART-PROTECT project, previously) to set up an industrialization of our simulation and calculation techniques in an operational context. For this, the creation of a consortium around Platrock and Siconos is under study. The industrialization part (marketing, maintenance, support) is ensured by the company HALIAS technologies.

9 Partnerships and cooperations

9.1 National initiatives

ANR SlimDisc

Participants: Felix Miranda-Villatoro, Bernard Brogliato.

SlimDisc (Sliding-Mode set-valued control and observation in finite and infinite dimensions: Discretization) is a project funded by The French National Research Agency (ANR) for the period October 2024–September 2026. It is a collaborative project between Ecole Central de Nantes, Inria Lille, and Inria Grenoble, SlimDisc. The start of the project is October 2024 for a duration of 4 years. This project follows two previous ANR projects on the same topic, with same partners (ChaSlim and DigitSlid). The main goals of SlimDisc are the analysis and the experimental validation of set-valued sliding-mode controllers and state observers/differentiators, in discrete time, as well as the design of a toolbox dedicated to the computation and implementation of discrete-time controllers and differentiators. This will be tackled using mainly the Euler implicit and semi-implicit discretization methods. The key feature of this project is the development of discretization methods for both finite- and infinite-dimensional control systems.

ANR SPECULAR

Participants: Olivier Goury.

SPECULAR (Simulation of Percutaneous Liver tumor Ablation in virtual Reality) is a project funded by The French National Research Agency (ANR) for the period January 2022–December 2025. The goal of the project is to develop an immersive simulation of needle-based procedures. It is a collaborative project between Inria Lille, Strasbourg University, Inria Nancy and the company InfinyTech3D. Olivier Goury is responsible of Work Package 2 in collaboration with DEFROST at Inria Lille where the focus is onto speed up the numerical simulation using reduced-order modeling techniques and parallel programming. This project is coordinated by Stéphane Cotin at Inria Nancy and Hadrien Courtecuisse at Strasbourg University.

PEPR Risques (Ex-IRIMA)

Participants: Vincent Acary, Franck Bourrier, Olivier Goury.

The IRiMa PEPR (integrated risk management for more resilient societies in an era of global change) is co-piloted by BRGM, CNRS and Grenoble-Alpes University. This exploratory PEPR, with a budget of €51.9 million over 8 years, brings together more than 30 partner institutions and laboratories. Within this PEPR, we are actively involved in the "Mountain" targeted project (PC) with the ANR IRIMONT funding application entitled "Assessment and mitigation of risks related to natural hazards in mountain territories in the global change context". The IRIMONT project looks at all the physical and social dimensions of natural hazards in mountain areas, from the characterisation of processes to decision-making and adaptation in a context of climate change and socio-environmental dynamics. The project is structured in 3 work packges. Guillaume Chambon (INRAE/IGE), Marc Peruzetto (BRGM) and Vincent Acary are responsible for WP1 - Analysis and understanding of mountain risks and their components. This work package targets the gaps in knowledge and the scientific barriers concerning mountain risks and their components (hazards, vulnerability, exposure). To this end, it includes the acquisition and analysis of new data (instrumental, historical, etc.) and the development of new predictive models (mechanical, stochastic, decisional). This work package thus constitutes the "toolbox" of the IRIMONT project. However, as in the IRIMONT project as a whole, we are favoring an approach in which the questions are linked to the mountain terrain and its specific features, rather than a more disciplinary/methodological approach, and we are focusing on developments that can be best integrated with the expectations of the other work packages.

PEPR MAth Vives - Mathematics for Life, Environment and Society – Complexflows

Participants: Vincent Acary, Franck Bourrier.

PEPR Maths-Vives is a national programme, funded as part of the France 2030 plan, which is investing 50 MEUR over ten years to promote dialogue between mathematics and other disciplines. Its aim is to respond to major contemporary challenges — life, the environment, society — through modeling, simulation and mathematical analysis of complex phenomena. The programme is organised around three thematic areas: ‘Life’ (biology, health, ecology, etc.), ‘Environment’ (climate, biodiversity, energy, etc.), and ‘Society’ (mobility, urban planning, social dynamics, economics, etc.). The ComplexFlows project (part of the PEPR Maths-Vives programme) aims to improve our understanding of complex natural free-surface flows — such as landslides, mudslides, debris avalanches, rockfalls, coastal erosion, etc. It uses multidisciplinary approaches combining mathematics and physics to model these phenomena, which involve solid particles densely suspended in one or more fluids. The project focuses on four main areas: macroscopic modeling based on interactions at different scales, consideration of free interfaces and variable bottoms, the role of elastic or acoustic waves in flow, and modeling of mixtures with exchanges between phases. The fundamental objective is to understand why natural landslides can be extremely mobile, to remove the barriers related to the rheology (mechanical behaviour) of granular and fluid media, and to develop theoretical or numerical tools that can help predict the dynamics, extent or speed of potentially dangerous events. The project is funded to the tune of approximately 1 MEUR over five years and coordinated by teams from the University of Savoie Mont Blanc (mathematics) and the CNRS (physics)/University of Montpellier. A half PhD thesis has been granted for the team within this project.

MIAI Chair: Artificial Intelligence and Mechanics (AIM)

Participants: Vincent Acary, Filippo Masi, Franck Bourrier.

The chair AIM (Artificial Intelligence and Mechanics for scale-bridging in complex materials) is funded by the MIAI Cluster and ANR through the France 2030 programme (Grant agreement ANR-23-IACL-0006), for the period June 2025 – September 2029. Principal investigators: V. Acary and F. Masi. The research is carried out jointly by four teams: the INRIA Grenoble TRIPOP team (V. Acary and F. Masi), the THOTH team (M. Arbel), the Geomechanics group at 3SR-UGA (G. Viggiani), and the ECRINS team at INRAE-IGE (F. Bourrier, T. Faug). AIM's interdisciplinary methodology bridges applied mathematics, mechanics, and artificial intelligence to better understand, model, and predict the mechanics and dynamics of granular media. By combining high-fidelity particle-scale simulations, cutting-edge in-operando experiments, and developing AI methods constrained by the fundamental principles from statistical physics and non-equilibrium thermodynamics, AIM will deliver a proof of principle to robustly and accurately predict the fine- and large-scale behaviour of granular systems. The project will also establish an innovation consortium dedicated to open-source software and build an interdisciplinary training program at the intersection of AI and mechanics, equipping the next generation of scientists and engineers. For more details, refer to the website.

10 Dissemination

Participants: Vincent Acary, Franck Bourrier, Bernard Brogliato, Olivier Goury, Filippo Masi, Arnaud Tonnelier, Felix Miranda Villatoro, Franck Pérignon, Antoine Cordoba, Guillaume Mestdagh, Louis Guillet, Florian Vincent.

10.1 Promoting scientific activities

10.1.1 Journal

Member of the editorial boards

- V. Acary is editor and co-founder of the Journal of Theoretical, Computational and Applied Mechanics (JTCAM).
- F. Bourrier is a member of the Editorial Board of the Journal Landslides.
- B. Brogliato was Guest Managing Editor of a special issue of Nonlinear Analysis Hybrid Systems: Nonsmooth Dynamical Systems: Analysis, Control and Optimization, in the framework of the European Network on NonSmooth Dynamics.

Reviewer - reviewing activities

- V. Acary. Invited reviewer for: Computational Mechanics, Computer Methods in Applied Mechanics and Enginering, Mechanics of materials.
- F. Bourrier. Invited reviewer for: Computers and Geotechnics, Geomorphology, Engineering Geology, Granular Matter.

- B. Brogliato. Invited reviewer for: IEEE Transactions on Automatic Control, Automatica, SIAM Journal on Control and Optimization, IFAC World Congress, IEEE Conference on Decision and Control, Mechatronics, International Journal of Nonlinear and Robust Systems, IEEE Control Letters, Control Engineering Practice, European Journal of Control.
- O. Goury. Invited reviewer for: Computer Graphics Forum, Eurographics, IEEE Control Systems Letters, IEEE International Conference on Robotics & Automation (ICRA), IEEE/RSJ International Conference on Intelligent Robots and Systems, IEEE Robotics and Automation Letters, IEEE-RAS International Conference on Soft Robotics.
- F. Masi. Invited reviewer for: Computational Mechanics – Computer Methods in Applied Mechanics and Enginering – Computer and Geotechnics – Géotechnique – International Journal for Numerical and Analytical Methods in Geomechanics – International Journal for Numerical Methods in Engineering – Journal of the Mechanics and Physics of Solids – Reliability Engineering & System Safety.
- A. Tonnelier. Invited reviewer for: Qualitative Theory of Dynamical System, Journal of Mathematical Analysis and Applications.
- F. Miranda Villatoro. Invited reviewer for: IEEE Transactions on Automatic Control, Automatica, Nonlinear Analysis: Hybrid Systems, IEEE Transactions on Control of Network Systems, IEEE Control Systems Letters, 23rd IFAC World Congress.

10.1.2 Invited talks

- V. Acary. Invited seminar at EPFL Civil Engineering Seminar Series, Lausanne, March: Nonsmooth dynamics of extrinsic cohesive models for fracture.
- V. Acary. Invited seminar at GeM Laboratory, Nantes University, Saint-Nazaire, June: An introduction to nonsmooth dynamics and its applications in geomechanics.
- V. Acary. Keynote lecture at ICCCM 2025, 8th International Conference on Computational Contact Mechanics, Munich, July.
- V. Acary. Invited seminar at Centre d'Automatique et des Systèmes (CAS), Mines de Paris, October: Nonsmooth dynamics and optimization. Solving the contact problem with friction using an interior-point method and the asymptotic numerical method
- V. Acary. Invited seminar at Laboratoire de Mécanique de Paris-Saclay (LMPS), ENS Paris-Saclay, December: Nonsmooth dynamical systems. Solving the contact problem with friction using methods derived from large-scale optimization.
- F. Masi. Invited speaker at 2025 Mécamat National Congress "*Homogénéisation du comportement mécanique des matériaux hétérogènes*" (programme, Aussois, France), January: *Réseaux de neurones artificiels basés sur la thermodynamique et modélisation multi-échelle*.
- F. Masi. Invited speaker at GdR MePhy & GdR I-Gaia Day "Machine Learning in Mechanics and Physics" (programme, ENSAM Paris, France), December: Discovering constitutive models from data and physics via hard constraints.
- F. Miranda Villatoro. Invited speaker at the 12th Annual Symposium of the European Network for Nonsmooth Dynamics, Germany, Erlangen, October: Discrete-time multivalued port-Hamiltonian systems with sliding motions.

10.1.3 Leadership within the scientific community

- V. Acary is member of the Comité National Français de Mécanique (CNFM), which is the French member organization of the International Union of Theoretical and Applied Mechanics (IUTAM).
- V. Acary is member of the Strategic Advisory Committee of Episciences (CCSD).

- V. Acary is co-coordinator, with R. Leine, of the European Network for Nonsmooth Dynamics (ENNSD)
- O. Goury is member of Inria's Evaluation Committee (CE).

10.2 Teaching - Supervision - Juries - Educational and pedagogical outreach

10.2.1 Teaching

- Bachelor: F. Masi, "Introduction to Numerical Methods" (19 h ETD) – Bachelor's degree in Mathematics (L3M "Mathématiques" and L3MAA "Mathématiques Avec Approfondissement"), Université Grenoble Alpes.
- Bachelor: A. Cordoba, "Software Project" (30 h ETD) – Bachelor's degree in Mathematics and Computer Science (L2), Université Grenoble Alpes.
- Bachelor: F. Vincent, Practical sessions in "Convex Optimization" (15 h ETD) – 2nd-year engineering students, Grenoble INP – Ensimag.
- Bachelor: F. Vincent, Practical sessions in "Introduction to Numerical Methods" (13 h ETD), Bachelor's degree in Mathematics (L3), Université Grenoble Alpes.
- Master: F. Bourrier, Rockfall modeling (5 h ETD), Master GAIA, Université Savoie Mont-Blanc.
- Master: F. Bourrier, "Slope stability" (25 h ETD), Polytech Grenoble 4th year, Université Grenoble Alpes.
- Master: F. Bourrier. "Structural analysis" (34 h ETD), Polytech Grenoble, 4th year, Université Grenoble Alpes.
- Master: F. Miranda Villatoro, "Distributed Optimization" (15 h ETD), Master M2 Phitem, Université Grenoble Alpes.
- Master: F. Vincent, "Data Science" (24 h ETD) – Master M1 SSD, Université Grenoble Alpes.
- Master: F. Vincent, Practical sessions in "Introduction to Operations Research" (12 h ETD) – Master M2 SSD, Université Grenoble Alpes.
- PhD: V. Acary, Introduction to Optimization (15 h ETD), Doctoral school.
- PhD: F. Masi, "Introduction to Machine Learning" & "Constitutive Modelling Meets Machine Learning: Theory and Applications" & "Hands-on Session" (3 h ETD), ALERT Olek Zienkiewicz Doctoral school (Prague, Czech Republic): programme and pedagogic material.
- PhD: F. Pérignon, "Outils collaboratifs/Git/Gitlab", "Des sources à l'exécutable", "Introduction au calcul parallèle" (52 h ETD). Collège des écoles doctorales, Université Grenoble Alpes: pedagogic material.
- PhD: F. Pérignon, "Forges logicielles et workflows" (one day) at thematic School "Open Science for the Humanities and Social Sciences: Scripts, Codes, and Software": programme.
- PhD: F. Miranda Villatoro. "Convex Optimization" (15 h ETD), Doctoral School.

10.2.2 Supervision

- Internship: Adrien Candela (May–Jun 2025) supervised by b. Brogliato.
- Internship: Rémi Ferrato (Jul–Aug 2025) supervised by F. Masi.
- Master: Anton Denisenko (May–Aug 2025) supervised by V. Acary and F. Bourrier.
- Master: Kseniia Ovchinnikova (Jun–Aug 2025) supervised by O. Goury.

- PhD: Chloé Gergely (Nov 2024 –), supervised by V. Acary and F. Bourrier.
- PhD: Louis Guillet (Jan 2023 – Dec 2025), supervised by V. Acary, F. Bourrier, and O. Goury.
- PhD: Henri Leroy (Nov 2025 –), supervised by F. Masi and V. Acary.
- PhD: Mattéo Oziol (Oct 2023 –), supervised by F. Bourrier, T. Faug, and V. Acary.
- PhD: Quang Hung Pham (Dec 2022 – Nov 2025), supervised by B. Brogliato and F. Miranda Villatoro.
- PhD: Florian Vincent (Oct 2023 –), supervised by V. Acary, F. Masi, and J. Malick (CNRS, LJK).
- Postdoc: Antoine Cordoba (Sep 2024 – Feb 2026), supervised by A. Tonnelier in collaboration with S. Fenet and P.Y. Longaretti (INRIA - STEEP).
- Postdoc: Guillaume Mestdagh (May 2025 –), supervised by V. Acary.
- Postdoc: Van Nam Vo (Oct 2025 –), supervised by B. Brogliato and F. Miranda Villatoro.

10.2.3 Juries

- F. Bourrier was member of the PhD committee of Katarina Radišić under the supervision of A. Vidard and C. Lauvernet, Université Grenoble Alpes.
- B. Brogliato was president of the PhD committee of Min Li under the superivision of A. Polyakov and Gang Zheng, École Centrale de Lille.
- F. Masi was member of the PhD committee of Pierre Hembert under the supervision of F. Chinesta and C. Ghnatios, ENSAM Paris.
- O. Goury was member of the jury SRP4ERC at Inria.

10.3 Popularization

10.3.1 Productions (articles, videos, podcasts, serious games, ...)

- B. Brogliato participated in the clip "*Commander la machine avant qu'elle ne dérape*" (video) for *L'esprit Sorcier* TV channel.
- F. Masi participated in the MAI Days event and presented the MIAI chair AIM: video.
- The article [11] is featured in the cover of the October 2025 issue of IEEE Control Systems Magazine.
- A. Cordoba participated in the Pizza Tech event by Inria | UGA in September 2025: "*Le rapport du club de Rome*".
- G. Mestdagh participated in the *Fête de la Science* outreach day at LJK with high-school classes in October 2025.
- L. Guillet participated in the Pizza Tech event by Inria | UGA in October 2025: "*Modélisation des géo-matériaux*".

11 Scientific production

11.1 Major publications

[1] V. Acary and B. Brogliato. *Numerical methods for nonsmooth dynamical systems. Applications in mechanics and electronics*. English. Lecture Notes in Applied and Computational Mechanics 35. Berlin: Springer. xxi, 525~p., 2008.

[2] B. Brogliato. *Nonsmooth Mechanics: Models, Dynamics and Control*. Springer International Publishing Switzerland; Communications and Control Engineering, 2016. DOI: `10.1007/978-3-319-28664-8`. URL: `https://inria.hal.science/hal-01236953` (cit. on pp. 6, 23).

11.2 Publications of the year

International journals

[3] V. Acary and N. A. Collins-Craft. ‘On the Moreau–Jean scheme with the Frémond impact law: energy conservation and dissipation properties for elastodynamics with contact, impact and friction’. In: *Journal of Theoretical, Computational and Applied Mechanics* (30th June 2025), pp. 1–30. DOI: 10.46298/jtcam.13480. URL: https://inria.hal.science/hal-04230941 (cit. on pp. 20, 21).

[4] B. Brogliato, F. Miranda-Villatoro and A. Younes. ‘Backstepping passivity-based trajectory tracking control of frictional oscillators: continuous and discrete time analyses’. In: *Automatica* (2026). URL: https://inria.hal.science/hal-04842497.

[5] B. Brogliato and A. Tanwani. ‘Passivity Preservation in Interconnections of Linear Cone Complementarity Systems with State Jumps’. In: *Nonlinear Analysis: Hybrid Systems* 60 (2026), p. 101682. DOI: 10.1016/j.nahs.2026.101682. URL: https://inria.hal.science/hal-04137144 (cit. on p. 24).

[6] F. Castaños, F. A. Miranda-Villatoro and B. Brogliato. ‘Multivalued Hamiltonian Systems with Sliding Motions: Analysis of the Backward-Euler Discretisation’. In: *Automatica* 182 (Dec. 2025), p. 112567. DOI: 10.1016/j.automatica.2025.112567. URL: https://inria.hal.science/hal-04625231 (cit. on p. 24).

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