RESEARCH CENTER
Saclay - Île-de-France

FIELD

Activity Report 2012

Section New Results

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AMIB Project-Team

5. New Results

5.1. RNA structures

5.1.1. RNA structure alignment

It is widely accepted that, for a large number of RNA families, the structure is more conserved than the sequence. Therefore, any reasonable notion of homology should consider the similarity in the secondary structure, i.e. how well the base-pairing positions in two structures can be put in correspondence, or aligned. In collaboration with a significant part of the French bioinformatics community, an assessment of the quality of existing algorithms for the problem was proposed [6]. Furthermore, a review of the state-of-the-art in RNA comparison algorithms is to be published [11], and a chapter in a forthcoming book on RNA computational biology was written in collaboration with Robert Giegerich (University Bielefeld) during his stay.

Most existing alignment tools rely on the assumption that the RNA structure is free of pseudoknots, i.e. free of crossing interactions. This condition naturally arises from the intractability of the unconstrained version of the problem. In a joint work, A. Denise, Ph. Rinaudo and Y. Ponty worked around this issue by proposing a parameterized complexity algorithmic solution for the unconstrained version of the problem. One of the key feature of this algorithm is that, although exponential in the worst-case scenario, it naturally adapts its complexity to the level of intricacy of the aligned structures, and remains polynomial for large classes of pseudoknots. Preliminary results of this work were presented at the WABI'12 conference [35].

5.1.2. Energy-weighted RNA algorithmics

We complemented previous studies led within AMIB on RNA structures with restricted classes of pseudoknots by showing, in a collaboration with Rolf Backofen (Freiburg University), that the computational hardness of RNA folding with general pseudoknots is extremely robust to the choice of a precise energy model. It was shown that the problem is completely unapproximable when expressive – yet realistic – energy models are taken into consideration. These results were presented at CPM'12 [37] (Helsinki, Finland).

Moreover, using an interpolation technique introduced at the RECOMB'11 conference, we were able to improve both the sequential and parallel complexities of the `RNAbor` algorithm developed within P. Clote's lab. The resulting algorithm and its application to the detection of conformational switches in sequence lengths that were previously unreachable by the algorithm, are described in a manuscript accepted in *Plos One*.

5.1.3. RNA knowledge-based potentials and 3D studies

The building of an RNA potential proved much harder and interesting than we initially expected. A non-redundant dataset had first to be extracted from the literature as the available dataset were not suitable for our study even the very recent ones. From the collected distance data, the building of a knowledge-based potential was usually done using histograms; and the histogram interval size and data fitting was an issue. In our 2012 study, we showed that the best solution to build potentials with no interval issue is by using Dirichlet Process Mixture Models (DPMs) [24]. We also benefited of the group experience in modeling the dynamics of RNA and normal-mode experiments to obtain two good decoy sets which complemented the well-known Farna study. We also showed that in many case our high-resolution predictions were better than the Farna/Rosetta standard.

5.1.4. RNA 3D structure prediction

In collaboration with PRISM at Versailles and Westhof's group at Strasbourg, we addressed the problem of ab initio prediction of RNA three-dimensional structure. We developed an algorithm for automatically predicting the topological family of any RNA three-way junction, an thus its coarse-grained local geometry, given only the information from the secondary structure: the sequence and the Watson–Crick pairings. Additionally, we showed that the results are noticeably improved if homology information is used [14]. The resulting software, `Cartaj`, is available online and downloadable at http://cartaj.lri.fr. Then we investigated a new approach for the global prediction of the coarse-grain 3D structure of RNA molecules. We model a molecule as being made of helices and junctions. Using our results above, we are able to classify junctions into topological families that determine their preferred 3D shapes. All the parts of the molecule are then allowed to establish long-distance contacts that induce a threedimensional folding of the molecule. An algorithm relying on game-theory was proposed to discover such long-distance contacts that allow the molecule to reach a Nash equilibrium. As reported by our experiments, this approach allows one to predict the global shape of large molecules of several hundreds of nucleotides that are out of reach of the state-of-the-art methods [15].

A graph-theoretic approach has been successfully used for classification and structure prediction of transmembrane beta-barrel proteins[23], [25].

5.2. Proteins structures and interactions

5.2.1. Protein-protein interaction

Adrien Guilhot, PhD candidate in our project worked on a modified scoring function for the Rosetta software suite. After an extensive conformation generation for the two recently published benchmarks, we now have a model for protein-RNA semi-flexible docking which is currently being tested.

The prediction of the network of protein-protein interactions (PPI) of an organism is crucial for the understanding of biological processes and for the development of new drugs. Machine learning methods have been successfully applied to the prediction of PPI in yeast by the integration of multiple direct and indirect biological data sources. However, experimental data are not available for most organisms. We propose in [9] an ensemble machine learning approach for the prediction of PPI that depends solely on features independent from experimental data. New estimators of the coevolution between proteins have been developed and combined them in an ensemble learning procedure.

This method has been applied to a dataset of known co-complexed proteins in *Escherichia coli* and compared it to previously published methods. Our method allows prediction of PPI with an unprecedented precision of 95.5% for the first 200 sorted pairs of proteins compared to 28.5% on the same dataset with the previous best method.

A close inspection of the best predicted pairs allowed us to detect new or recently discovered interactions between chemotactic components, the flagellar apparatus and RNA polymerase complexes in *E. coli*.

5.3. Combinatorics and Annotation

5.3.1. Word counting and random generation

A long-term research on word enumeration has been realized by the team, in order to calculate a statistical significance for a pattern occurrence according to a given background model. As a part of E. Furletova's thesis, defended in February 2012, co-advised by M. Roytberg (IMPB, Puschino, Russia) and M. Régnier, an extension to Hidden Markov Models, `SufPref`, has been proposed. It relies on a new concept of overlap graphs that efficiently overcomes the main difficulty - overlapping occurrences - in probabilities computation. An implementation is available at http://server2.lpm.org.ru/bio/online/sf/. This algorithm provides a significant space improvement over a previous algorithm, `AhoPro` developed with our former associate team MIGEC. Word statistics were used to identify mRNA targets for miRNAs involved in carcinogenesis [13].

Large deviation results have been derived in [41] that take advantage of general combinatorial properties of words. First, an approximation is derived for the double strands counting problem that refers to a counting of a given pattern in a set of sequences that arise from both strands of the genome. Here dependencies between a sequence and its complement plays a fundamental role. Second, sets of small sequences, with non-identical distributions, are addressed. Possible applications are the search of cis-acting elements in regulatory sequences that may be known, for example from ChIP-chip or ChipSeq experiments, as being under a similar regulatory control.

In [21], we developed a new algorithm for generating uniformly at random words of any regular language L. When using floating point arithmetics, its bit-complexity is $O(q \log n)$ in space and $O(qn \log n)$ in time, where n stands for the length of the word, and q stands for the number of states of a finite deterministic automaton of L. We implemented the algorithm and compared its behavior to the state-of-the-art algorithms, on a set of large automata from the VLTS benchmark suite. Both theoretical and experimental results show that our algorithm offers an excellent compromise in terms of space and time requirements, compared to the known best alternatives. In particular, it is the only method that can generate long paths in large automata. Moreover, in [10], in collaboration with the Fortesse group at LRI, we presented several randomised algorithms for generating paths in large models according to a given coverage criterion. This work opens new perspectives for future studies of statistical testing and model checking, mainly to fight the combinatorial explosion problem.

5.3.2. Analysis and design of weighted combinatorial models

Weighted context-free grammars are natural – yet powerful – random models for biological sequence and structures. We furthered our developments on these objects, and applied them to the study of the Boltzmann ensemble of low-energy in RNA.

In collaboration with P. Clote (Boston College), we used such analytic combinatorics to establish that the average geometric distance between the terminal ends of an RNA sequence, once folded, is asymptotically constant [8].

Furthermore, in collaboration with C. Banderier, O. Bodini and H. Tafat (LIPN), we constructively showed that any predefined distribution of pattern could be attained by a (possibly ambiguous) regular expressions. We also designed a dynamic-programming algorithm to automatically build such models, adopting a segmentation approach based on a parsimony principle. This work was presented at the ANALCO'12 conference [30].

Finally, we continued with D. Gardy and J. Du Boisberranger (PRISM, Université de Versailles-St Quentin) a joint study of collisions in weighted random generation. Indeed, while performing a random generation within large collections of weighted objects, the probability of any sample can be exactly and efficiently computed. Therefore, any redundancy in the sampled set is uninformative (contrasting with situations where the probability is also estimated by the sampling procedure). Following previous results presented at GASCOM'10 (Montreal), we presented at the AOFA'12 (Montreal, Canada) conference [33], a new close formula for the waiting-time of the coupon collector problem, i.e. the average number of words that one must draw to obtain the full collection. The framework defined here has direct applications in the context of RNA : approaches based on sampling are preferred to deterministic optimizations, and algorithmic efficiency of the methods can be critically affected by the redundancy of sampled sets. .

5.3.3. Scientific Workflows

Several Scientific workflow systems have been designed to support users in the tasks of designing, managing, monitoring, and executing in-silico experiments. Such systems are now equipped of provenance modules able to collect data produced and consumed during workflow runs to enhance reproducibility. In this context, we have worked in two directions. First, we have worked on the problem of reuse between scientific workflows. In particular, we have identified the presence of common or similar (sub-)workflows and workflow elements, and have deeply studied, for the first time in the literature, the problem of cross-author reuse [38].

Second, we have worked on studying the structure of scientific workflows. More precisely, we have focused on the series-parallel graph structures. Designing sub-workflows, querying or monitoring workflows leads to perform graph sub-isomorphism. This problem is NP-complete when general DAGs are considered but can

be solved in polynomial time when graphs restricted to SP graphs are considered. We have designed and implemented the SPFlow algorithm that rewrites any workflow into an SP workflow while ensuring that the provenance of the rewritten workflow is the same as the original [32], [39].

We are currently working on identifying the reasons why some scientific workflows have a non SP structure. Our long-term goal is to design a *distilling procedure* for scientific workflows offering users the ability of naturally designing workflows having a structure close to SP structures. This work is done in close collaboration with the University of Manchester [31].

5.4. Systems Biology

5.4.1. Reasoning on knowledge to build signaling networks:

We have introduced a logic-based method to infer molecular networks and show how it allows inferring signalling networks from the design of a knowledge base. Provenance of inferred data has been carefully collected, allowing quality evaluation. Our method (i) takes into account various kinds of biological experiments and their origin; (ii) mimics the scientist's reasoning within a first-order logic setting; (iii) specifies precisely the kind of interaction between the molecules; (iv) provides the user with the provenance of each interaction; (v) automatically builds and draws the inferred network [29].

5.4.2. Metabolic pathways

The topological analyse of metabolic networks is a first step to understand their behaviours and is described in term of fluxes analyses. We work on the elaboration of a stoichiometric model of *Bacillus subtilis* where its fluxes analyse predicted transcriptional regulation to be more important for the dynamics induced by glucose than by malate [7].

In metabolic pathway analyses, the metabolic networks are described in term of biochemical reactions and metabolites. The integration of structural data is required for a comprehensive understanding of the metabolic networks. We represent the metabolic networks with the functional connectivity between the protein functional domains to make more relevant analyses. We used BioΨ, a formal multi-level description based on elementary actions, to assign functions on structural domains and the elementary flux modes theory to check if the already known pathways remain presents and to identify new ones.

A new version of the software has `Mpas` (Metabolic Pathway Analyser Software) been developed during a Master2'internship by Gh. Fievet. Meanwhile we have also introduced in the landscape of the cell its membranes and the numerous pumps that facilitate ions transfers, hence taking into account the pH of the cytoplasm, a parameter that fits the cell mytosis cycle and which proves to separate the cancerous/normal status of cells [22]. We now aim at study larger and more elaborate metabolic systems, including the Krebs cycle and the mitochondria influence, thus enhancing the scalability of our method [17].

5.4.3. Bacterial phenotypic adaptation

We attempt to re-interpret a major event, the initiation of chromosome replication in *Escherichia coli*, in the light of scales of equilibria. This entails thinking in terms of hyperstructures as responsible for intensity sensing and quantity sensing and how this sensing might help explain the role of the DnaA protein in initiation of replication. We outline experiments and an automaton approach to the cell cycle that should test and refine the scales concept [19].

Another possible direction to study the mechanisms used by cells to integrate and respond to their environment is to search for a link between two large hyperstructures: the cytoskeleton and the general metabolic activity of the cell. There is extensive evidence for the interaction of metabolic enzymes with the eukaryotic cytoskeleton. We state the hypothesis that the cytoskeleton senses and integrates the general metabolic activity of the cell. The physical and chemical effects arising from metabolic sensing by the cytoskeleton would have major consequences on cell shape, dynamics and cell cycle progression. The hypothesis provides a framework that helps the significance of the enzyme-decorated cytoskeleton be determined [18].

In order to test these hypotheses, we have added many features to the HSIM simulation software. The main addition being a way to get both the power of expression of the "entity-centred" paradigm and the computational efficiency of global methods, such that Gillespie-like stochastic simulation algorithm (SSA). To achieve this, we have implemented two new algorithms. The first one concerns the possibility to take into account the interactions between two classes of molecules: the one we want to follow the spatial location over time (entities) and the one for which only the evolution of the number of copies over time is relevant.

The second algorithm is an enhancement of the tau-leap variant of the exact Gillespie SSA; This allows to take into account the interactions between globally treated molecules. The HSIM-SSA algorithm performs an adaptive processing of the number of reactions which may have been triggered during the time step. At each time step, the fast reactions are averaged while the slow reactions are fully stochastically treated. This allows HSIM-SSA to be more than 10 times faster than the other tau-leap SSA implementations [28].

5.4.4. Use of bacteria for biotechnology

Another center of interest has been to find a way to use bacteria as a mean to help us to engineer new biomolecules with specific characteristics. It is sometimes speculated that the equivalent of the polymerase chain reaction might be developed for identification of peptides, proteins or other molecules. Natural amplification systems do exist as in the case of certain autoinducer systems in bacteria. We have been outlined a possible, generic method, *the mimic chain reaction*, for obtaining peptides with 3-D structures that mimic the 3-D structure of their targets. These targets would include a variety of molecules, including proteins. There are therefore two categories of applications: the ability via amplification firstly to detect a known protein or other target at an extremely low concentration and secondly to obtain a set of peptides that mimic the structure of an unknown target and that can be used to obtain a *photofit* [20].

AVIZ Project-Team

6. New Results

6.1. Tangible Visualization

Participants: Pierre Dragicevic [correspondant], Petra Isenberg, Yvonne Jansen, Jean-Daniel Fekete.

The goal of tangible visualization is to move data and controls to the physical world in order to exploit peoples' natural abilities to perceive and to manipulate objects, and to collaborate through these objects. This is a new topic in information visualization. Physical objects can be used either to represent data (physical visualizations) or to interact with data (physical controls). We studied both.

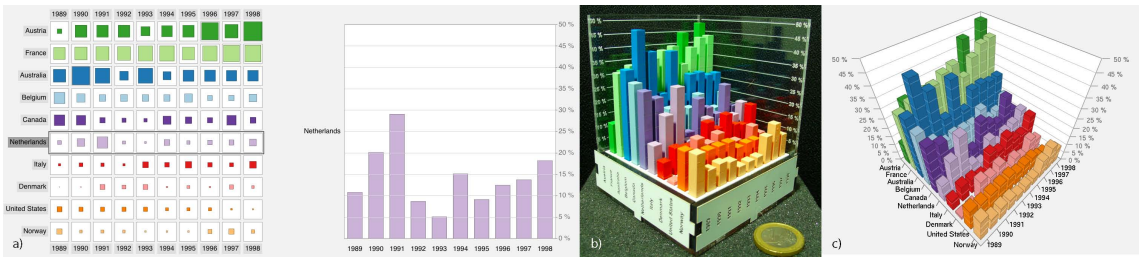

Figure 8. Education expenses of Country by Year shown under three conditions: a) on-screen 2D control; b) on-screen 3D bar chart; c) physical 3D bar chart.

Physical visualizations already exist in the form of data sculptures. Data sculptures are an increasingly popular data-driven media whose purposes are essentially artistic, communicative or educational. We are maintaining a public list of such visualizations at http://www.aviz.fr/Research/PassivePhysicalVisualizations (for passive visualizations) and at http://www.aviz.fr/Research/ActivePhysicalVisualizations (for visualizations with electronics included). But despite prolific work from the art and design communities, physical visualizations have been largely ignored in infovis research. In particular, there is no study on whether physical visualizations can help carry out actual information visualization tasks. We carried out the first infovis study comparing physical to on-screen visualizations. We focused on 3D visualizations, as these are common among physical visualizations but known to be problematic on computers. Taking 3D bar charts as an example (Figure 8), we showed that moving visualizations to the physical world can improve users' efficiency at information retrieval tasks. In contrast, augmenting on-screen visualizations with stereoscopic rendering alone or with prop-based manipulation was of limited help. Our work suggests that the efficiency of physical visualizations stem from features that are unique to physical objects, such as their ability to be touched and their perfect visual realism. These findings provide empirical motivation for current research on fast digital fabrication and self-reconfiguring materials.

We also studied how physical artifacts can help explore and interact with on-screen visualizations. One project consisted in building customizable tangible remote controllers for interacting with visualizations on wall-sized displays [34] (see http://www.aviz.fr/trc). Such controllers are especially suited to visual exploration tasks where users need to move to see details of complex visualizations. In addition, we conducted a controlled user study suggesting that tangibles make it easier for users to focus on the visual display while they interact. Another project explored the concept of stackable tangibles designed to support faceted information seeking in a variety of contexts (see http://www.aviz.fr/stackables). Each Stackable tangible represents search parameters

that can be shared amongst collaborators, modified during an information seeking process, and stored and transferred. Stackables were designed to support collaborative browsing and search in large data spaces. They are useful in meetings, for sharing results from individual search activities, and for realistic datasets including multiple facets with large value ranges.

For more information, see http://www.aviz.fr/phys.

Figure 9. Four stackables. The left shows Stackables with their filter selection interface. The right two show the selected filters.

6.2. EVE : Evolutionary Visual Exploration

Participants: Evelyne Lutton [correspondant], Nadia Boukehlifa, Waldo Cancino, Anastasia Bezerianos.

Evolutionary Visual Exploration (EVE) is a new approach that combines visual analytics with stochastic optimisation to aid the exploration of multidimensional datasets characterised by a large number of possible views or projections. A prototype tool (EvoGraphDice) has been built as an extension of GraphDice, this work has been funded by the System@tics project CSDL, see Figure 10 .

Starting from dimensions whose values are automatically calculated by a PCA, an interactive evolutionary algorithm progressively builds (or evolves) non-trivial viewpoints in the form of linear and non-linear dimension combinations, to help users discover new interesting views and relationships in their data. The criteria for evolving new dimensions is not known a priori and is partially specified by the user via an interactive interface: (i) The user selects views with meaningful or interesting visual patterns and provides a satisfaction score. (ii) The system calibrates a fitness function (optimised by the evolutionary algorithm) to take into account the user input, and then calculates new views. Our method leverages automatic tools to detect interesting visual features and human interpretation to derive meaning, validate the findings and guide the exploration without having to grasp advanced statistical concepts. To validate our method, we conducted an observational study with five domain experts. Our results show that EvoGraphDice can help users quantify qualitative hypotheses and try out different scenarios to dynamically transform their data. Importantly, it allowed our experts to think laterally, better formulate their research questions and build new hypotheses for further investigation.

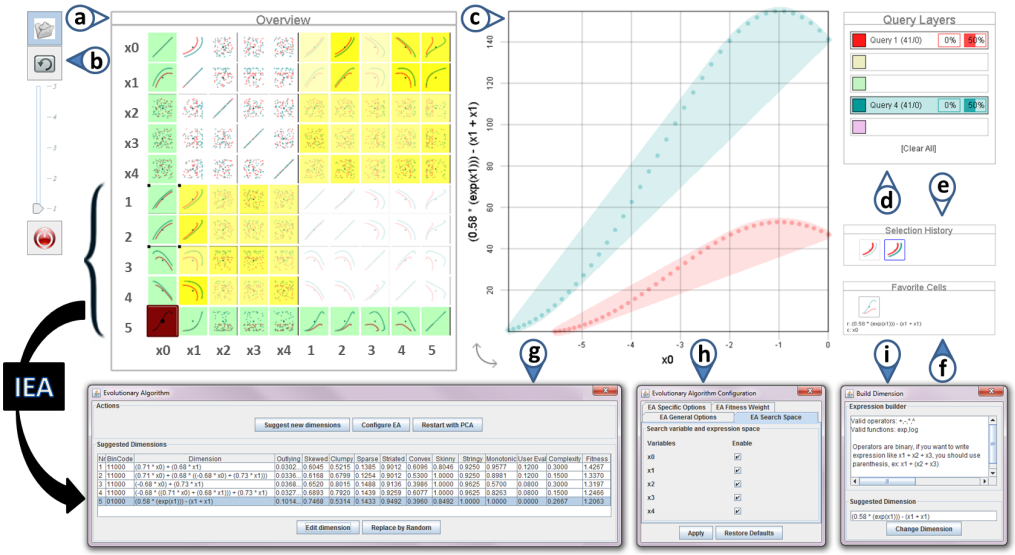

Figure 10. EvoGraphDice prototype showing an exploration session of a synthetic dataset. New extensions to the GraphDice system are indicated by coloured label arrows. Widgets: (a) an overview scatterplot matrix showing the original data set of 5 dimensions (x0..x4) and the new dimensions (1..5) as suggested by the evolutionary algorithm. (b) a tool bar with (top to bottom)"favourite" toggle button, "evolve" button , a slider to evaluate cells and a restart (PCA) button. (c) main plot view. (d) the selection query window. (e) the selection history tool. (f) the favourite cells window. (g) IEA main control window. (h) window to "limit the search space". (i) dimension editor.

6.3. Perception of Visual Variables on Wall-Sized Displays

Participants: Anastasia Bezerianos [correspondant], Petra Isenberg.

We ran two user studies on the perception of visual variables on tiled high-resolution wall-sized displays [9]. We contribute an understanding of, and indicators predicting how, large variations in viewing distances and viewing angles affect the accurate perception of angles, areas, and lengths. Our work, thus, helps visualization researchers with design considerations on how to create effective visualizations for these spaces. The first study showed that perception accuracy was impacted most when viewers were close to the wall but differently for each variable (angle, area , length). Our second study examined the effect of perception when participants could move freely compared to when they had a static viewpoint. We found that a far but static viewpoint was as accurate but less time consuming than one that included free motion. Based on our findings, we recommend encouraging viewers to stand further back from the display when conducting perception estimation tasks. If tasks need to be conducted close to the wall display, important information should be placed directly in front of the viewer or above, and viewers should be provided with an estimation of the distortion effects predicted by our work—or encouraged to physically navigate the wall in specific ways to reduce judgement error. For more information, see http://www.aviz.fr/Research/WallVariables.

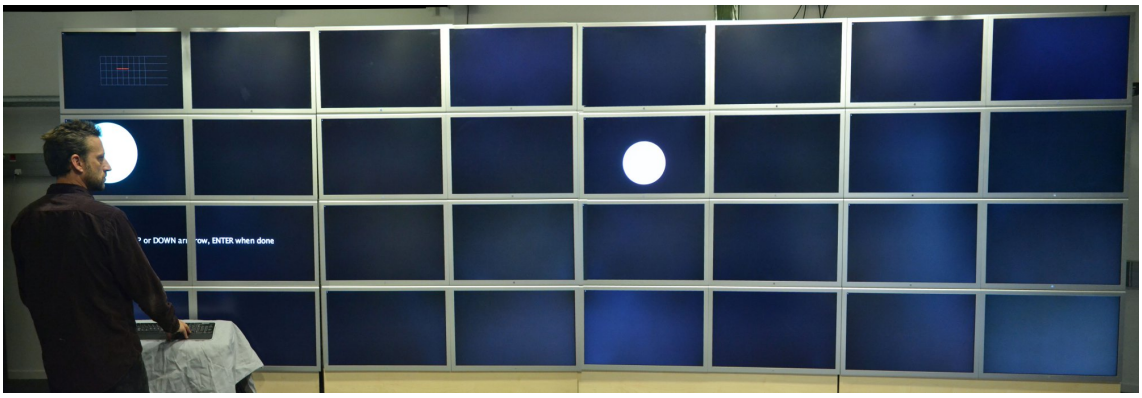

Figure 11. A participant during one trial of a user study on the WILD wall-sized display.

6.4. Sketchyness in Visualization

Participants: Tobias Isenberg [correspondant], Petra Isenberg, Jo Wood, Jason Dykes, Aidan Slingsby, Nadia Boukhelifa, Anastasia Bezerianos, Jean-Daniel Fekete.

AVIZ, in collaboration with City University London, studied how sketchiness can be used, both as a visual style and as a way to represent qualitative uncertainty.

We first studied Handy, an alternative renderer for the Processing graphics environment developed by our collaborators at the City University London [24]. It allows higher-level graphical features such as bar charts, line charts, treemaps and node-link diagrams to be drawn in a sketchy style with a specified degree of sketchiness. Our evaluation concentrated on two core aspects: the perception of sketchiness as a visual variable and higher-level impact of sketchiness on the perception of a whole graphic drawn in this style. Results suggest relative area judgment is compromised by sketchy rendering and that its influence is dependent on the shape being rendered. We showed that degree of sketchiness may be judged on an ordinal scale but that its judgement varies strongly between individuals. We evaluated higher-level impacts of sketchiness through user testing of scenarios that encourage user engagement with data visualization and willingness to critique visualization design. Results suggest that where a visualization is clearly sketchy, engagement may be increased and

that attitudes to participating in visualization annotation are more positive. The results of this work have implications for effective information visualization design that go beyond the traditional role of sketching as a tool for prototyping or its use for an indication of general uncertainty.

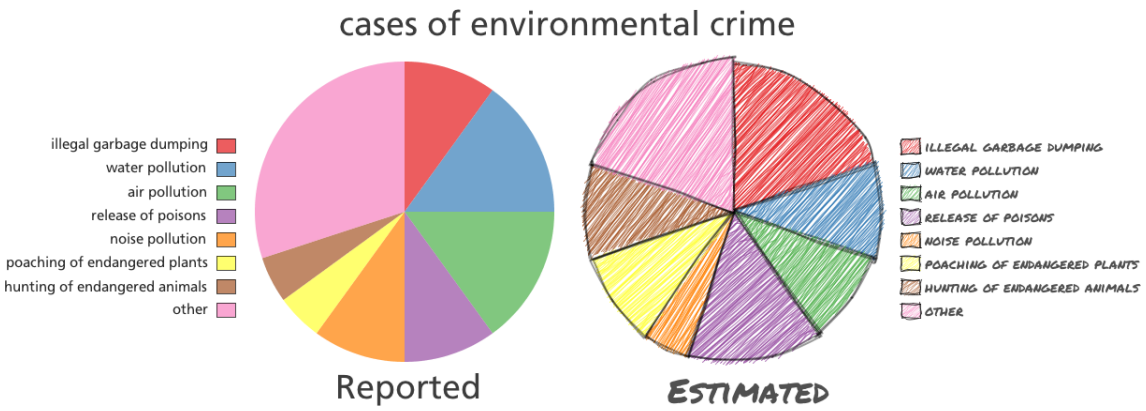

Figure 12. A pie chart drawn in regular and sketchy style.

On this last issue, we have studied whether sketchiness was an effective rendering style for conveying qualitative uncertainty [10]. We compared sketchiness to blur, intensity and dashes and obtained mixed results, showing that sketchiness is not worse than the other visual encodings but that none of them are intuitive and all of them are very limited in range, although still usable for common cases. More work is needed to asses how sketchiness can be best used and to find out more effective encodings for conveying uncertainty in a spontaneous/intuitive way.

For more information, see http://www.aviz.fr/Research/SketchyRendering and http://www.aviz.fr/Research/UncertaintySketchy.

6.5. Supporting Judgment and Decision Making with Visualizations

Participants: Pierre Dragicevic [correspondant], Luana Micallef, Jean-Daniel Fekete.

People have difficulty understanding statistical information and are unaware of their wrong judgments. Cognitive biases abound, particularly in Bayesian reasoning (see http://youtu.be/D8VZqxcu0I0 for a classic example). Psychology studies suggest that the way Bayesian problems are represented can impact comprehension, but few visual designs have been evaluated and only populations with a specific background have been involved. We conducted a study where a textual and six visual representations for three classic problems were compared using a diverse subject pool through crowdsourcing []. Visualizations included area-proportional Euler diagrams, glyph representations, and hybrid diagrams combining both. Our findings were inconsistent with previous studies in that subjects' accuracy was remarkably low and did not significantly improve when a visualization was provided with the text. A follow-up experiment confirmed that simply adding a visualization to a textual Bayesian problem is of little help for crowdsource workers. It however revealed that communicating statistical information with a diagram, giving no numbers and using text to merely set the scene significantly reduces probability estimation errors. Thus, novel representations that holistically combine text and visualizations and that promote the use of estimation rather than calculation need to be investigated. We also argued for the need to carry out more studies in settings that better capture real-life rapid decision making than laboratories. We proposed the use of crowdsourcing to partly address this concern, as crowdsourcing captures a more diverse and less intensely focused population than university students. Doing so, we hope that appropriate representations that facilitate reasoning for both laymen and professionals, independent of

their background, knowledge, abilities and age will be identified. By effectively communicating statistical and probabilistic information, physicians will interpret diagnostic results more adequately, patients will take more informed decisions when choosing medical treatments, and juries will convict criminals and acquit innocent defendants more reliably.

For more information, see http://www.aviz.fr/bayes.

BYMOORE Exploratory Action

4. New Results

4.1. BenchNN: On the Broad Potential Application Scope of Hardware Neural Network Accelerators

The emergence of high-performance applications like Recognition, Mining, and Synthesis (RMS) suggest that the potential application scope of a hardware neural network accelerator would be broad. We have highlighted that a hardware neural network accelerator is indeed compatible with many of the emerging high-performance workloads, currently accepted as benchmarks for high-performance micro-architectures. For that purpose, we develop and evaluate software neural network implementations of 5 (out of 12) RMS applications from the PARSEC Benchmark Suite. Our results show that neural network implementations can achieve competitive results, with respect to application-specific quality metrics, on these 5 RMS applications.

4.2. A Defect-Tolerant Accelerator for Emerging High-Performance Applications

Due to the evolution of technology constraints, especially energy constraints which may lead to heterogeneous multicores, and the increasing number of defects, the design of defect-tolerant accelerators for heterogeneous multi-cores may become a major micro-architecture research issue. Most custom circuits are highly defect sensitive, a single transistor can wreck such circuits. On the contrary, artificial neural networks (ANNs) are inherently error tolerant algorithms. And the emergence of high-performance applications implementing recognition and mining tasks, for which competitive ANN-based algorithms exist, drastically expands the potential application scope of a hardware ANN accelerator. However, while the error tolerance of ANN algorithms is well documented, there are few in-depth attempts at demonstrating that an actual hardware ANN would be tolerant to faulty transistors. Most fault models are abstract and cannot demonstrate that the error tolerance of ANN algorithms can be translated into the defect tolerance of hardware ANN accelerators. In this article, we introduce a hardware ANN geared towards defect tolerance and energy efficiency, by spatially expanding the ANN. In order to precisely assess the defect tolerance capability of this hardware ANN, we introduce defects at the level of transistors, and then assess the impact of such defects on the hardware ANN functional behavior. We empirically show that the conceptual error tolerance of neural networks does translate into the defect tolerance of hardware neural networks, paving the way for their introduction in heterogeneous multi-cores as intrinsically defect-tolerant and energy-efficient accelerators.

4.3. Design of a Hardware Spiking Neural Network

Hardware Spiking Neurons Design: Analog or Digital? Neurons can be implemented either as analog or digital components. While the respective advantages of each approach are well known, i.e., digital designs are more simple but analog neurons are more energy efficient, there exists no clear and precise quantitative comparison of both designs. In this paper, we compare the digital and analog implementations of the same Leaky Integrate-and-Fire neuron model at the same technology node (CMOS 65 nm) with the same level of performance (SNR and maximum spiking rate), in terms of area and energy. We show that the analog implementation requires 5 times less area, and consumes 20 times less energy than the digital design. As a result, the analog neuron, in spite of its greater design complexity, is a serious contender for future large-scale silicon neural systems.

Configurable Conduction Delay Circuits for High Spiking Rates. The conduction delay in neural systems has been proven to play an important role in processing neural information. In hardware spiking neural networks (SNN), emulating conduction delays consists of intercepting and buffering spikes for a certain amount of time during their transfer. The complexity of the conduction delay implementation increases with high spiking rates; it implies (1) storing a large number of spikes into memory cells and (2) conserving the required time resolution while processing the delays. As a result, the circuit size becomes very large and difficult to integrate into large scale SNN systems. In this paper, we highlight the trade-offs of an efficient digital delay circuit design supporting high neuron firing rates. The key issue resides in conserving spikes and spike timings while limiting storage requirements. We present a digital implementation of a configurable delay circuit supporting spiking rates of up to 1Meps (Mega events per second) and a delay range going from 1μs to 50ms with a time resolution less than 5% of the configured delay time. Synthesis results show that, using the CMOS 65nm technology, the required silicon area is 1600um2.

4.4. 3D-Stacked Implementation of Neural Networks

In order to cope with increasingly stringent power and variability constraints, architects need to investigate alternative paradigms. Neuromorphic architectures are increasingly considered (especially spike-based neurons) because of their inherent robustness and their energy efficiency. Yet, they have two limitations: the massive parallelism among neurons is hampered by 2D planar circuits, and the most cost-effective hardware neurons are analog implementations that require large capacitors, We show that 3D stacking with Through-Silicon-Vias applied to neuromorphic architectures can solve both issues: not only by providing massive parallelism between layers, but also by turning the parasitic capacitances of TSVs into useful capacitive storage.

4.5. Iterative Optimization for the Data Center (Alchemy-related research)

This result corresponds to research started within Alchemy, and it is less related to the objectives of ByMoore itself.

Iterative optimization is a simple but powerful approach that searches for the best possible combination of compiler optimizations for a given workload. However, each program, if not each data set, potentially favors a different combination. As a result, iterative optimization is plagued by several practical issues that prevent it from being widely used in practice: a large number of runs are required for finding the best combination; the process is inherently data set sensitive; and the exploration process incurs significant overhead that needs to be compensated for by performance benefits. Therefore, while iterative optimization has been shown to have significant performance potential, it is seldomly used in production compilers.

We propose [5] Iterative Optimization for the Data Center (IODC): we show that servers and data centers offer a context in which all of the above hurdles can be overcome. The basic idea is to spawn different combinations across workers and recollect performance statistics at the master, which then evolves to the optimum combination of compiler optimizations. IODC carefully manages costs and benefits, and is transparent to the end user.

We evaluate IODC using both MapReduce and throughput server applications. In order to reflect the large number of users interacting with the system, we gather a very large collection of data sets (at least 1000 and up to several million unique data sets per program), for a total storage of 10.7TB, and 568 days of CPU time. We report an average performance improvement of 1.48×, and up to 2.08×, for the MapReduce applications, and 1.14×, and up to 1.39×, for the throughput server applications.

4.6. Statistical Performance Comparisons of Computers (Alchemy-related research)

This result corresponds to research started within Alchemy, and it is less related to the objectives of ByMoore itself.

As a fundamental task in computer architecture research, performance comparison has been continuously hampered by the variability of computer performance. In traditional performance comparisons, the impact of performance variability is usually ignored (i.e., the means of performance measurements are compared regardless of the variability), or in the few cases where it is factored in using parametric confidence techniques, the confidence is either erroneously computed based on the distribution of performance measurements, instead of the distribution of sample mean of performance measurem ents, or too few measurements are considered for the distribution of sample mean to be normal. We first illustrate how such erroneous practices can lead to incorrect comparisons.

Then, we propose [4] a non-parametric Hierarchical Performance Testing (HPT) framework for performance comparison, which is significantly more practical than standard p arametric confidence tests because it does not require to collect a large number of measurements in order to achieve a normal distribution of the sample mean. This HPT framework has been implemented as an open-source software.

COMETE Project-Team

6. New Results

6.1. Foundations of information hiding

Information hiding refers to the problem of protecting private information while performing certain tasks or interactions, and trying to avoid that an adversary can infer such information. This is one of the main areas of research in Comète; we are exploring several topics, described below.

6.1.1. Measuring information leakage

A fundamental concern in computer security is to control information flow, whether to protect confidential information from being leaked, or to protect trusted information from being tainted. In view of the pragmatic difficulty of preventing undesirable flows completely, there is now much interest in theories that allow information flow to be quantified, so that "small" leaks can be tolerated. In [19] we introduced g-leakage, a rich generalization of the min-entropy model of quantitative information flow. In g-leakage, the benefit that an adversary derives from a certain guess about a secret is specified using a gain function g. Gain functions allow a wide variety of operational scenarios to be modeled, including those where the adversary benefits from guessing a value close to the secret, guessing a part of the secret, guessing a property of the secret, or guessing the secret within some number of tries. We proved important properties of g-leakage, including bounds between min-capacity, g-capacity, and Shannon capacity. We also showed a deep connection between a strong leakage ordering on two channels, C1 and C2, and the possibility of factoring C1 into C2 C3 , for some C3 . Based on this connection, we proposed a generalization of the Lattice of Information from deterministic to probabilistic channels.

6.1.2. Interactive systems

In [12] we have considered systems where secrets and observables can alternate during the computation. We have shown that the information-theoretic approach which interprets such systems as (simple) noisy channels is not valid anymore. However, the principle can be recovered if we consider more complicated types of channels, that in Information Theory are known as channels with memory and feedback. We have shown that there is a complete correspondence between interactive systems and such kind of channels. Furthermore, we have shown that the capacity of the channels associated to such systems is a continuous function of the Kantorovich metric.

6.1.3. Unlinkability

Unlinkability is a privacy property of crucial importance for several systems (such as RFID or voting systems). Informally, unlinkability states that, given two events/items in a system, an attacker is not able to infer whether they are related to each other. However, in the literature we find several definitions for this notion, which are apparently unrelated and shows a potentially problematic lack of agreement. In [22] we shed new light on unlinkability by comparing different ways of defining it and showing that in many practical situations the various definitions coincide. It does so by (a) expressing in a unifying framework four definitions of unlinkability from the literature (b) demonstrating how these definitions are different yet related to each other and to their dual notion of "inseparability" and (c) by identifying conditions under which all these definitions become equivalent. We argued that the conditions are reasonable to expect in identification systems, and we prove that they hold for a generic class of protocols.

6.1.4. A compositional method to compute the sensitivity of differentially private queries

Differential privacy is a modern approach in privacy-preserving data analysis to control the amount of information that can be inferred about an individual by querying a database. The most common techniques are based on the introduction of probabilistic noise, often defined as a Laplacian parametric on the sensitivity of the query. In order to maximize the utility of the query, it is crucial to estimate the sensitivity as precisely as possible.

In [28] we considered relational algebra, the classical language for expressing queries in relational databases, and we proposed a method for computing a bound on the sensitivity of queries in an intuitive and compositional way. We used constraint-based techniques to accumulate the information on the possible values for attributes provided by the various components of the query, thus making it possible to compute tight bounds on the sensitivity.

6.1.5. A differentially private mechanism of optimal utility for a region of priors

Differential privacy (already introduced in the previous section) is usually achieved by using mechanisms that add random noise to the query answer. Thus, privacy is obtained at the cost of reducing the accuracy, and therefore the utility, of the answer. Since the utility depends on the user's side information, commonly modeled as a prior distribution, a natural goal is to design mechanisms that are optimal for every prior. However, it has been shown in the literature that such mechanisms do not exist for any query other than counting queries.

Given the above negative result, in [38] we considered the problem of identifying a restricted class of priors for which an optimal mechanism does exist. Given an arbitrary query and a privacy parameter, we geometrically characterized a special region of priors as a convex polytope in the priors space. We then derived upper bounds for utility as well as for min-entropy leakage for the priors in this region. Finally we defined what we call the tight-constraints mechanism and we discussed the conditions for its existence. This mechanism has the property of reaching the bounds for all the priors of the region, and thus it is optimal on the whole region.

6.1.6. Differential privacy with general metrics

Differential privacy, already described above, is a formal privacy guarantee that ensures that sensitive information relative to individuals cannot be easily inferred by disclosing answers to aggregate queries. If two databases are adjacent, i.e. differ only for an individual, then querying them should not allow to tell them apart by more than a certain factor. The transitive application of this property induces a bound also on the distinguishability of two generic databases, which is determined by their distance on the Hamming graph of the adjacency relation.

In [37] we lifted the restriction relative to the Hamming graphs and we explored the implications of differential privacy when the indistinguishability requirement depends on an arbitrary notion of distance. We showed that we can express, in this way, (protection against) kinds of privacy threats that cannot be naturally represented with the standard notion. We gave an intuitive characterization of these threats in terms of Bayesian adversaries, which generalizes the characterization of (standard) differential privacy from the literature. Next, we revisited the well-known result on the non-existence of universally optimal mechanisms for any query other than counting queries. We showed that in our setting, for certain kinds of distances, there are many more queries for which universally optimal mechanisms exist: Notably sum, average, and percentile queries. Finally, we showed some applications in various domains: statistical databases where the units of protection are groups (rather than individuals), geolocation, and smart metering.

6.1.7. Privacy for location-based systems

The growing popularity of location-based systems, allowing unknown/untrusted servers to easily collect and process huge amounts of users' information regarding their location, has recently started raising serious concerns about the privacy of this kind of sensitive information. In [36] we studied geo-indistinguishability, a formal notion of privacy for location-based systems that protects the exact location of a user, while still allowing approximate information - typically needed to obtain a certain desired service - to be released.

Our privacy definition formalizes the intuitive notion of protecting the user's location within a radius r with a level of privacy that depends on r. We presented three equivalent characterizations of this notion, one of which corresponds to a generalized version [37] of the well-known concept of differential privacy. Furthermore, we presented a perturbation technique for achieving geo-indistinguishability by adding controlled random noise to the user's location, drawn from a planar Laplace distribution. We demonstrated the applicability of our technique through two case studies: First, we showed how to enhance applications for location-based services with privacy guarantees by implementing our technique on the client side of the application. Second,

we showed how to apply our technique to sanitize location-based sensible information collected by the US Census Bureau.

6.1.8. Compositional analysis of information hiding

Systems concerned with information hiding often use randomization to obfuscate the link between the observables and the information to be protected. The degree of protection provided by a system can be expressed in terms of the probability of error associated to the inference of the secret information. In [15] we considered a probabilistic process calculus to specify such systems, and we studied how the operators affect the probability of error. In particular, we characterized constructs that have the property of not decreasing the degree of protection, and that can therefore be considered safe in the modular construction of these systems. As a case study, we applied these techniques to the Dining Cryptographers, and we derived a generalization of Chaum's strong anonymity result.

In [29], a similar framework was proposed for reasoning about the degree of differential privacy provided by such systems. In particular, we investigated the preservation of the degree of privacy under composition via the various operators. We illustrated our idea by proving an anonymity-preservation property for a variant of the Crowds protocol for which the standard analyses from the literature are inapplicable. Finally, we made some preliminary steps towards automatically computing the degree of privacy of a system in a compositional way.

6.1.9. Anonymous and route-secure communication systems

Incentives to Cooperation. Anonymity systems have a broad range of users, ranging from ordinary citizens who want to avoid being profiled for targeted advertisements, to companies trying to hide information from their competitors, to entities requiring untraceable communication over the Internet. With these many potential users, it would seem that anonymity services based on a consumer/provider users will naturally be well-resourced and able to operate efficiently. However, cooperation cannot be taken for granted. Current deployed systems show that some users will indeed act selfishly, and only use the system to send their messages whilst ignoring the requests to forward others' messages. Obviously, with not enough cooperative users, the systems will hardly operate at all, and will certainly not be able to afford adequate anonymity guarantees. It is therefore vital that these systems are able to deploy incentives to encourage users' cooperation and so make the anonymity provision effective. Some interesting approaches to achieve that have been proposed, such as make running relays easier and provide better forwarding performance.

To evaluate whether these approaches are effective, we need a framework which empowers us to analyze them, as well as provide guidelines and some mechanism design principles for incentive schemes. This much we have provided in [30], exploiting notions and techniques from Game Theory. We proposed a game theoretic framework and used it to analyze users' behaviours and also predict what strategies users will choose under different circumstances and according to their exact balance of preferences among factors such as anonymity, performance (message delivery time) and cost. Significantly, we also used the model to assess the effectiveness of the gold-star incentive mechanism, which was introduced in Tor network to encourage users to act as cooperative relays, and thus enhance the service performance for well-behaved forwarders.

Trust in anonymity networks. Trust metrics are used in anonymity networks to support and enhance reliability in the absence of verifiable identities, and a variety of security attacks currently focus on degrading a user's trustworthiness in the eyes of the other users. In [16] we have presented an enhancement of the Crowds anonymity protocol via a notion of trust which allows crowd members to route their traffic according to their perceived degree of trustworthiness of each other member of the crowd. Such trust relations express a measure of an individual's belief that another user may become compromised by an attacker, either by a direct attempt to corrupt or by a denial-of-service attack. Our protocol variation has the potential of improving the overall trustworthiness of data exchanges in anonymity networks, which cannot normally be taken for granted in a context where users are actively trying to conceal their identities. Using such formalization, in the paper we have then analyzed quantitatively the privacy properties of the protocol under standard and adaptive attacks.

6.2. Foundations of Concurrency

Distributed systems have changed substantially in the recent past with the advent of phenomena like social networks and cloud computing. In the previous incarnation of distributed computing the emphasis was on consistency, fault tolerance, resource management and related topics; these were all characterized by *interaction between processes*. Research proceeded along two lines: the algorithmic side which dominated the Principles Of Distributed Computing conferences and the more process algebraic approach epitomized by CONCUR where the emphasis was on developing compositional reasoning principles. What marks the new era of distributed systems is an emphasis on managing access to information to a much greater degree than before.

The *Concurrent constraint programming (ccp)* paradigm focuses on information access and therefore it is suited for this new era of concurrent systems. Ccp singles out the fundamental aspects of asynchronous systems whose agents (or processes) evolve by accessing information in a global medium. In the works [20], [21], [31], [26] described below we developed algorithms and extended the foundations of ccp.

6.2.1. Spatial and Epistemic Modalities for Constraint-based Calculi

Epistemic concepts were crucial in distributed computing as was realized in the mid 1980s with Halpern and Moses' groundbreaking paper on common knowledge. This led to a flurry of activity in the next few years with many distributed protocols being understood from an epistemic point of view. The impact of epistemic ideas in the concurrency theory community was slower in coming. We believe that epistemic ideas need to be exploited more by concurrency theorists and we did so in the following works.

In [26] we introduced spatial and epistemic process calculi for reasoning about spatial information and knowledge distributed among the agents of a system. We also introduced domain-theoretical structures to represent spatial and epistemic information. Finally we provided operational and denotational techniques for reasoning about the potentially infinite behaviour of spatial and epistemic processes. We also gave compact representations of infinite objects that can be used by processes to simulate announcements of common knowledge and global information. We also developed an interpreter of these calculi in [31].

6.2.2. Bisimilarity for Constraint-based Calculi

Bisimilarity is a standard behavioural equivalence in concurrency theory, but a well-behaved notion of bisimilarity for ccp has been proposed only recently. When the state space of a system is finite, the ordinary notion of bisimilarity can be computed via the well-known partition refinement algorithm, but unfortunately, this algorithm does not work for ccp bisimilarity. In [20] we proposed a variation of the partition refinement algorithm for verifying ccp bisimilarity. To the best of our knowledge this is the first work providing for the automatic verification of program equivalence for ccp.

In [20] we only studied the strong version of bisimilarity. Weak bisimiliarity is obtained from the strong case by taking into account only the actions that are observable in the system. Typically, the standard partition refinement can also be used for deciding weak bisimilarity simply by using Milner's reduction from weak to strong bisimilarity; a technique referred to as saturation. In [21] we showed that, because of its involved labeled transitions, the above-mentioned saturation technique does not work for ccp. We also gave an alternative reduction from weak ccp bisimilarity to the strong one that allows us to use the ccp partition refinement algorithm for deciding this equivalence.

In the more traditional setting of the pi-calculus we have also proposed an approach to restrict access to information.

6.2.3. Locality in the Pi-Calculus

In [25] we enriched the pi-calculus with an operator for confidentiality (hide), whose main effect is to restrict the access to the object of the communication, thus representing confidentiality in a natural way. The hide operator is meant for local communication, and it differs from new in that it forbids the extrusion of the name and hence has a static scope. Consequently, a communication channel in the scope of a hide can be implemented as a dedicated channel, and it is more secure than one in the scope of a new. To emphasize the

difference, we introduced a spy context that represents a side-channel attack and breaks some of the standard security equations for new. To formally reason on the security guarantees provided by the hide construct, we also introduced an observational theory and establish stronger equivalences by relying on a proof technique based on bisimulation semantics.

6.2.4. Foundations of Probabilistic Concurrent Systems

In [17] we have solved an open problem in the literature by proving that two known semantics for the probabilistic mu-calculus, a denotational semantics and a two-player stochastic game semantics, coincide on all models.

In [18] we have improved the result of [17] by introducing a new logic called probabilistic mu-calculus with independent product. We have proved that two semantics coincide in all models: a denotational semantics and a two-player game semantics based on a novel class of concurrent games. Furthermore, we have shown how the new logic is strictly more expressive than the other. This allows the encoding of other important temporal logics for probabilistic concurrent systems such as PCTL.

In [27] we have introduced a proof system designed for supporting human-aided verification of properties (expressed as probabilistic mu-calculus formulas ([17]) of concurrent probabilistic processes described by SOS-style operational semantics.

6.2.5. Interference metrics for Mobile ad-hoc networks (MANETs)

Mobile ad-hoc networks consist of a collection of nodes that communicate with each other through wireless links without a pre-established networking infrastructure. A common feature of most of these networks is free node mobility. Each device will therefore change its links to other devices frequently. These frequent changes in the network topology can cause the nodes to continuously enter and exit each other transmission area. Hence, highly dynamic routing algorithms are needed to ensure the connectivity. Moreover, mobile devices may have strict requirements on the energy consumption because their expected life-time often depends on the energy stored in a battery or other exhaustible power sources. For these reasons, finding a good trade-off between network connectivity, power saving and interference reduction is one of the most critical challenges in managing mobile ad hoc networks. In [23], we have proposed an effective framework for analysing protocol connectivity and measuring the level of interference and, based on that for developing novel interference-aware communication strategies. Though other models exist in the literature, to our best knowledge, our framework is the most comprehensive and effective for the behavioral analysis and a quantitative assessment of interference for wireless networks in the presence of node mobility.

COMMANDS Project-Team

6. New Results

6.1. Optimal control of partial differential equations

6.1.1. Optimal control of a semilinear parabolic equation with singular arcs

Participant: Frédéric Bonnans.

This paper, published as Inria report 8099 [25], develops a theory of singular arc, and the corresponding second order necessary and sufficient conditions, for the optimal control of a semilinear parabolic equation with scalar control applied on the r.h.s. We obtain in particular an extension of Kelley's condition, and the characterization of a quadratic growth property for a weak norm.

6.2. Trajectory optimization

6.2.1. First and second order optimality conditions for optimal control problems of state constrained integral equations

Participants: Frédéric Bonnans, Xavier Dupuis.

In this work performed with Constanza De La Vega (U. Buenos Aires), and published as Inria report 7961 [26], we deal with optimal control problems of integral equations, with initial-final and running state constraints. The order of a running state constraint is defined in the setting of integral dynamics, and we work here with constraints of arbitrary high orders. First and second-order necessary conditions of optimality are obtained, as well as second-order sufficient conditions.

6.2.2. Sensitivity analysis for relaxed optimal control problems with final-state constraints

Participants: Frédéric Bonnans, Laurent Pfeiffer.

In this work, performed with Oana Serea (U. Perpignan), and published as Inria report 7977 [27], we compute a second-order expansion of the value function of a family of relaxed optimal control problems with final-state constraints, parameterized by a perturbation variable. The sensitivity analysis is performed for controls that we call R-strong solutions. They are optimal solutions with respect to the set of feasible controls with a uniform norm smaller than a given R and having an associated trajectory in a small neighborhood for the uniform norm. In this framework, relaxation enables us to consider a wide class of perturbations and therefore to derive sharp estimates of the value function.

6.2.3. Sensitivity analysis for the outages of nuclear power plants

Participants: Frédéric Bonnans, Laurent Pfeiffer.

In this work, performed with Kengy Barty (EDF), and published as Inria report 7884 [24]. Nuclear power plants must be regularly shut down in order to perform refueling and maintenance operations. The scheduling of the outages is the first problem to be solved in electricity production management. It is a hard combinatorial problem for which an exact solving is impossible.

Our approach consists in modelling the problem by a two-level problem. First, we fix a feasible schedule of the dates of the outages. Then, we solve a low-level problem of optimization of elecricity production, by respecting the initial planning. In our model, the low-level problem is a deterministic convex optimal control problem.

Given the set of solutions and Lagrange multipliers of the low-level problem, we can perform a sensitivity analysis with respect to dates of the outages. The approximation of the value function which is obtained could be used for the optimization of the schedule with a local search algorithm.

6.2.4. Optimization of the anaerobic digestion of microalgae in a coupled process

Participant: Pierre Martinon.

In this work in collaboration with Terence Bayen (U. Monptellier) and Francis Mairet (Inria Sophia), submitted to ECC13 [30], we study the maximization of the production of methane in a bioreactor coupling an anaerobic digester and a culture of micro-algae limited by light. The decision parameter is the dilution rate which is chosen as a control, and we enforce periodic constraints in order to repeat the same operation every day. The system is gathered into a three-dimensional system taking into account a day-night model of the light in the culture of micro-algae. Applying Pontryagin maximum principle, the necessary conditions on optimal trajectories indicate that the control consists of bang and/or singular arcs. We provide numerical simulations by both direct and indirect methods, which show the link between the light model and the structure of optimal solutions.

6.3. Stochastic programming

6.3.1. Solving multi-stage stochastic mixed integer linear programs by the dual dynamic programming approach

Participants: Frédéric Bonnans, Zhihao Cen.

In this work performed in the framework of the PhD thesis of Zhihao Cen, and published as an Inria report RR-7868 [29], We consider a model of medium-term commodity contracts management. Randomness takes place only in the prices on which the commodities are exchanged, whilst state variable is multi-dimensional, and decision variable is integer. In our previous article, we proposed an algorithm based on the quantization of random process and a dual dynamic programming type approach to solve the continuous relaxation problem. In this paper, we study the multi-stage stochastic mixed integer linear program (SMILP) and show the difficulty when using dual programming type algorithm. We propose an approach based on the cutting plane method combined with the algorithm in our previous article, which gives an upper and a lower bound of the optimal value and a sub-optimal integer solution. Finally, a numerical test on a real problem in energy market is performed.

6.3.2. Two methods of pruning Benders' cuts and their application to the management of a gas portfolio

Participant: Laurent Pfeiffer.

This report, coauthored with R. Apparigliato and S. Auchapt (Gdf Suez), and published as Inria report 8133 [31], describes a gas portfolio management problem, which is solved with the SDDP (Stochastic Dual Dynamic Programming) algorithm. We present some improvements of this algorithm and focus on methods of pruning Benders' cuts, that is to say, methods of picking out the most relevant cuts among those which have been computed. Our territory algorithm allows a quick selection and a great reduction of the number of cuts. Our second method only deletes cuts which do not contribute to the approximation of the value function, thanks to a test of usefulness. Numerical results are presented.

6.4. Hamilton-Jacobi approach

6.4.1. Hamilton-Jacobi equations in singular domains

Participants: Zhiping Rao, Hasnaa Zidani.

A good deal of attention has been devoted to the analysis of Hamilton–Jacobi equations adapted to unconventional domains, particularly in view of application to control problems and traffic models. The topic is new and capable of interesting developments, the results so far obtained have allowed to clarify under reasonable assumptions, basic items as the right notion of viscosity solution to be adopted and the validity of comparison principles.

- The work [19], co-authored with C. Imbert (LAMA, U. Paris-Est) and R. Monneau (Cermics, ENPC), focuses on a Hamilton-Jacobi approach to junction problems with applications to traffic flows. More specifically, the paper is concerned with the study of a model case of

first order Hamilton-Jacobi equations posed on a *junction*, that is to say the union of a finite number of half-lines with a unique common point. The main result is a comparison principle. We also prove existence and stability of solutions. The two challenging difficulties are the singular geometry of the domain and the discontinuity of the Hamiltonian. As far as discontinuous Hamiltonians are concerned, these results seem to be new. They are applied to the study of some models arising in traffic flows. The techniques developed here provide new powerful tools for the analysis of such problems.

- This work deals with deterministic control problems where the dynamic can be completely different in multi-complementary domains of the space $I\!R^d$. As a consequence, the dynamics present discontinuities at the interfaces of these domains. This leads to a complex interplay that has to be analyzed among transmission conditions to "glue" the propagation of the value function on the interfaces. Several questions arise: how to define properly the value function and what is the right Bellman Equation associated to this problem?. In the case of finite horizon problems without runing cost, a jonction condition is derived on the interfaces, and a precise viscosity notion is provided in a paper in progress. Moreover, a uniqueness result of a viscosity solution is shown.

6.4.2. A general Hamilton-Jacobi framework for nonlinear state-constrained control problems

Participants: Olivier Bokanowski, Hasnaa Zidani.

This work [10], co-authored with Albert Altarovici, deals with deterministic optimal control problem with state constraints and nonlinear dynamics. It is known for such a problem that the value function is in general discontinuous and its characterization by means of an HJ equation requires some controllability assumptions involving the dynamics and the set of state constraints. Here, we first adopt the viability point of view and look at the value function as its epigraph. Then, we prove that this epigraph can always be described by an auxiliary optimal control problem free of state constraints, and for which the value function is Lipschitz continuous and can be characterized, without any additional assumptions, as the unique viscosity solution of a Hamilton-Jacobi equation. The idea introduced in this paper bypasses the regularity issues on the value function of the constrained control problem and leads to a constructive way to compute its epigraph by a large panel of numerical schemes. Our approach can be extended to more general control problems. We study in this paper the extension to the infinite horizon problem as well as for the two-player game setting. Finally, an illustrative numerical example is given to show the relevance of the approach.

6.4.3. State-constrained optimal control problems of impulsive differential equations

Participants: Nicolas Forcadel, Zhiping Rao, Hasnaa Zidani.

The research report [35] presents a study on optimal control problems governed by measure driven differential systems and in presence of state constraints. The first result shows that using the graph completion of the measure, the optimal solutions can be obtained by solving a reparametrized control problem of absolutely continuous trajectories but with time-dependent state-constraints. The second result shows that it is possible to characterize the epigraph of the reparametrized value function by a Hamilton-Jacobi equation without assuming any controllability assumption

6.4.4. Level-set approach for reachability analysis of hybrid systems under lag constraints

Participants: Giovanni Granato, Hasnaa Zidani.

The study in [36] aims at characterizing a reachable set of a hybrid dynamical system with a lag constraint in the switch control. The setting does not consider any controllability assumptions and uses a level-set approach. The approach consists in the introduction of an adequate hybrid optimal control problem with lag constraints on the switch control whose value function allows a characterization of the reachable set. The value function is in turn characterized by a system of quasi-variational inequalities (SQVI). We prove a comparison principle for the SQVI which shows uniqueness of its solution. A class of numerical finite differences schemes for solving the system of inequalities is proposed and the convergence of the numerical solution towards the value function is studied using the comparison principle. Some numerical examples illustrating the method are presented. Our study is motivated by an industrial application, namely, that of range extender electric vehicles. This class of

electric vehicles uses an additional module *the range extender* as an extra source of energy in addition to its main source a high voltage battery. The methodolgy presented in [36] is used to establish the maximum range of a Hybrid vehicle, see [22].

6.5. Collision avoidance and motion planning

6.5.1. Collision analysis for a UAV

Participants: Anna Désilles, Hasnaa Zidani.

The *Sense and Avoid* capacity of Unmanned Aerial Vehicles (UAV) is one of the key elements to open the access to airspace for UAVs. In order to replace a pilot's *See and Avoid* capacity such a system has to be certified "as safe as a human pilot on-board". The problem is to prove that an unmanned aircraft equipped with a S&A system can comply with the actual air transportation regulations. A paper in progress aims to provide mathematical and numerical tools to link together the safety objectives and sensors specifications. Our approach starts with the natural idea of a specified "safety volume" around the aircraft: the safety objective is to guarantee that no other aircraft can penetrate this volume. We use a general reachability and viability concepts to define nested sets which are meaningful to allocate sensor performances and manoeuvring capabilities necessary to protect the safety volume. Using the general framework of HJB equations for the optimal control and differential games, we give a rigorous mathematical characterization of these sets. Our approach allows also to take into account some uncertainties in the measures of the parameters of the incoming traffic. We also provide numerical tools to compute the defined sets, so that the technical specifications of a S&A system can be derived in accordance with a small set of intuitive parameters. We consider several dynamical models corresponding to the different choices of maneuvers (lateral, longitudinal and mixed). Our numerical simulations show clearly that the nature of used maneuvers is an important factor in the specifications of sensor's performances.

6.6. Numerical methods for HJ equations

6.6.1. An adaptive sparse grid semi-lagrangian scheme for first order Hamilton-Jacobi Bellman equations

Participant: Olivier Bokanowski.

The paper [14], co-authored with M. Griebel (Fraunhofer SCAI & Univ. Bonn), J. Garcke and I. Klopmpaker (TUB, Berlin) proposes a semi-Lagrangian scheme using a spatially adaptive sparse grid to deal with non-linear time-dependent Hamilton-Jacobi Bellman equations. We focus in particular on front propagation models in higher dimensions which are related to control problems. We test the numerical efficiency of the method on several benchmark problems up to space dimension $d = 8$, and give evidence of convergence towards the exact viscosity solution. In addition, we study how the complexity and precision scale with the dimension of the problem.

6.6.2. A discontinuous Galerkin scheme for front propagation with obstacles

Participant: Olivier Bokanowski.

In [33], co-authored with C.-W. Shu (Brown Univ.) and Y. Cheng (Michigan Univ.), some front propagation problems in the presence of obstacles are analysed. We extend a previous work (Bokanowski, Cheng and Shu, SIAM J. Scient. Comput., 2011), to propose a simple and direct discontinuous Galerkin (DG) method adapted to such front propagation problems. We follow the formulation of (Bokanowski, Forcadel and Zidani, SIAM J. Control Optim. 2010), leading to a level set formulation driven by $\min(u_t + H(x, \nabla u), u - g(x)) = 0$, where $g(x)$ is an obstacle function. The DG scheme is motivated by the variational formulation when the Hamiltonian H is a linear function of ∇u, corresponding to linear convection problems in the presence of obstacles. The scheme is then generalized to nonlinear equations, written in an explicit form. Stability analysis is performed for the linear case with Euler forward, a Heun scheme and a Runge-Kutta third order time discretization using the technique proposed in (Zhang and Shu, SIAM J. Control and Optim., 2010). Several numerical examples are provided to demonstrate the robustness of the method. Finally, a narrow band approach is considered in order to reduce the computational cost.

6.6.3. Semi-Lagrangian discontinuous Galerkin schemes for some first and second order PDEs

Participant: Olivier Bokanowski.

Explicit, unconditionally stable, high order schemes for the approximation of some first and second order linear, time-dependent partial differential equations (PDEs) are proposed in [34], in collaboration with G. Simarmata (internship 2011, currently in RI dep. of Rabobank). The schemes are based on a weak formulation of a semi-Lagrangian scheme using discontinuous Galerkin elements. It follows the ideas of the recent works of Crouseilles, Mehrenberger and Vecil (2010) and of Qiu and Shu (2011), for first order equations, based on exact integration, quadrature rules, and splitting techniques. In particular we obtain high order schemes, unconditionally stable and convergent, in the case of linear second order PDEs with constant coefficients. In the case of non-constant coefficients, we construct "almost" unconditionally stable second order schemes and give precise convergence results. The schemes are tested on several academic examples, including the Black and Scholes PDE in finance.

DAHU Project-Team

5. New Results

5.1. Distributed data management

Participants: Serge Abiteboul, Émilien Antoine, Cristina Sirangelo, Nadime Francis, Luc Segoufin.

Distributed knowledge base. We are developing the system Webdamlog [16], [13], [14] to address the challenges faced by everyday Web users, who interact with inherently heterogeneous and distributed information. Managing such data is currently beyond the skills of casual users. In Webdamlog, we see the Web as a knowledge base consisting of distributed logical facts and rules. The objective is to enable automated reasoning over this knowledge base, ultimately improving the quality of service and of data. The system supports the Webdamlog language, a Datalog style language with rule delegation.

Deduction in uncertain worlds. Motivated by reasoning in distributed environments in which disagreements arise between different actors, we study in [17] deduction (captured by datalog programs) in the presence of inconsistencies (induced by functional dependency (FD) violations). We adopt an operational semantics for datalog with FDs based on inferring facts one at a time, while never violating the FDs. This yields a set of possible worlds that we capture by c-tables of possibly exponential size. We propose to use probabilities to measure this nondeterminism and define a probabilistic semantics that can be captured by probabilistic conditional tables. Not surprisingly, we show that computing the probability of a query answer in our setting is expensive, which leads us to introduce a sampling algorithm to estimate answer probabilities. We then turn our attention to the problem of explaining why a particular answer holds. This leads us to consider two novel notions: the most influential extensional facts, and the most likely proofs for an answer. We study algorithms for ranking facts and proofs based on their contribution to the derivation of an answer. Finally, we consider how our framework can be adapted to a distributed setting, and in particular, how sampling can be performed in a distributed manner.

Access rights in a distributed setting. We started considering access right issues in Webdamlog. This is related to specifying access right on views in standard databases. There is also the issues of controlling rules that are run locally but were specified by other peers.

Incomplete information in Web data. Incomplete information often arises from the integration of different Web data sources, as well as from the exchange of data between communicating Web applications. The semantics of incompleteness (i.e. which possible complete databases are represented by an incomplete one) depends on the context and the particular scenario where incompleteness raises from. We have studied how to deal with the presence of incomplete information under different possible semantics. We have in particular studied in which condition it is possible to query incomplete data “naively", i.e. as if it were complete. We have exhibited “natural" fragments of first order logic for which naive evaluation is possible, under different semantics.

Graph data management. Graph structured data can be found in new emerging applications such as RDF and linked data, or social networks. The peculiarity of queries over graphs is that they are interested in both data carried by the graph and in the graph topology; they are often based on reachability patterns. In a distributed setting it is very common to be able to query only a partial description or a "view" of the graph. We studied the problem of answering queries using only the information provided by the views. The presence of a form of recursion in views and queries presents new challenges. We found restricted classes of graph views and queries that allow efficient query answering over views.

5.2. Tree automata theory

Participants: Luc Segoufin, Serge Abiteboul, M Praveen.

Tree automata We studied the expressive power of a subclass of regular tree languages. We gave a decidable characterization of those languages that are "piecewise testable", i.e. definable using boolean combination of existential first-order formulas [12].

Automata with counters. We studied extending techniques used in standard Petri nets to other models. We extended the Rackoff technique to decide coverability and boundedness problems for Strongly Increasing Affine nets, a subclass of Affine nets [20].

Languages on trees. We studied in [18] highly expressive query languages for unordered data trees, using as formal vehicles Active XML and extensions of languages in the while family. All languages may be seen as adding some form of control on top of a set of basic pattern queries. The results highlight the impact and interplay of different factors: the expressive power of basic queries, the embedding of computation into data (as in Active XML), and the use of deterministic vs. nondeterministic control. All languages are Turing complete, but not necessarily query complete in the sense of Chandra and Harel. Indeed, we show that some combinations of features yield serious limitations, analogous to FOk definability in the relational context. On the other hand, the limitations come with benefits such as the existence of powerful normal forms. Other languages are "almost" complete, but fall short because of subtle limitations reminiscent of the copy elimination problem in object databases.

Probabilistic XML. In [15], we study the problem of, given a corpus of XML documents and its schema, finding an optimal (generative) probabilistic model, where optimality here means maximizing the like- lihood of the particular corpus to be generated. Focusing first on the structure of documents, we present an efficient algorithm for finding the best generative probabilistic model, in the absence of constraints. We further study the problem in the presence of integrity constraints, namely key, inclusion, and domain constraints. We study in this case two different kinds of generators. First, we consider a continuation-test generator that performs, while generating documents, tests of schema satisfiability ; these tests prevent from generating a document violating the constraints but, as we will see, they are computationally expensive. We also study a restart generator that may generate an invalid document and, when this is the case, restarts and tries again. Finally, we consider the injection of data values into the structure, to obtain a full XML document. We study different approaches for generating these values.

Infinite alphabet. We studied the complexity of satisfiability of linear temporal logics extended to reason about repetitions of values from an infinite data domain. We refined an existing result that reduced this problem to Petri net reachability, and showed that it can be reduced to the coverability problem. Using this refinement, we gave the precise complexity of the satisfiability problem. We also characterized the complexity of satisfiability for many fragments and extensions of the logic.

DEFI Project-Team

6. New Results

6.1. Qualitative methods for inverse scattering problems

6.1.1. Sampling methods with time dependent data

Participant: Houssem Haddar.

Together with A. Lechleiter and S. Marmorat we proposed and analyzed a time domain linear sampling method as an algorithm to solve the inverse scattering problem of reconstructing an obstacle with Robin or Neumann boundary condition from time-dependent near-field measurements of scattered waves. Our algorithm is based on our earlier work to solve a similar inverse scattering problem for obstacles with Dirichlet boundary conditions. In addition to the analysis of a different scattering problem, we provided a substantial improvement of the method on both theoretical and numerical levels. More specifically, we analyzed the method for incident waves generated by pulses with bounded spectrum. Moreover, adapting the function space setting to this type of data allowed us to provide a simpler analysis. On the numerical side, we presented a fast implementation of the inversion algorithm that relies on a FFT-based evaluation of the near-field operator [34].

6.1.2. Inverse problems for periodic penetrable media

Participant: Dinh Liem Nguyen.

Imaging periodic penetrable scattering objects is of interest for non-destructive testing of photonic devices. The problem is motivated by the decreasing size of periodic structures in photonic devices, together with an increasing demand in fast non-destructive testing. In this project, we considered the problem of imaging a periodic penetrable structure from measurements of scattered electromagnetic waves. As a continuation of earlier work jointly with A. Lechleiter [24], [25], [23], we considered an electromagnetic problem for transverse magnetic waves (previous work treats transverse electric fields), and also the full Maxwell equations. In both cases, we treat the direct problem by a volumetric integral equation approach and construct a Factorization method [4], [44], [43], [48].

6.1.3. Transmission Eigenvalues and their application to the identification problem

Participant: Houssem Haddar.

The so-called interior transmission problem plays an important role in the study of inverse scattering problems from (anisotropic) inhomogeneities. Solutions to this problem associated with singular sources can be used for instance to establish uniqueness for the imaging of anisotropic inclusions from muti-static data at a fixed frequency. It is also well known that the injectivity of the far field operator used in sampling methods is related to the uniqueness of solutions to this problem. The frequencies for which this uniqueness fails are called transmission eigenvalues. We are currently developing approaches where these frequencies can be used in identifying (qualitative informations on) the medium properties. Our research on this topic is mainly done in the framework of the associate team ISIP http://www-direction.inria.fr/international/PHP/Networks/LiEA.php with the University of Delaware. A review article on the state of art concerning the transmission eigenvalue problem has been written in collaboration with F. Cakoni [32]. We are also in the process of editing a spacial issue of the journal Inverse Problems dedicated to the use of these transmission eigenvalues in inverse problems. Our recent contributions are the following:

- In collaboration with M. Fares and F. Collino from CERFACS and A. Cossonnière from ENSIEETA we finalized our work on the use of a surface integral equation approach to numerically compute transmission eigenvalues for inclusions with piecewise constant index. The main difficulty behind this procedure is the compactness of the obtained integral operator in usual Sobolev spaces associated with the forward scattering problem. We solved this difficulty by introducing a preconditioning

operator associated with a "coercive" transmission problem. On the theoretical side, together with A; Cossonnière we also finalyzed the analysis of the Fredholm properties of the interior transmission problem for the cases where the index contrast changes sign outside the boundary by using the surface integral equation approach [16].

- With G. Giorgi, we developed a method that give estimates on the material properties using the first transmission eigenvalue. This method is based on reformulating the interior transmission eigenvalue problem into an eigenvalue problem for the material coefficients. We validated our methodology for homogeneous and inhomogeneous inclusions and backgrounds. We also treated the case of a background with absorption and the case of scatterers with multiple connected components of different refractive indexes [21].
- With F. Cakoni and D. Colton we initiated the study of transmission eigenvalues for absorbing media. In particular, we showed that, in the case of absorbing media, transmission eigenvalues form a discrete set, exist for sufficiently small absorption and for spherically stratified media exist without this assumption. For constant index of refraction, we also obtained regions in the complex plane where the transmission eigenvalues cannot exist and obtain a priori estimates for real transmission eigenvalues [14].
- With F. Cakoni and A. Cossonnière we considered the interior transmission problem corresponding to the inverse scattering by an inhomogeneous (possibly anisotropic) media in which an impenetrable obstacle with Dirichlet boundary conditions is embedded. Our main focus is to understand the associated eigenvalue problem, more specifically to prove that the transmission eigenvalues form a discrete set and show that they exist. The presence of Dirichlet obstacle brings new difficulties to already complicated situation dealing with a non-selfadjoint eigenvalue problem. In this work we employed a variety of variational techniques under various assumptions on the index of refraction as well as the size of the Dirichlet obstacle [15].

6.1.4. The factorization method for inverse scattering problems

6.1.4.1. The factorization method for cracks with impedance boundary conditions

Participants: Yosra Boukari, Houssem Haddar.

We use the Factorization method to retrieve the shape of cracks with impedance boundary conditions from farfields associated with incident plane waves at a fixed fre- quency. This work is an extension of the study initiated by Kirsch and Ritter [Inverse Problems, 16, pp. 89-105, 2000] where the case of sound soft cracks is considered. We address here the scalar problem and provide theoretical validation of the method when the impedance boundary conditions hold on both sides of the crack. We then deduce an inversion algorithm and present some validating numerical results in the case of simply and multiply connected cracks [38].

6.1.4.2. The factorization method for EIT with uncertain background

Participants: Giovanni Migliorati, Houssem Haddar.

We extended the Factorization Method for Electrical Impedance Tomography to the case of background featuring uncertainty. This work is based on our earlier algorithm for known but inhomogeneous backgrounds. We developed three methodologies to apply the Factorization Method to the more difficult case of piece-wise constant but uncertain background. The first one is based on a recovery of the background through an optimization scheme and is well adapted to relatively low dimensional random variables describing the background. The second one is based on a weighted combination of the indicator functions provided by the Factorization Method for different realiza- tions of the random variables describing the uncertain background. We show through numerical experiments that this procedure is well suited to the case where many real-izations of the measurement operators are available. The third strategy is a variant of the previous one when measurements for the inclusion-free background are available. In that case, a single pair of measurements is sufficient to achieve comparable accuracy to the deterministic case [42].

6.1.4.3. The factorization method for GIBC

Participants: Mathieu Chamaillard, Nicolas Chaulet, Houssem Haddar.

We are concerned with the identification of some obstacle and some Generalized Impedance Boundary Conditions (GIBC) on the boundary of such obstacle from far field measurements generated by the scattering of harmonic incident waves. The GIBCs are approximate models for thin coatings, corrugated surfaces, rough surfaces or imperfectly conducting media.

We justified the use of the Factorization method to solve the inverse obstacle problem in the presence of GIBCs. This method gives a uniqueness proof as well as a fast algorithm to reconstruct the obstacle from the knowledge of the far field produced by incident plane waves for all the directions of incidence at a given frequency. We also provided some numerical reconstructions of obstacles for several impedance operators.

6.2. Iterative Methods for Non-linear Inverse Problems

6.2.1. Inverse medium problem for axisymmetric eddy current models

Participants: Houssem Haddar, Zixian Jiang, Kamel Riahi.

We continued our developments of shape optimization methods for inclusion detection in an axisymmetric eddy current model. This problem is motivated by non-destructive testing methodologies for steam generators. We are finalizing our joint work with A. Lechleiter on numerical methods for the solution of the direct problem in weighted Sobolev spaces using approriate Dirichlet-to-Neumann mappings to bound the computational domain. We are also finalizing jointly with M. El Guedri the work on inverse solver using a regularized steepest descent method for the problem of identifying a magnetite deposits using axial eddy current probe.

We are currently investigating two research directions:

- The development of asymptotic models to identify thin highly conducting deposits. We derived three possible asymptotic models that can be exploited in the inverse problem. The numerical validation is under study.
- The extension of this work to 3D configurations with axisymmetric configuration at infinity, which has been started with the PostDoc of K. Riahi.

6.2.2. A min-max formulation for inverse scattering problems

Participants: Grégoire Allaire, Houssem Haddar, Dimitri Nicolas.

After having developed an inverse solver combining the use of Level-Set method and topological garadient method for multistatic inverse scattering problem and numerically showed how convergence can be achieved with intial guess provided by the Linear Sampling Method, we explored the use of an objective function that would lead to quicker and more stable reconstructions. This has been achieved through maximizing the least-square difference with respect to the Herglotz kernel of used incident wave while minimizing with respect to the geometrical parameters. Premliminary numerical experimentations showed that this procedure is viable and lead to quicker inversion algorithms [5].

6.2.3. The conformal mapping method and inverse scattering at low frequencies

Participant: Houssem Haddar.

Together with R. Kress we have employed a conformal mapping technique for the inverse problem to reconstruct a perfectly conducting inclusion in a homogeneous background medium from Cauchy data for electrostatic imaging, that is, for solving an inverse boundary value problem for the Laplace equation. In a recent work [41] we proposed an extension of this approach to inverse obstacle scattering for time-harmonic waves, that is, to the solution of an inverse boundary value problem for the Helmholtz equation. The main idea is to use the conformal mapping algorithm in an iterative procedure to obtain Cauchy data for a Laplace problem from the given Cauchy data for the Helmholtz problem. We presented the foundations of the method together with a convergence result and exhibit the feasibility of the method via numerical examples.

6.2.4. A steepest descent method for inverse electromagnetic scattering problems

Participants: Houssem Haddar, Nicolas Chaulet.

In a continuation of our earlier work jointly with L. Bourgeois [13], we studied the application of non linear optimization techniques to solve the inverse scattering problems for the $3D$ Maxwell's equations with generalized impedance boundary conditions. We characterized the shape derivative in the case where the GIBC is defined by a second order surface operator. We then applied a boundary variation method based on a regularized steepest descent to solve the 3-D inverse problem with partial farfield data. The obtained numerical results demonstrated the possibility of identifying the shape of coated objects as well as the parameters of the coating in the $3D$ Maxwell case.

6.3. Shape and topology optimization

6.3.1. Geometric constraints in shape and topology optimization

Participant: Grégoire Allaire.

With François Jouve (LJLL) and Georgios Michailidis (Renault and CMAP), we propose a method to handle geometric constraints in shape and topology optimization. In the framework of the level-set method we rely on a notion of local thickness which is computed using the signed-distance function to the boundary of the shape. We implement this method in two and three space dimensions for a model of linear elasticity. We consider various formulations of the constrained optimization problem and compute a shape derivative to advect the shape. We discuss different ways to handle the constraints. The resulting optimized shape is strongly dependent on the initial guess and on the way the constraints are being treated.

6.3.2. A hybrid optimization method

Participant: Grégoire Allaire.

With Charles Dapogny (Renault and LJLL) and Pascal Frey (LJLL) we propose a method for structural optimization that relies on two alternative descriptions of shapes : on the one hand, they are exactly meshed so that mechanical evaluations by finite elements are accurate ; on the other hand, we resort to a level-set characterization to describe their deformation along the shape gradient. The key ingredient is a meshing algorithm for building a mesh, suitable for numerical computations, out of a piecewise linear level-set function on an unstructured mesh. Therefore, our approach is at the same time a geometric optimization method (since shapes are exactly meshed) and a topology optimization method (since the topology of successive shapes can change thanks to the power of the level-set method).

6.3.3. DeHomogenization

Participant: Olivier Pantz.

In most shape optimization problems, the optimal solution does not belong to the set of genuine shapes but is a composite structure. The homogenization method consists in relaxing the original problem thereby extending the set of admissible structures to composite shapes. From the numerical viewpoint, an important asset of the homogenization method with respect to traditional geometrical optimization is that the computed optimal shape is quite independent from the initial guess (even if only a partial relaxation is performed). Nevertheless, the optimal shape being a composite, a post-treatment is needed in order to produce an almost optimal non-composite (i.e. workable) shape. The classical approach consists in penalizing the intermediate densities of material, but the obtained result deeply depends on the underlying mesh used and the details level is not controllable. We proposed in [51] a new post-treatment method for the compliance minimization problem of an elastic structure. The main idea is to approximate the optimal composite shape with a locally periodic composite and to build a sequence of genuine shapes converging toward this composite structure. This method allows us to balance the level of details of the final shape and its optimality. Nevertheless, it was restricted to particular optimal shapes, depending on the topological structure of the lattice describing the arrangement of the holes of the composite. We lifted this restriction in order to extend our method to any optimal composite structure for the compliance minimization problem in [50]. Since, the method has been improved and a new article presenting the last results is in preparation. Moreover, we intend to extend this approach to other kinds of cost functions. A first attempt, based on a gradient method, has been made. Unfortunately, it was leading to local minima. Thus a new strategy has to be worked out. It will be mainly based on the same ideas than the one developed for the compliance minimization problem, but some difficulties are still to be overcome.

6.3.4. Level-Set Method

Participant: Olivier Pantz.

We have begin to work, with Gabriel Delagado, on a new level-set optimization method, based on a gradient method. The key idea consists in computing directly the derivative of the discretized cost functions. The main advantage is that it is usually more simple to implement than the standard approach (consisting in using a discretized version of the gradient of the cost function). Moreover, the results obtained are as good or even better than the one obtained in previous works. Nevertheless, this method has its drawbacks, since the cost function is only derivable almost everywhere (the zero level-set has to be transverse to the triangulation of the mesh). It follows that convergence toward the minimum by the gradient method is not granted. To overcome this problem, we intend to use a mix-formulation for the state function. Unfortunately, such a formulation, in the case of linear elasticity is quite difficult to obtain. We thus intend to begin with the simplest scalar case, for which a lot more hybrid formulations are available.

6.3.5. Robust Optimization

Participant: Olivier Pantz.

One of the main problem in shape optimization problems is due to the fact that the gradient is never computed exactly. When the current solution is far from a local optimum, this is not a problem: even a rough approximation of the gradient enable us to exhibit a descent direction. On the contrary, when close to a local optimal, a very precise computation of the gradient is needed. We intend, with G. Delgado, to use a-posteriori error estimates evaluate the errors made on the computation of the gradient and to ensure that at each step, a genuine descent direction is used in the gradient method.

6.3.6. Level-set method applied to structural optimization with contact

Participants: Houssem Haddar, Olivier Pantz.

The current study covers the design and implementation of a method for topological shape optimization in order to optimize multi-connected structures taking into account the contact that may arise between the different components. This project is motivated by the optimization of leaf springs, issue proposed by the company CORTEL and is conducted in the framework of the Master internship of M. Mahjoub. We proposed a method that relies on the use of a Level Set Method coupled with a penalty method to handle contact with different components. The level set function is used for instance to construct the penalization functional. Preliminary results showed that the method efficiently handle optimal design with a targeted non linear deformation behavior prescribed by the manufacturer.

6.3.7. Optimization of a sodium fast reactor core

Participants: Grégoire Allaire, Olivier Pantz.

In collaboration with D. Schmidt, G. Allaire and E. Dombre, we apply the geometrical shape optimization method for the design of a SFR (Sodium Fast reactor) core in order to minimize a thermal counter-reaction known as the sodium void effect. In this kind of reactor, by increasing the temperature, the core may become liable to a strong increase of reactivity ρ, a key-parameter governing the chain-reaction at quasi-static states. We first use the 1 group energy diffusion model and give the generalization to the 2 groups energy equation. We then give some numerical results in the case of the 1 group energy equation. Note that the application of our method leads to some designs whose interfaces can be parametrized by very smooth curves which can stand very far from realistic designs. We don't explain here the method that it would be possible to use for recovering an operational design but there exists several penalization methods that could be employed to this end. This work was partially sponsored by EDF. Our results will be published in the proceedings of the CEMRACS'11, during which part of the results have been obtained.

6.4. Asymptotic Analysis

6.4.1. Asymptotic analysis of the interior transmission eigenvalues related to coated obstacles

Participants: Nicolas Chaulet, Houssem Haddar.

This work is a collaboration with Fioralba Cakoni from the University of Delaware (USA). The interior transmission eigenvalues play an important role in the area of inverse scattering problems. These eigenvalues can actually be determined by multi-static far field data. Thus, they could be used for non destructive testing. We focused on the case where the obstacle is a perfectly conducting body coated by some thin dielectric material. We derived and justified the asymptotic expansion of the first interior transmission eigenvalue with respect to the thickness of the coating for the TM electromagnetic polarization. This expansion provided interesting qualitative information about the behavior of these eigenvalues and also gave an explicit formula to compute the thickness of the coating.

6.4.2. *Effective boundary conditions for thin periodic coatings*

Participants: Mathieu Chamaillard, Houssem Haddar.

This topic is the object of a collaboration with Patrick Joly and is a continuation of our earlier work on interface conditions done in the framework of the PhD thesis of Berangère Delourme [18], [17]. Th goal here is to derive effective conditions that model scattering from thin periodic coatings where the thickness and the periodicity are of the same length but very small compared to the wavelength. The originality of our work, compared to abundant literature is to consider the case of arbitrary geometry (2-D or 3-D) and to consider higher order approximate models. We formally derived third order effective conditions after exhibiting the full asymptotic expansion of the solution in terms of the periodicity length.

6.4.3. *Homogenization of thermal radiative transfer models in heterogeneous domains*

Participant: Grégoire Allaire.

With my former PhD student, Zakaria Habibi, we studied the homogenization of heat transfer in periodic porous media where the fluid part is made of long thin parallel cylinders, the diameter of which is of the same order than the period. The heat is transported by conduction in the solid part of the domain and by conduction, convection and radiative transfer in the fluid part (the cylinders). A non-local boundary condition models the radiative heat transfer on the cylinder walls. To obtain the homogenized problem we first use a formal two-scale asymptotic expansion method. The resulting effective model is a convection-diffusion equation posed in a homogeneous domain with homogenized coefficients evaluated by solving so-called cell problems where radiative transfer is taken into account. In a second step we rigorously justify the homogenization process by using the notion of two-scale convergence. One feature of this work is that it combines homogenization with a 3D to 2D asymptotic analysis since the radiative transfer in the limit cell problem is purely two-dimensional. Eventually, we provide some 3D numerical results in order to show the convergence and the computational advantages of our homogenization method. We also focused on the contribution of the so-called second order corrector. If the source term is a periodically oscillating function (which is the case in our application to nuclear reactor physics), a strong gradient of the temperature takes place in each periodicity cell, corresponding to a large heat flux between the sources and the perforations. This effect cannot be taken into account by the homogenized model, neither by the first order corrector. We show that this local gradient effect can be reproduced if the second order corrector is added to the reconstructed solution. Z. Habibi received the 2012 Paul Caseau PhD prize in the field "modélisation et simulation numérique", prize created by the Académie des technologies and EDF.

6.4.4. *Homogenization of complex flows in porous media*

Participant: Grégoire Allaire.

With Robert Brizzi (CMAP), Jean-François Dufrêche (Marcoule and Montpellier), Andro Mikelic (Lyon 1) and Andrey Piatnitski (Narvik) we studied the homogenization (or upscaling) of a system of partial differential equations describing the non-ideal transport of a N-component electrolyte in a dilute Newtonian solvent through a rigid porous medium. Non-ideal effects are taken into account by the mean spherical approximation (MSA) model. We first study the existence of equilibrium solutions in the absence of external forces. When the motion is governed by a small static electric field and a small hydrodynamic force, we generalize O'Brien's argument to deduce a linearized model. We then proceed to the homogenization of these linearized equations and prove that the effective tensor satisfies Onsager properties, namely is symmetric positive definite. We eventually make numerical comparisons with the ideal case.

With my PhD student Harsha Hutridurga we study the convection and diffusion of a solute in a porous medium in the presence of a linear chemical reaction of adsorption/desorption on the pore surfaces. The mathematical model is a system of two coupled convection-diffusion equations, one in the bulk of the saturated fluid flowing in the porous medium, the other on the pore surface, at the interface with the solid part of the porous medium. The coupling takes place through a linear reaction term expressing the exchange of mass between the bulk concentration and the surface concentration. By a method of two-scale asymptotic expansion with drift we obtain the homogenized problem in a moving frame. We rigorously justify our upscaling approach by using the notion of two-scale convergence with drift. Some 2-d numerical tests are performed in order to study the effect of variations of the adsorption rate constant and surface molecular diffusion on the effective dispersion tensor.

With Irina Pankratova (Narvik) and Andrey Piatnitski (Narvik) we consider the homogenization of a non-stationary convection-diffusion equation posed in a bounded domain with periodically oscillating coefficients and homogeneous Dirichlet boundary conditions. Assuming that the convection term is large, we give the asymptotic profile of the solution and determine its rate of decay. In particular, it allows us to characterize the “hot spot”, i.e., the precise asymptotic location of the solution maximum which lies close to the domain boundary and is also the point of concentration. Due to the competition between convection and diffusion, the position of the “hot spot” is not always intuitive as exemplified in some numerical tests.

6.4.5. Multiscale finite elements

Participant: Grégoire Allaire.

With my PhD student Franck Ouaki we introduced a new multiscale finite element method to solve convection-diffusion problems where both velocity and diffusion coefficient exhibit strong variations at a much smaller scale than the domain of resolution. In that case, classical discretization methods, used at the scale of the heterogeneities, turn out to be too costly or useless. Our method aims at solving this kind of problems on coarser grids with respect to the size of the heterogeneities by means of particular basis functions. These basis functions are solutions to cell problems and are designed to reproduce the variations of the solution on an underlying fine grid. Since all cell problems are independent from each other, these problems can be solved in parallel, which makes the method very efficient when used on parallel architectures. The convergence proof of our method is still in progress. But, on the basis of results of periodic homogenization, an a priori error estimate, that represents a first step in the proof, has already been proved. A 2-d numerical implementation in FreeFem++ has also been performed.

6.4.6. A new shell modeling modeling

Participant: Olivier Pantz.

Using a formal asymptotic expansion, we have proved with K. Trabelsi, that non-isotropic thin-structure could behave (when the thickness is small) like a shell combining both membrane and bending effects. It is the first time to our knowledge that such a model is derived. An article on this is currently under review.

6.4.7. A new Liouville type Rigidity Theorem

Participant: Olivier Pantz.

We have recently developed a new Liouville type Rigidity Theorem. Considering a cylindrical shaped solid, we prove that if the local area of the cross sections is preserved together with the length of the fibers, then the deformation is a combination of a planar deformation and a rigid motion. The results currently obtained are limited to regular deformations and we are currently working with B. Merlet to extend them. Nevertheless, we mainly focus on the case where the conditions imposed to the local area of the cross sections and the length of the fibers are only "almost" fulfilled. This will enable us to derive rigorously new non linear shell models combining both membrane and flexural effects that we have obtained using a formal approach. An article on this subject is currently in preparation.

6.4.8. Lattices

Participant: Olivier Pantz.

With A. Raoult and N. Meunier (Université Paris Descartes), we have compute the asymptotic limit of a square lattice with three-points interactions. Considering such interaction is important in the case of square lattices, because such lattices, if only endowed with two-points closest neighbor interactions, show no resistance to compression, what is quit restrictive. We prove in particular that under some symmetry assumptions on the type of elementary interactions, no micro-relaxation do occur and that the limit can be obtained by a mere quasiconvexication. Without those assumptions, the computation of the limit requires the resolution of a homogenization problem on an infinite number of cells, what is usually out of reach. Our work has been published in M3AS [26].

6.5. Diffusion MRI

Participants: Jing-Rebecca Li, Houssem Haddar, Julien Coatléven, Dang Van Nguyen, Hang Tuan Nguyen.

Diffusion Magnetic Resonance Imaging (DMRI) is a promising tool to obtain useful information on microscopic structure and has been extensively applied to biological tissues. In particular, we would like to focus on two applications:

- inferring from DMRI measurements changes in the cellular volume fraction occurring upon various physiological or pathological conditions.

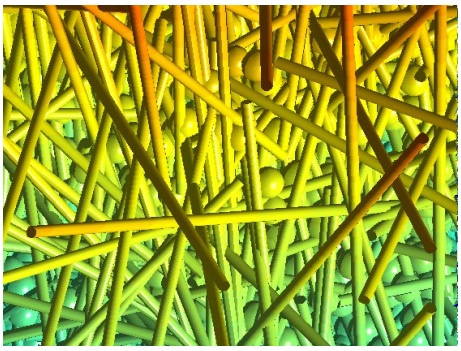

Figure 1. Computational domain for simulating diffusion in cerebral gray matter.

  This application is one of the first to show the promise of DMRI because it can detect acute cerebral ischemia (cell swelling) on the basis of lower than normal apparent diffusion coefficient a few minutes after stroke.

- estimating the average cell size in the case of tumor imaging

  This application is useful as a diagnostic tool as well as a tool for the evaluation of tumor treatments.

For both of the above applications we approach the problem via the following steps:

- Construct reduced models of the multiple-compartment Bloch-Torrey partial differential equation (PDE) using homogenization methods.
- Invert the resulting reduced models for the biological parameters of interest: the cellular volume fraction in the first case, and the average distance between neighboring cells in the second case.

We obtained the following results.

- We generated fairly complicated meshes that can be used to simulate diffusion in cerebral gray matter. In the Finite Elements code, this required using the mesh generation software Salome, developed at the CEA Saclay. We are working on the problem of increasing the cellular volume fraction to a physically realistic level, which is difficult for the mesh generator because of the very small distances between the neurons.

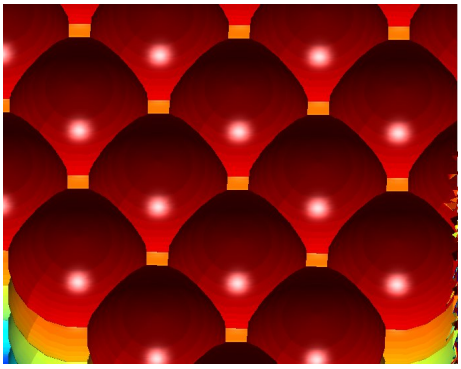

Figure 2. Computational domain for simulating tumor cells.

- We developed a homogenized model for the apparent diffusion coefficient (the slope of the log of the DMRI signal) of heterogenous cellular domains. An article on this topic has been submitted.
- We developed a reduce model of the complete DMRI signal (not just the slope as in the above) using more sophisticated homogenization methods. An article on this topic is under preparation.

DISCO Project-Team

6. New Results

6.1. Algorithmic study of linear functional systems

Participants: Alban Quadrat, Thomas Cluzeau [ENSIL, Univ. Limoges], Daniel Robertz [Univ. Aachen].

In [108], it is shown that every linear functional system (e.g., PD systems, differential time-delay systems, difference systems) is equivalent to a linear functional system defined by an upper block-triangular matrix of functional operators: each diagonal block is respectively formed by a generating set of the elements of the system satisfying a purely i-codimensional system. Hence, the system can be integrated in cascade by successively solving (inhomogeneous) i-codimensional linear functional systems to get a Monge parametrization of its solution space [110]. The results are based on an explicit construction of the grade/purity filtration of the module associated with the linear functional system. This new approach does not use involved spectral sequence arguments as is done in the literature of modern algebra [82], [83]. To our knowledge, the algorithm obtained in [34] is the most efficient algorithm existing in the literature of non-commutative algebra. It was implemented in the PURITYFILTRATION package developed in Maple (see Section 5.6) and in the `homalg` package of GAP 4 (see Section 5.7). Classes of overdetermined/underdetermined linear systems of partial differential equations which cannot be directly integrated by Maple can be solved using the PURITYFILTRATION package.

Given a linear multidimensional system (e.g., ordinary/partial differential systems, differential time-delay systems, difference systems), Serre's reduction aims at finding an equivalent linear multidimensional system which contains fewer equations and fewer unknowns. Finding Serre's reduction of a linear multidimensional system can generally simplify the study of structural properties and of different numerical analysis issues, and it can sometimes help solving the linear multidimensional system in closed form. In [13], Serre's reduction problem is studied for underdetermined linear systems of partial differential equations with either polynomial, formal power series or analytic coefficients and with holonomic adjoints in the sense of algebraic analysis [82], [83]. These linear partial differential systems are proved to be equivalent to a linear partial differential equation. In particular, an analytic linear ordinary differential system with at least one input is equivalent to a single ordinary differential equation. In the case of polynomial coefficients, we give an algorithm which computes the corresponding linear partial differential equation.

The connection between Serre's reduction and the decomposition problem [90], which aims at finding an equivalent linear functional system which is defined by a block diagonal matrix of functional operators, is algorithmically studied in [92].

In [111], algorithmic versions of Statford's results [114] (e.g., computation of unimodular elements, decomposition of modules, Serre's splitting-off theorem, Stafford's reduction, Bass' cancellation theorem, minimal number of generators) were obtained and implemented in the STAFFORD package. In particular, we show how a determined/overdetermined linear system of partial differential equations with either polynomial, rational, formal power series or locally convergent power series coefficients is equivalently to a linear system of partial differential in at most two unknowns. This result is a large generalization of the cyclic vector theorem which plays an important role in the theory of linear ordinary differential equations.

6.2. Boundary value problems for linear ordinary integro-differential equations

Participants: Alban Quadrat, Georg Regensburger.

In [61], we study algorithmic aspects of linear ordinary integro-differential operators with polynomial coefficients. Even though this algebra is not noetherian and has zero divisors, Bavula recently proved in [81] that it is coherent, which allows one to develop an algebraic systems theory. For an algorithmic approach to linear systems theory of integro-differential equations with boundary conditions, computing the kernel of matrices is a fundamental task. As a first step, we have to find annihilators, which is, in turn, related to polynomial solutions. We present an algorithmic approach for computing polynomial solutions and the index for a class of linear operators including integro-differential operators. A generating set for right annihilators can be constructed in terms of such polynomial solutions. For initial value problems, an involution of the algebra of integro-differential operators also allows us to compute left annihilators, which can be interpreted as compatibility conditions of integro-differential equations with boundary conditions. These results are implemented in MAPLE based on the `IntDiffOp` and `IntDiffOperations` packages. Finally, system-theoretic interpretations of these results are given and illustrated on integro-differential equations.

In [78], we develop linear algebra results needed for generalizing the composition of boundary problems to singular ones. We consider generalized inverses of linear operators and study the question when their product in reverse order is again a generalized inverse. This problem has been studied for various kinds of generalized inverses, especially for matrices. Motivated by our application to boundary problems, we use implicit representation of subspaces via "boundary conditions" from the dual space and this approach gives a new representation of the product of generalized inverses. Our results apply to arbitrary vector spaces and for Fredholm operators, the corresponding computations reduce to finite-dimensional problems, which is crucial for our implementation for boundary problem for linear ordinary differential equations.

In collaboration with Li Guo and Markus Rosenkranz [77], we study algebraic aspects of integro-differential algebras and their relation to so-called differential Rota-Baxter algebras. We generalize this concept to that of integro-differential algebras with weight. Based on free commutative Rota-Baxter algebras, we investigate the construction of free integro-differential algebras with weight generated by a regular differential algebra. The explicit construction is not only interesting from an algebraic point of view but is also an important step for algorithmic extensions of differential algebras to integro-differential algebras (compare with the related construction and the implementation of integro-differential polynomials in [72]). In this paper, we review also the construction of integro-differential operators, the algorithms for regular boundary problems and a prototype implementation in the Theorema system.

In [11], we adapt our factorization technique for boundary problems to study ruin probabilities and related quantities in renewal risk theory. The analysis is based on boundary problems for linear ordinary differential equations (on the half bounded interval from zero to infinity) with variable coefficients and the corresponding factorization of Green's operators. With this approach, we obtain closed-form and asymptotic expressions for discounted penalty functions under the more realistic assumption that the premium income depends on the present surplus of the insurance portfolio.

6.3. Symbolic methods for developing new domain decomposition algorithms

Participants: Thomas Cluzeau [ENSIL, Univ. Limoges], Victorita Dolean [Univ. Nice - Sophia-Antipolis], Frédéric Nataf [CNRS, Paris 6], Alban Quadrat.

Some algorithmic aspects of systems of partial differential equations based simulations can be better clarified by means of symbolic computation techniques. This is very important since numerical simulations heavily rely on solving systems of partial differential equations. For the large-scale problems we deal with in today's standard applications, it is necessary to rely on iterative Krylov methods that are scalable (i.e., weakly dependent on the number of degrees on freedom and number of subdomains) and have limited memory requirements. They are preconditioned by domain decomposition methods, incomplete factorizations and multigrid preconditioners. These techniques are well understood and efficient for scalar symmetric equations (e.g., Laplacian, biLaplacian) and to some extent for non-symmetric equations (e.g., convection-diffusion). But they have poor performances and lack robustness when used for symmetric systems of partial differential equations, and even more so for non-symmetric complex systems (fluid mechanics, porous media, ...). As a general rule, the study of iterative solvers for systems of partial differential equations as opposed to scalar

partial differential equations is an underdeveloped subject. In [76], we aim at building new robust and efficient solvers, such as domain decomposition methods and preconditioners for some linear and well-known systems of partial differential equations based on algebraic techniques (e.g., Smith normal forms, Gröbner basis techniques, D-modules).

6.4. Noncommutative geometry approach to infinite-dimensional systems

Participant: Alban Quadrat.

In [105], [104], [103], it was shown how the fractional representation approach to analysis and synthesis problems developed by Vidyasagar, Desoer, Callier, Francis, Zames..., could be recast into a modern algebraic analysis approach based on module theory (e.g., fractional ideals, algebraic lattices) and the theory of Banach algebras. This new approach successfully solved open questions in the literature. Basing ourselves on this new approach, we explain in [107] why the non-commutative geometry developed by Alain Connes is a natural framework for the study of stabilizing problems of infinite-dimensional systems. Using the 1-dimensional quantized calculus developed in non-commutative geometry and results obtained in [105], [104], [103], we show that every stabilizable system and their stabilizing controllers naturally admit geometric structures such as connections, curvatures, Chern classes, ... These results developed in [59] are the first steps toward the use of the natural geometry of the stabilizable systems and their stabilizing controllers in the study of the important H_∞ and H_2-problems.

6.5. Stabilization of time-delay systems

Participants: Alban Quadrat, Arnaud Quadrat [SAGEM, MASSY].

In [60], we study the stabilization problem of a linear system formed by a simple integrator and a time-delay. We show that the stabilizing controllers of such a system can be be rewritten as the closed-loop system defined by the stabilizing controllers of the simple integrator and a distributed delay. This result is used to study tracking problems appearing in the study of inertially stabilized platforms for optical imaging systems.

6.6. Stabilization of MISO fractional systems with delays

Participants: Catherine Bonnet, Le Ha Vy Nguyen.

In order to yield the set of all the stabilizing controllers of a class of MISO fractional systems with delays by mean of Youla-Kucera parametrization regarding H_∞-stability, we are interested in determining coprime factorizations of the transfer function. Explicit expressions of left coprime factorizations and left Bézout factors are derived in [51]. On the other hand, right coprime factorizations exist, and we have obtained explicit expressions for several particular cases of the studied systems.

6.7. Stability analysis of (fractional) neutral systems with commensurate delays

Participants: Catherine Bonnet, Andre Fioravanti [UNICAMP], Le Ha Vy Nguyen.

Neutral time-delay systems may have chains of poles asymptotic to the imaginary axis. As the chains approach the axis, some systems are H_∞-unstable even though all the poles are in the left-half plane. For a class of such systems, H_∞-stability conditions were presented in [84]. While systems with no more than one chain of poles asymptotic to a set of points on the imaginary axis were exhaustedly studied, only a particular case of systems with multiple chains were considered. We continue the stability analysis for more general cases of the latter systems. Primary results on pole locations are obtained [53], [52]. Based on these results, H_∞-stability conditions have also been derived.

6.8. Matrix Norm Approach for Control of Linear Time-Delay Systems

Participants: Catherine Bonnet, André Fioravanti [UNICAMP], José Claudio Geromel [UNICAMP], Silviu Niculescu.

In [94], we have treated the time-delay linear systems control design in the framework of complete and partial information. We were able to find linear controllers that increase the first stability window imposing at the same time that the delay-free system is stable using some properties about the norms of the state-space matrices. Our method treated the design problem by numeric routines based on Linear Matrix Inequalities (LMI) arisen from classical linear time invariant system theory coupled together with a unidimensional search. Both the state and output feedback design, were solved. We have this year tried our method on a 'high-dimensional' example for which no existing direct method would be computationnally feasible.

6.9. Interval observer

Participants: Frederic Mazenc, Silviu Niculescu, Thach Ngoc Dinh, Olivier Bernard [Inria - Sophia-Antipolis], Eric Walter [CNRS - L2S - Supelec], Michel Kieffer [CNRS - L2S - Supelec].

We made several progresses in the domain of the construction of state estimators called interval observers.
1) We presented the design of families of interval observers for continuous-time linear systems with a point-wise delay after showing that classical interval observers for systems without delays are not robust with respect to the presence of delays and that, in general, for linear systems with delay, the classical interval observers endowed with a point-wise delay are unstable. We proposed a new type of design of interval observers enabling to circumvent these obstacles. It incorporates distributed delay terms [26].

2) We considered a family of continuous-time systems that can be transformed through a change of coordinates into triangular systems. By extensively using this property, we constructed interval observers for nonlinear systems which are not cooperative and not globally Lipschitz. For a narrower family of systems, the interval observers possess the Input to State Stability property with respect to the bounds of the uncertainties [42], [21].

3) For the first time, we addressed in [44] the problem of constructing interval observers for discrete-time systems. Under a strong assumption, we proposed time-invariant interval observers for a very broad family of systems. In a second step, we have shown that, for any time-invariant exponentially stable discrete-time linear system with additive disturbances, time-varying exponentially stable discrete-time interval observers can be constructed. The latter result relies on the design of time-varying changes of coordinates which transform a linear system into a nonnegative one.

4) We considered continuous-time linear systems with additive disturbances and discrete-time measurements. First, we constructed a standard observer, which converges to the state trajectory of the linear system when the maximum time interval between two consecutive measurements is sufficiently small and there are no disturbances. Second, we constructed interval observers allowing to determine, for any solution, a set that is guaranteed to contain the actual state of the system when bounded disturbances are present [46].

6.10. New reduction model approach

Participants: Frederic Mazenc, Silviu Niculescu, Mounir Bekaik, Dorothee Normand-Cyrot [CNRS - L2S - Supelec], Claudio de Persis [Sapienza University of Rome], Miroslav Krstic [Univ. of California].

We considered several distinct problems entailing to the reduction model approach. Let us recall that this technique makes it possible to stabilize systems with arbitrarily large pointwise or distributed delay.

1) We proposed a new construction of exponentially stabilizing sampled feedbacks for continuous-time linear time-invariant systems with an arbitrarily large constant pointwise delay in the inputs. Stability is guaranteed under an assumption on the size of the largest sampling interval. The proposed design is based on an adaptation of the reduction model approach. The stability of the closed loop systems is proved through a Lyapunov-Krasovskii functional of a new type, from which is derived a robustness result [28], [50].

2) For linear systems with pointwise or distributed delays in the inputs which are stabilized through the reduction approach, we proposed a new technique of construction of Lyapunov-Krasovskii functionals. These functionals allow us to establish the ISS property of the closed-loop systems relative to additive disturbances [27], [49].

3) We proposed a solution to the problem of stabilizing nonlinear systems with input with a constant pointwise delay and state-dependent sampling. It relies on a recursive construction of the sampling instants and on a recent variant of the classical reduction model approach. The state feedbacks that are obtained do not incorporate distributed terms [43].

6.11. Analysis of neutral systems

Participants: Frederic Mazenc, Hiroshi Ito [Kyushu Institute of Technology].

1) For nonlinear systems with delay of neutral type, we developped a new technique of stability and robustness analysis. It relies on the construction of functionals which make it possible to establish estimates of the solutions different from, but very similar to, estimates of ISS or iISS type. These functionals are themselves different from, but very similar to, ISS or iISS Lyapunov-Krasovskii functionals. The approach applies to systems which do not have a globally Lipschitz vector field and are not necessarily locally exponentially stable. We apply this technique to carry out a backstepping design of stabilizing control laws for a family of neutral nonlinear systems [22], [45].

2) We extended the previous result to the problem of deriving the iISS property for dynamical networks with neutral, retarded and communication delay [41].

6.12. Hyperbolic systems

Participants: Frederic Mazenc, Christophe Prieur [GIPSA-Lab CNRS].

We considered a family of time-varying hyperbolic systems of balance laws. The partial differential equations of this family can be stabilized by selecting suitable boundary conditions. For the stabilized systems, the classical technique of construction of Lyapunov functions provides a function whose derivative along the trajectories of the systems may be not negative definite. In order to obtain a Lyapunov function with a negative definite derivative along the trajectories, we transform this function through a so-called "strictification" approach, which gives a time-varying strict Lyapunov function. It allows us to establish asymptotic stability in the general case and a robustness property with respect to additive disturbances of Input-to-State Stability type [32].

6.13. Time-varying systems with delay

Participants: Frederic Mazenc, Silviu Niculescu, Mounir Bekaik, Michael Malisoff [Departement of Mathematics - LSU].

1) We solved aproblem of state feedback stabilization of time-varying feedforward systems with a pointwise delay in the input. The approach relies on a time-varying change of coordinates and Lyapunov-Krasovskii functionals. The result applies for any given constant delay, and provides uniformly globally asymptotically stabilizing controllers of arbitrarily small amplitude. The closed-loop systems enjoy Input-to-State Stability properties with respect to additive uncertainty on the controllers. The work is illustrated through a tracking problem for a model for high level formation flight of unmanned air vehicles [48], [24].

2) We addressed the problem of stabilizing systems belonging to a family of time-varying nonlinear systems with distributed input delay through state feedbacks without retarded term. The approach we adopted is based on a new technique that is inspired by the reduction model technique. The control laws we obtained are nonlinear and time-varying. They globally uniformly exponentially stabilize the origin of the considered system. We illustrate the construction with a networked control system [25].

6.14. Positive invariance for time delay systems

Participants: Sorin Olaru [correspondent], Silviu Niculescu [CNRS (LSS)], Georges Bitsoris [University of Patras, Greece].

A new concept of positive invariance has been established in the original state space for discrete time dynamical systems. Furthermore, the necessary and sufficient algebraic condition for such properties have been derived allowing a direct test using basic linear programming arguments. In a recent work, the rigid positive invariance has been relaxed toward a cyclic invariant concept [18].

6.15. Predictive control for networked control systems

Participants: Sorin Olaru [correspondent], Silviu Niculescu [CNRS (LSS)], Warody Lombardi [INSA Lyon].

The work on the networked control system modeling lead to the establishement of a solid framework based on linear difference inclusion. Subsequently via set invariance and optimization based techniques, a design procedure has been proposed to deal with the real time constrained feedback control. Is worth to be mentioned that the robust feasibility and control performances are enforced via inverse optimality principles [19].

6.16. Reduced order H_∞-controllers synthesis with explicit constraints handling

Participants: Guillaume Sandou [correspondent], Gilles Duc [Suplec (E3S), Control Department], Mohamed Yagoubi [Ecole des Mines de Nantes].

Efficient dedicated methods have been developed for Hinfinity controller synthesis. However, such methods require translating the design objectives using weighting filters, whose tuning is not easy; in addition they lead to high order controllers which have to be reduced. Previous works have dealt with these two problems separately with the help of Particle Swarm Optimization: optimization of filter tunings for a full order synthesis and reduced order synthesis with fixed filters. In recent works, we have considered the solution to both problems in one shot. The constraints of the problem are explicitly taken into account in the synthesis problem, thanks to the use of Particle swarm optimization which does not require any specific expression for costs and constraints [63].

6.17. Robust optimization for energy management

Participants: Guillaume Sandou [correspondent], Philippe Dessante [Suplec (E3S), Energy Department], Marc Petit [Suplec (E3S), Energy Department].

The optimization of energy networks and the solution to Unit Commitment problems are one of the main collaborations between the Control and Energy Departments of Supelec. Robust optimization has been used to take into account the uncertainties which are observed on the consumer demand, the cost function, and the maximum capacity [66], [73].

6.18. Firefly optimization for the synthesis of controllers and the identification of systems

Participants: Guillaume Sandou [correspondent], Alfonso Goches Sanchez [Suplec (E3S), Control Department].

Firefly optimization is a new optimization algorithm which has appeared in 2009. This algorithm belongs to the class of metaheuristic algorithms. As such algorithms can optimized any cost and functions, firefly optimization has been tested for the optimization of PID controllers (with no reformulations of specifications) and the identification of nonlinear systems.

6.19. Receding horizon based controllers for the energy management in complex systems

Participants: Guillaume Sandou [correspondent], Sorin Olaru, Silviu Niculescu, Emmanuel Witrant [GipsLab, Grenoble].

The use of receding horizon based controllers is a good trend to extend the optimization results of a complex system in a closed loop framework. To prove the viability and the efficiency of the approach, several real life examples have been tested. Among them are the district heating networks and the mining ventilation system.

6.20. Particle Swarm Optimization for the optimization of feasibility domain volumes

Participants: Guillaume Sandou [correspondent], Mohamad-Taki Asghar [Suplec (E3S), Control Department].

It is a well-known fact that using mu-analysis for the computation of a guaranteed stability domain gives the largest hyper-rectangle included in the real stability domain (which is impossible to compute). However, the results strongly depend on the choice which has been made for the nominal system and the parameterization of the uncertainties. In this study, these choices are considered as optimization variables. The goal is now to find the best parameterization of the problem to get the largest stability domain. The optimization has been done using Particle Swarm Optimization.

6.21. Model of reaction networks

Participants: Georg Regensburger, Stefan Müller [RICAM, Linz].

In [100], we propose a notion of generalized mass action systems that could serve as a more realistic model for reaction networks in intracellular environments; classical mass action systems capture chemical reaction networks in homogeneous and dilute solutions. We show that several results of chemical reaction network theory carry over to the case of generalized mass action kinetics. Our main result gives conditions for the existence of a unique positive steady state for arbitrary initial conditions and independent of rate constants in this generalized setting. The conditions are formulated in terms of sign vectors (oriented matroids) of the stoichiometric and kinetic-order subspace and face lattices of related cones. We also give necessary and sufficient conditions for multistationarity, which is an important property in many applications, for example, in connection with cell differentiation.

6.22. Control of aircraft dynamics

Participants: Frederic Mazenc, Michael Malisoff [Departement of Mathematics - LSU], Aleksandra Gruszka [Departement of Mathematics - LSU].

We have worked on several models describing physical devices.

1) We studied a kinematic model that is suitable for control design for high level formation flight of UAVs [16], [40]. We designed controllers that give robust global tracking for a wide class of reference trajectories in the sense of input-to-state stability while satisfying amplitude and rate constraints on the inputs.

2) We studied feedback tracking problems for the planar vertical takeoff and landing (PVTOL) aircraft dynamics, which is a benchmark model in aerospace engineering. We provided a survey of the literature on the model. Then we constructed new feedback stabilizers for the PVTOL tracking dynamics. The novelty of our work is in the boundedness of our feedback controllers and their applicability to cases where the velocity measurements may not be available, coupled with the uniform global asymptotic stability and uniform local exponential stability of the closed loop tracking dynamics, and the input-to-state stable performance of the closed loop tracking dynamics with respect to actuator errors [15].

3) We solved a stabilization problem for an important class of feedback controllers that arise in curve tracking problems for robotics. Previous experimental results suggested the robust performance of the control laws under perturbations. Consequently, we used input-to-state stability to prove predictable tolerance and safety bounds that ensure robust performance under perturbations and time delays. Our proofs are based on an invariant polygon argument and a new strict Lyapunov function design [20].

6.23. Study of chemostat models

Participants: Frederic Mazenc, Michael Malisoff [Departement of Mathematics - LSU].

We provided a study of chemostat models in which two or more species compete for two or more limiting nutrients. First we considered the case where the nutrient flow and species removal rates and input nutrient concentrations are all given positive constants. In that case, we used Brouwer fixed point theory to give conditions guaranteeing that the models admit globally asymptotically stable componentwise positive equilibrium points. For cases where the dilution rate and input nutrient concentrations can be selected as controls, we used Lyapunov methods to prove that many different possible componentwise positive equilibria can be made globally asymptotically stable. We demonstrated our methods in simulations [23].

6.24. Modeling and control of Acute Myeloid Leukemia

Participants: José Luis Avila Alonso, Annabelle Ballesta [BANG project-team], Frédéric Bonnans [COMMANDS project-team], Catherine Bonnet, Jean Clairambault [BANG project-team], Xavier Dupuis [COMMANDS project-team], Pierre Hirsch [INSERM Paris (Team18 of UMR 872) Cordeliers Research Centre and St. Antoine Hospital, Paris], Jean-Pierre Marie [INSERM Paris (Team18 of UMR 872) Cordeliers Research Centre and St. Antoine Hospital, Paris], Faten Merhi [INSERM Paris (Team18 of UMR 872) Cordeliers Research Centre and St. Antoine Hospital, Paris], Silviu Niculescu, Hitay Özbay [Bilkent University, Ankara, Turkey], Ruoping Tang [INSERM Paris (Team18 of UMR 872) Cordeliers Research Centre and St. Antoine Hospital, Paris].

We have continued this year our work on modeling healthy and pathological hematopoiesis [36]. A. Ballesta has performed some experiments on patient fresh cell cultures in order to identify parameters of our model of acute myeloblastic leukemia (AML). To evaluate therapies, she also considered patient fresh cell cultures under anticancer drugs.

GALEN Team

6. New Results

6.1. Machine Learning & Optimization

Participants: Andreas Argyriou, Matthew Blaschko, Pawan Kumar.

- **Sparse Prediction & Convex Optimization Decomposition** [*Andreas Argyriou*]
  In [36], we have introduced a new regularization penalty for sparse prediction, the k-support norm. This norm corresponds to the tightest convex relaxation of sparsity combined with an ℓ_2 penalty. We have shown that this new norm provides a tighter relaxation than the elastic net, and is thus a good replacement for the Lasso or the elastic net in sparse prediction problems. In [41], motivated by learning problems we proposed a novel optimization algorithm for minimizing a convex objective which decomposes into three parts: a smooth part, a simple non-smooth Lipschitz part, and a simple non-smooth non-Lipschitz part.
- **Learning Optimization for NP-complete Inference** [*Matthew Blaschko*]
  In [14] an optimization strategy for learning to optimize boolean satisfiability (SAT) solvers is given. Applications to real-world SAT problems show improved computational performance as a result of the learning algorithm.
- **Max-Margin Min-Entropy Models & Dissimilarity Coefficient based Learning** [*Pawan Kumar*]
  In [22] we proposed the family of max-margin min-entropy (M3E) models, which predicts a structured output for a given input by minimizing the Renyi entropy. The parameters of M3E are learned by minimizing an upper bound on a user-defined loss. We demonstrated the efficacy of M3E on two problems using publicly available datasets: motif finding and image classification. In [19] we proposed a novel structured prediction framework for weakly supervised datasets. The framework minimizes a dissimilarity coefficient between the predictor and a conditional distribution over the missing information. We demonstrated the efficacy of our approach on two problems using publicly available datasets: object detection and action detection.

6.2. Computational Vision & Perception

Participants: Matthew Blaschko, Iasonas Kokkinos, Pawan Kumar, Nikos Paragios.

- **Structured Output Ranking & Detailed Understanding of Objects in Computer Vision** [*Matthew Blaschko*]
  In [23] we proposed a novel method for efficiently optimizing an objective that ranks structured outputs by their loss. Based on the observation that structured output spaces [9] in computer vision problems can be well-modeled by a small number of loss values, our algorithm is able to optimize a quadratic number of pairwise constraints in linear time. In [38] we detail the research activities of a summer workshop hosted by Johns Hopkins University on learning a detailed understanding of objects and scenes in natural images. We worked on automatic verification of annotations provided through Amazon Mechanical Turk [35], texture categorization, and dependence modeling for bottom up proposals.
- **Efficient inference and learning for structured probabilistic models of deformable objects** [*Iasonas Kokkinos*, Haithem Boussaid & Stavros Tsogkas]

We have developed novel features to describe surface points intrinsically through the Intrinsic Shape Context (ISC) descriptor published in [17]. This method has delivered state-of-the-art results in surface point matching and we will explore its use for surface correspondence. The implementation of these descriptors is publicly available. In [32] we proposed a learning-based approach to symmetry detection by fusing multiple cues related to image intensity, color and texture, which delivered state-of-the-art results. We intend to extend this approach to 3D image analysis, and in particular for medical images. The implementation of these detectors is publicly available. In [27] we introduce a grouping-based method to learn and detect action classes in spatio-temporal data. Our method can both classify actions and indicate the spatio-temporal structures which provide support for the decision. The implementation of our front-end is publicly available. In [40] we have extended our work on efficient algorithms for object detection to accommodate fast methods for computing the part scores in a principled optimization framework, while he have thoroughly presented it in [40] and made the implementation publicly available.

- **Multi-view Image Segmentation & Parsing** [*Nikos Paragios*]
  In [28] a method for image matching was proposed that exploits hierarchical image representations through higher order graphs. The matching was achieved through a graph-based theoretical framework where the similarity and spatial consistency of the image semantic objects is encoded in a graph of commute times that is also endowed with singleton terms through shape descriptors. Many-to-many matching of regions are specially challenging due to the instability of the segmentation under slight image changes, and we explicitly handle it through high order potentials. These ideas were further explored in the context of co-segmentation [29] where a method to determine a consistent partition of multiple images was introduced through a multi-scale multiple-image generative model based on region matching that exploits inter-image information and establishes correspondences between the common objects that appear in the scene. Last, but not least in [24] a method that combines bottom up (visual information, visual descriptors, elements detection) information and top-town models (hierarchical shape grammars) was considered towards automatic facade parsing though reinforcement learning while in [30] a method for 3D image parsing was proposed based on a hierarchical grammar that was performing explicit 3D modeling of the scene through a combination of multi-image segmentation and a depth reconstruction process. The problem optimal combination of these two concurrent terms was addressed trough a pareto-driven criterion while the optimization was addressed through an evolutionary computation algorithm.

6.3. Biomedical Image Analysis

Participant: Nikos Paragios.

- **Image Reconstruction** [*Nikos Paragios* & Hellene Langet]
  In [21] a novel iterative reconstruction algorithm based on compressed sensing was proposed for Digital Subtraction Rotational Angiography (DSRA) that exploits both spatial and temporal sparsity through a proximal implementation that accommodates multiple $L-1$-penalties. These ideas was further explored in [20] where we introduced a three-dimensional reconstruction of tomographic acquisitions in C-arm-based rotational angiography was proposed that was able to deal with the temporal variations due to intra-arterial injections through a compressed-sensing approach leading to significant motion artifacts reduction in spite of the cone-beam geometry, the short-scan acquisition, and the truncated and subsampled data.
- **Image Segmentation** [*Nikos Paragios*, Pierre-Yves Baudin, Xiang Bo & Sarah Parisot]
  In [11] the problem of human skeletal muscle segmentation was considered through a graph-based approach (random walker). An automatic seed placement framework was introduced through a graph-theoretic formulation. Towards accounting for anatomical constraints, the Random Walker algorithm was endowed with a liner sub-space statistical prior towards improving segmentation robustness on missing and incomplete data [12]. The same formulation was extended to cope with non-linear priors through a Gaussian-like local prior model penalizing the deviations of the

coefficients of the random walker diffusion matrix from the ones learned from the training data [13]. In [25] a novel graph-based prior was considered towards modeling the distribution of low-glioma brain tumors and spatially characterizing them through a sparse hierarchical graph. Such a prior model was integrated to an image-driven voxel-like segmentation framework where image separation was achieved through a machine learning method towards automatic detection, characterization and segmentation of brain tumors. Furthermore, towards encoding pose invariance in the context of knowledge-based segmentation in [33] where a higher order graph-based implicit pose invariant formulation was introduced for cardiac segmentation. The formulation was endowed with higher order cliques allowing (i) the estimation of boundary and regional image support and (ii) the implicit modeling of local deformations with respect to a prior statistical model while being invariant to linear transformations.

- **Image Registration** [*Nikos Paragios*, Nicolas Honnorat & Sarah Parisot]
  In [15] the problem of organ-driven registration was addressed through simultaneous combined fusion of multi-modal images in the context of guide-wire segmentation through fluoroscopic and contrast enhanced images. To this end, a graphical model was considered that was segmenting and registering the guide-wire in the two modalities while establishing correspondences between the associated curves as well. Similar philosophy was used in the [26] where a method for one shot deformable brain registration and tumor segmentation was proposed between a healthy anatomical atlas and a diseased patient. Both tasks were addressed through a discrete formulation (pair-wise MRF using grid-like deformation models and machine learning discriminative frameworks for the separation of healthy versus diseased tissues) while interconnections between the two graphs were used to alleviate the registration requirement on tumor areas. The problem of symmetric registration was studied in [31] through a common grid deforming in both directions according to a symmetric manner towards minimizing the image similarity criterion between the source and the target image while guaranteeing the expected diffeomorphic nature of the deformation field.
- **Computational Anatomy** [*Nikos Paragios*]
  In [16] we introduced a novel approach for detecting the presence of white matter lesions in periventricular areas of the brain using manifold-constrained embeddings. The proposed method uses locally linear embedding (LLE) to create ”normality” distributions of the brain where deviations from the manifolds are estimated by calculating geodesic distances along locally linear planes in the embedding. Experiments highlight the need of nonlinear techniques to learn the studied data leading to outstanding detection rates when comparing individuals to a specific pathological pattern.

GECO Team

6. New Results

6.1. New results: geometric control

We start by presenting some results on the design of motion planning and tracking algorithms.

- In [10] we present an iterative steering algorithm for nonholonomic systems (also called driftless control-affine systems) and we prove its global convergence under the sole assumption that the Lie Algebraic Rank Condition (LARC) holds true everywhere. That algorithm is an extension of the one introduced in [65] for regular systems. The first novelty here consists in the explicit algebraic construction, starting from the original control system, of a lifted control system which is regular. The second contribution of the paper is an exact motion planning method for nilpotent systems, which makes use of sinusoidal control laws and which is a generalization of the algorithm described in [83] for chained-form systems.
- [6] and [5] are about motion planning for kinematic systems, and more particularly ε-approximations of non-admissible trajectories by admissible ones. This is done in a certain optimal sense. The resolution of this motion planing problem is showcased through the thorough treatment of the ball with a trailer kinematic system, which is a non-holonomic system with flag of type $(2, 3, 5, 6)$.

Application-oriented results about motion planning are contained in [15]. The paper proposes in particular a strategy for providing Unmanned Aerial Vehicles with a certain degree of autonomy, via autonomous planification/replanification strategies.

Let us list some new results in sub-Riemannian geometry and hypoellitpic diffusion.

- In [1] we study the Radon-Nikodym derivative of the spherical Hausdorff measure with respect to a smooth volume for a regular sub-Riemannian manifold . We prove that this is the volume of the unit ball in the nilpotent approximation and it is always a continuous function. We then prove that up to dimension 4 it is smooth, while starting from dimension 5, in corank 1 case, it is C^3 (and C^4 on every smooth curve) but in general not C^5. These results answer to a question addressed by Montgomery about the relation between two intrinsic volumes that can be defined in a sub-Riemannian manifold, namely the Popp and the Hausdorff volume. If the nilpotent approximation depends on the point (that may happen starting from dimension 5), then they are not proportional, in general.
- In [9] we study the Laplace–Beltrami operator on generalized Riemannian structures on orientable surfaces for which a local orthonormal frame is given by a pair of vector fields that can become collinear. Under the assumption that the structure is 2-step Lie bracket generating, we prove that the Laplace–Beltrami operator is essentially self-adjoint and has discrete spectrum. As a consequence, a quantum particle cannot cross the singular set (i.e., the set where the vector fields become collinear) and the heat cannot flow through the singularity.
- For an equiregular sub-Riemannian manifold M, Popp's volume is a smooth volume which is canonically associated with the sub-Riemannian structure, and it is a natural generalization of the Riemannian one. In [4] we prove a general formula for Popp's volume, written in terms of a frame adapted to the sub-Riemannian distribution. As a first application of this result, we prove an explicit formula for the canonical sub-Laplacian, namely the one associated with Popp's volume. Finally, we discuss sub-Riemannian isometries, and we prove that they preserve Popp's volume. We also show that, under some hypotheses on the action of the isometry group of M, Popp's volume is essentially the unique volume with such a property.

- In [21], for a sub-Riemannian manifold provided with a smooth volume, we relate the small time asymptotics of the heat kernel at a point y of the cut locus from x with roughly "how much" y is conjugate to x. This is done under the hypothesis that all minimizers connecting x to y are strongly normal, i.e. all pieces of the trajectory are not abnormal. Our result is a refinement of the one of Leandre $4t \log p_t(x,y) \to -d^2(x,y)$ for $t \to 0$, in which only the leading exponential term is detected. Our results are obtained by extending an idea of Molchanov from the Riemannian to the sub-Riemannian case, and some details we get appear to be new even in the Riemannian context. These results permit us to obtain properties of the sub-Riemannian distance starting from those of the heat kernel and vice versa. For the Grushin plane endowed with the Euclidean volume we get the expansion $p_t(x,y) \sim t^{-5/4} \exp\left(-d^2(x,y)/4t\right)$ where y is reached from a Riemannian point x by a minimizing geodesic which is conjugate at y. In [22] we investigate the small time heat kernel asymptotics on the cut locus on the class of two-spheres of revolution, which is the simplest class of 2-dimensional Riemannian manifolds different from the sphere with nontrivial cut-conjugate locus. We determine the degeneracy of the exponential map near a cut-conjugate point and present the consequences of this result to the small time heat kernel asymptotics at this point. These results give a first example where the minimal degeneration of the asymptotic expansion at the cut locus is attained.
- In [24] we studied normal forms for 2-dimensional almost-Riemannian structures. The latter are generalized Riemannian structures on surfaces for which a local orthonormal frame is given by a Lie bracket generating pair of vector fields that can become collinear. Generically, there are three types of points: Riemannian points where the two vector fields are linearly independent, Grushin points where the two vector fields are collinear but their Lie bracket is not, and tangency points where the two vector fields and their Lie bracket are collinear and the missing direction is obtained with one more bracket. In [24] we consider the problem of finding normal forms and functional invariants at each type of point. We also require that functional invariants are complete, in the sense that they permit to recognize locally isometric structures. The problem happens to be equivalent to the one of finding a smooth canonical parameterized curve passing through the point and being transversal to the distribution. For Riemannian points such that the gradient of the Gaussian curvature K is different from zero, we use the level set of K as support of the parameterized curve. For Riemannian points such that the gradient of the curvature vanishes (and under additional generic conditions), we use a curve which is found by looking for crests and valleys of the curvature. For Grushin points we use the set where the vector fields are parallel. Tangency points are the most complicated to deal with. The cut locus from the tangency point is not a good candidate as canonical parameterized curve since it is known to be non-smooth. Thus, we analyse the cut locus from the singular set and we prove that it is not smooth either. A good candidate happens to be a curve which is found by looking for crests and valleys of the Gaussian curvature. We prove that the support of such a curve is uniquely determined and has a canonical parametrization.

6.2. New results: quantum control

New results have been obtained for the control of the bilinear Schrödinger equation.

- In [16] we obtained a sufficient condition for approximate controllability of the bilinear discrete-spectrum Schrödinger equation exploiting the use of more than one control. The controllability result extends to simultaneous controllability, approximate controllability in H^s, and tracking in modulus. The result is more general than those present in the literature even in the case of one control and permits to treat situations in which the spectrum of the uncontrolled operator is very degenerate (e.g. multiple eigenvalues or presence of equal gaps among eigenvalues). These results are applied to the case of a rotating polar linear molecule in the space, driven by three external fields. A remarkable property of this model is the presence of infinitely many degeneracies and resonances in the spectrum preventing the application of the results in the literature.
- In [19] we present a constructive method to control the bilinear Schrödinger equation by means of

two or three controlled external fields. The method is based on adiabatic techniques and works if the spectrum of the Hamiltonian admits eigenvalue intersections, with respect to variations of the controls, and if the latter are conical. We provide sharp estimates of the relation between the error and the controllability time.

- In [18] we consider the minimum time population transfer problem for a two level quantum system driven by two external fields with bounded amplitude. The controls are modeled as real functions and we do not use the Rotating Wave Approximation. After projection on the Bloch sphere, we tackle the time-optimal control problem with techniques of optimal synthesis on 2-D manifolds. Based on the Pontryagin Maximum Principle, we characterize a restricted set of candidate optimal trajectories. Properties on this set, crucial for complete optimal synthesis, are illustrated by numerical simulations. Furthermore, when the two controls have the same bound and this bound is small with respect to the difference of the two energy levels, we get a complete optimal synthesis up to a small neighborhood of the antipodal point of the starting point.

6.3. New results: neurophysiology

- In [17] we study the global properties of an optimal control model of geometry of vision due to Petitot, Citti and Sarti. In particular, we consider the problem of minimizing $\int_0^L \sqrt{\xi^2 + K^2(s)}\, ds$ for a planar curve having fixed initial and final positions and directions. The total length L is free. Here s is the variable of arclength parametrization, $K(s)$ is the curvature of the curve and $\xi > 0$ a parameter. The main feature of the problem is that, if for a certain choice of boundary conditions there exists a minimizer, then this minimizer is smooth and has no cusp. However, not for all choices of boundary conditions there is a global minimizer. We study existence of local and global minimizers for this problem. We prove that if for a certain choice of boundary conditions there is no global minimizer, then there is neither a local minimizer nor a stationary curve (geodesic). We give properties of the set of boundary conditions for which there exists a solution to the problem. Finally, we present numerical computations of this set.
- In [2] we studied the general problem of reconstructing the cost from the observation of trajectories, in a problem of optimal control. It is motivated by the problem of determining what is the cost minimized in human locomotion. This applied question is very similar to the following applied problem, concerning HALE drones: one would like them to decide by themselves for their trajectories, and to behave at least as a good human pilot. These starting points are the reasons for the particular classes of control systems and of costs under consideration. To summarize, our conclusion is that in general, inside these classes, three experiments visiting the same values of the control are needed to reconstruct the cost, and two experiments are in general not enough. The method is constructive. The proof of these results is mostly based upon the Thom's transversality theory.

6.4. New results: switched systems

- In [12] we study the phenomenon of polynomial instability of switched systems. Stability properties for continuous-time linear switched systems are at first determined by the (largest) Lyapunov exponent associated with the system, which is the analogue of the joint spectral radius for the discrete-time case. We provided a characterization of marginally unstable systems, i.e., systems for which the Lyapunov exponent is equal to zero and such that there exists an unbounded trajectory. We also analyzed the asymptotic behavior of their trajectories. Our main contribution consists in pointing out a resonance phenomenon associated with marginal instability. In the course of our study, we derived an upper bound of the state at time t, which is polynomial in t and whose degree is computed from the resonance structure of the system. We also derived analogous results for discrete-time linear switched systems.
- The paper [13] is concerned with the stability of planar linear singularly perturbed switched systems of the type $\dot{x}(t) = \sigma(t)A_1^\epsilon x(t) + (1-\sigma(t))A_2^\epsilon x(t)$, where $\sigma : [0, +\infty) \to \{0, 1\}$, A_1^ϵ and A_2^ϵ are real matrices which represent singularly perturbed modes. By ϵ we denote here the parameter of

singular perturbation. We propose a characterization of the stability properties of such singularly perturbed switched systems based on the results given in [47]. More generally, we study transitions as ϵ varies and we restrict their number and nature. Finally, we compare the results obtained in this way with the Tikhonov-type results for differential inclusions obtained in the literature.

GEOMETRICA Project-Team

6. New Results

6.1. Mesh Generation and Geometry Processing

6.1.1. New bounds on the size of optimal meshes

Participant: Donald Sheehy.

The theory of optimal size meshes gives a method for analyzing the output size (number of simplices) of a Delaunay refinement mesh in terms of the integral of a sizing function over the input domain. The input points define a maximal such sizing function called the feature size. This work aims to find a way to bound the feature size integral in terms of an easy to compute property of a suitable ordering of the point set. The key idea is to consider the pacing of an ordered point set, a measure of the rate of change in the feature size as points are added one at a time. In previous work, Miller et al. showed that if an ordered point set has pacing ϕ, then the number of vertices in an optimal mesh will be $O(\phi^d n)$, where d is the input dimension. We give a new analysis of this integral showing that the output size is only $\Theta(n + n \log \phi)$. The new analysis tightens bounds from several previous results and provides matching lower bounds. Moreover, it precisely characterizes inputs that yield outputs of size $O(n)$ [20].

6.1.2. State of the art in quad meshing

Participant: David Bommes.

Triangle meshes have been nearly ubiquitous in computer graphics, and a large body of data structures and geometry processing algorithms based on them has been developed in the literature. At the same time, quadrilateral meshes, especially semi-regular ones, have advantages for many applications, and significant progress was made in quadrilateral mesh generation and processing during the last several years. In this work, we discuss the advantages and problems of techniques operating on quadrilateral meshes, including surface analysis and mesh quality, simplification, adaptive refinement, alignment with features, parametrization, and remeshing [23].

6.1.3. Meshing the hyperbolic octagon

Participants: Mathieu Schmitt, Monique Teillaud.

We propose a practical method to compute a mesh of the octagon, in the Poincaré disk, that respects its symmetries. This is obtained by meshing the Schwartz triangle $T(8, 3, 2)$ and applying relevant hyperbolic symmetries (ie., Euclidean reflexions or inversions). The implementation is based on CGAL 2D meshes and on the ongoing implementation on CGAL hyperbolic Delaunay triangulations [44]. Further work will include solving robutsness issues and generalizing the method to any Schwartz triangle [62].

6.1.4. Index-based data structure for 3D polytopal complexes

Participant: David Bommes.

OpenVolumeMesh is a data structure which is able to represent heterogeneous 3-dimensional polytopal cell complexes and is general enough to also represent non-manifolds without incurring undue overhead [30]. Extending the idea of half-edge based data structures for two-manifold surface meshes, all faces, i.e. the two-dimensional entities of a mesh, are represented by a pair of oriented half-faces. The concept of using directed half-entities enables inducing an orientation to the meshes in an intuitive and easy to use manner. We pursue the idea of encoding connectivity by storing first-order top-down incidence relations per entity, i.e. for each entity of dimension d, a list of links to the respective incident entities is stored. For instance, each half-face as well as its orientation is uniquely determined by a tuple of links to its incident half-edges or each 3D cell by the set of incident half-faces. This representation allows for handling non-manifolds as well as mixed-dimensional mesh configurations. No entity is duplicated according to its valence, instead, it is shared by all incident entities in order to reduce memory consumption. Furthermore, an array-based storage layout is used in combination with direct index-based access. This guarantees constant access time to the entities of a mesh. Although bottom-up incidence relations are implied by the top-down incidences, our data structure provides the option to explicitly generate and cache them in a transparent manner. This allows for accelerated navigation in the local neighbor- hood of an entity. We provide an open-source and platform-independent implementation of the proposed data structure written in C++ using dynamic typing paradigms. The library is equipped with a set of STL compliant iterators, a generic property system to dynamically attach properties to all entities at run-time, and a serializer/deserializer supporting a simple file format. Due to its similarity to the OpenMesh data structure, it is easy to use, in particular for those familiar with OpenMesh. Since the presented data structure is compact, intuitive, and efficient, it is suitable for a variety of applications, such as meshing, visualization, and numerical analysis. OpenVolumeMesh is open-source software licensed under the terms of the LGPL [29].

6.1.5. *Editable SQuad representation for triangle meshes*

Participant: Olivier Devillers.

In collaboration with Luca Castelli Aleardi (LIX, Palaiseau) and Jarek Rossignac (Georgia Tech).

We consider the problem of designing space efficient solutions for representing the connectivity information of manifold triangle meshes. Most mesh data structures are quite redundant, storing a large amount of information in order to efficiently support mesh traversal operators. Several compact data structures have been proposed to reduce storage cost while supporting constant-time mesh traversal. Some recent solutions are based on a global re-ordering approach, which allows to implicitly encode a map between vertices and faces. Unfortunately, these compact representations do not support efficient updates, because local connectivity changes (such as edge-contractions, edge-flips or vertex insertions) require re-ordering the entire mesh. Our main contribution is to propose a new way of designing compact data structures which can be dynamically maintained. In our solution, we push further the limits of the re-ordering approaches: the main novelty is to allow to re-order vertex data (such as vertex coordinates), and to exploit this vertex permutation to easily maintain the connectivity under local changes. We describe a new class of data structures, called Editable SQuad (ESQ), offering the same navigational and storage performance as previous works, while supporting local editing in amortized constant time. As far as we know, our solution provides the most compact dynamic data structure for triangle meshes. We propose a linear-time and linear-space construction algorithm, and provide worst-case bounds for storage and time cost [25].

6.1.6. *Surface reconstruction through point set structuring*

Participants: Pierre Alliez, Florent Lafarge.

We present a method for reconstructing surfaces from point sets. The main novelty lies into a structure-preserving approach where the input point set is first consolidated by structuring and resampling the planar components, before reconstructing the surface from both the consolidated components and the unstructured points. The final surface is obtained through solving a graph-cut problem formulated on the 3D Delaunay triangulation of the structured point set where the tetrahedra are labeled as inside or outside cells. Structuring facilitates the surface reconstruction as the point set is substantially reduced and the points are enriched with structural meaning related to adjacency between primitives. Our approach departs from the common dichotomy between smooth/piecewise-smooth and primitive-based representations by gracefully combining

canonical parts from detected primitives and free-form parts of the inferred shape. Our experiments on a variety of inputs illustrate the potential of our approach in terms of robustness, flexibility and efficiency [59].

6.1.7. Feature-preserving surface reconstruction and simplification from defect-laden point sets

Participants: Pierre Alliez, David Cohen-Steiner, Julie Digne.

In collaboration with Fernando de Goes and Mathieu Desbrun from Caltech.

We introduce a robust and feature-capturing surface reconstruction and simplification method that turns an input point set into a low triangle-count simplicial complex. Our approach starts with a (possibly non-manifold) simplicial complex filtered from a 3D Delaunay triangulation of the input points. This initial approximation is iteratively simplified based on an error metric that measures, through optimal transport, the distance between the input points and the current simplicial complex, both seen as mass distributions. Our approach is shown to exhibit both robustness to noise and outliers, as well as preservation of sharp features and boundaries (Figure 1). Our new feature-sensitive metric between point sets and triangle meshes can also be used as a post-processing tool that, from the smooth output of a reconstruction method, recovers sharp features and boundaries present in the initial point set [58].

6.1.8. Similarity based filtering of point clouds

Participant: Julie Digne.

Denoising surfaces is a crucial step in the surface processing pipeline. This is even more challenging when no underlying structure of the surface is known, that is when the surface is represented as a set of unorganized points. We introduce a denoising method based on *local similarities*. The contributions are threefold: first, we do not denoise directly the point positions but use a low/high frequency decomposition and denoise only the high frequency. Second, we introduce a local surface parameterization which is proved stable. Finally, this method works directly on point clouds, thus avoiding building a mesh of a noisy surface which is a difficult problem. Our approach is based on denoising a height vector field by comparing the neighborhood of the point with neighborhoods of other points on the surface (Figure 2). It falls into the non-local denoising framework that has been extensively used in image processing, but extends it to unorganized point clouds [26].

6.1.9. Progressive compression of manifold polygon meshes

Participant: Pierre Alliez.

In collaboration with Adrien Maglo, Clément Courbet and Céline Hudelot from Ecole Centrale Paris.

We present a new algorithm for the progressive compression of surface polygon meshes. The input surface is decimated by several traversals that generate successive levels of detail through a specific patch decimation operator which combines vertex removal and local remeshing. This operator encodes the mesh connectivity through a transformation that generates two lists of Boolean symbols during face and edge removals. The geometry is encoded with a barycentric error prediction of the removed vertex coordinates. In order to further reduce the size of the geometry and connectivity data, we propose a curvature prediction method and a connectivity prediction scheme based on the mesh geometry. We also include two methods that improve the rate-distortion performance: a wavelet formulation with a lifting scheme and an adaptive quantization technique. Experimental results demonstrate the effectiveness of our approach in terms of compression rates and rate-distortion performance. Our approach compares favorably to compression schemes specialized to triangle meshes [31].

6.2. Topological and Geometric Inference

6.2.1. Homological reconstruction and simplification in $\mathbb{R}^3$

Participants: Olivier Devillers, Marc Glisse.

In collaboration with Dominique Attali (Gipsa-lab), Ulrich Bauer (Göttingen Univ.), and André Lieutier (Dassault Systèmes).

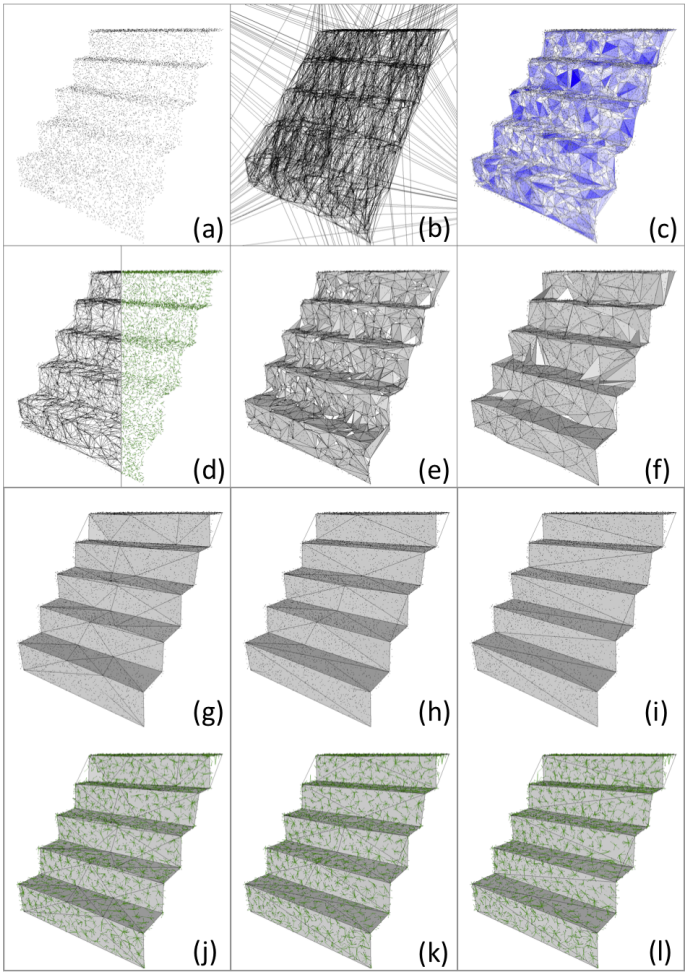

Figure 1. Steps of our algorithm: (a) Initial point set; (b) 3D Delaunay triangulation of a random subset containing 10% *of the input points; (c) Initial simplicial complex constructed from facets of the 3D triangulation with non-zero measure; (d) Initial transport plan assigning point samples to bin centroids (green arrows); (e-f) Intermediary decimation steps; (g-i) Reconstruction with 100, 50, and 22 vertices, respectively; (j-l) Final transport plan with 100, 50, and 22 vertices, respectively.*

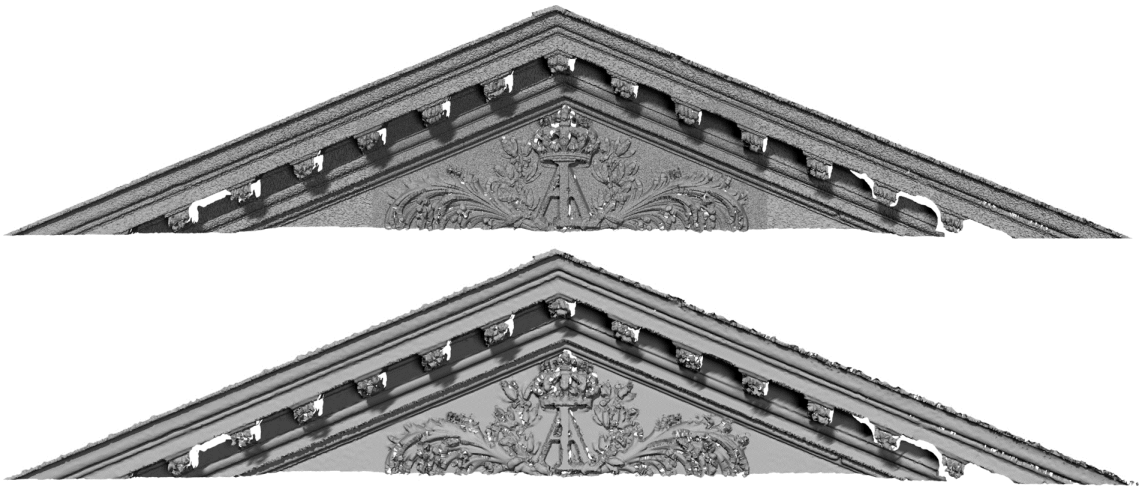

Figure 2. Similarity-based denoising. Top: input point set. Bottom: point set after denoising.

We consider the problem of deciding whether the persistent homology group of a simplicial pair (K, L) can be realized as the homology $H_*(X)$ of some space X with $L \subset X \subset K$. We show that this problem is NP-complete even if K is embedded in $\mathbb{R}^3$.

As a consequence, we show that it is NP-hard to simplify level and sublevel sets of scalar functions on $\mathbb{S}^3$ within a given tolerance constraint. This problem has relevance to the visualization of medical images by isosurfaces. We also show an implication to the theory of well groups of scalar functions: not every well group can be realized by some level set, and deciding whether a well group can be realized is NP-complete [43].

6.2.2. The structure and stability of persistence modules

Participants: Frédéric Chazal, Marc Glisse, Steve Oudot.

In collaboration with Vin de Silva (Pomona College)

We give a self-contained treatment of the theory of persistence modules indexed over the real line. We give new proofs of the standard results. Persistence diagrams are constructed using measure theory. Linear algebra lemmas are simplified using a new notation for calculations on quiver representations. We show that the stringent finiteness conditions required by traditional methods are not necessary to prove the existence and stability of the persistence diagram. We introduce weaker hypotheses for taming persistence modules, which are met in practice and are strong enough for the theory still to work. The constructions and proofs enabled by our framework are, we claim, cleaner and simpler [54].

6.2.3. Persistence stability for geometric complexes

Participants: Frédéric Chazal, Steve Oudot.

In collaboration with Vin de Silva (Pomona College)

We study the properties of the homology of different geometric filtered complexes (such as Vietoris–Rips, Čech and witness complexes) built on top of precompact spaces. Using recent developments in the theory of topological persistence [54] we provide simple and natural proofs of the stability of the persistent homology of such complexes with respect to the Gromov–Hausdorff distance. We also exhibit a few noteworthy properties of the homology of the Rips and Čech complexes built on top of compact spaces [53].

6.2.4. Zigzag zoology: rips zigzags for homology inference

Participants: Steve Oudot, Donald Sheehy.

For points sampled near a compact set X, the persistence barcode of the Rips filtration built from the sample contains information about the homology of X as long as X satisfies some geometric assumptions. The Rips filtration is prohibitively large, however zigzag persistence can be used to keep the size linear. We present several species of Rips-like zigzags and compare them with respect to the signal-to-noise ratio, a measure of how well the underlying homology is represented in the persistence barcode relative to the noise in the barcode at the relevant scales. Some of these Rips-like zigzags have been available as part of the Dionysus library for several years while others are new. Interestingly, we show that some species of Rips zigzags will exhibit less noise than the (non-zigzag) Rips filtration itself. Thus, the Rips zigzag can offer improvements in both size complexity and signal-to-noise ratio.

Along the way, we develop new techniques for manipulating and comparing persistence barcodes from zigzag modules. We give methods for reversing arrows and removing spaces from a zigzag. We also discuss factoring zigzags and a kind of interleaving of two zigzags that allows their barcodes to be compared. These techniques were developed to provide our theoretical analysis of the signal-to-noise ratio of Rips-like zigzags, but they are of independent interest as they apply to zigzag modules generally [60].

6.2.5. A space and time efficient implementation for computing persistent homology

Participants: Jean-Daniel Boissonnat, Clément Maria.

In collaboration with Tamal Dey (Ohio State University)

The persistent homology with Z_2-coefficients coincides with the same for cohomology because of duality. Recently, it has been observed that the cohomology based algorithms perform much better in practice than the originally proposed homology based persistence algorithm. We have implemented a cohomology based algorithm that attaches binary labels called annotations with the simplices. This algorithm fits very naturally with our recently developed data structure called simplex tree to represent simplicial complexes [49], [22]. By taking advantages of several practical tricks such as representing annotations compactly with memory words, using a union-find structure that eliminates duplicate annotation vectors, and a lazy evaluation, we save both space and time cost for computations. The complexity of the procedure, in practice, depends almost linearly on the size of the simplicial complex and on the variables related to the maximal dimension of the local homology groups we maintain during the computation, which remain small in practice. We provide a theoretical analysis as well as a detailed experimental study of our implementation. Experimental results show that our implementation performs several times better than the existing state-of-the-art software for computing persistent homology in terms of both time and memory requirements and can handle very large (several hundred million simplices in high-dimension) complexes efficiently [45].

6.2.6. Minimax rates for homology inference

Participant: Donald Sheehy.

In collaboration with Sivaraman Balakrishnan and Alessandro Rinaldo and Aarti Singh and Larry A. Wasserman (Carnegie Mellon University)

Often, high dimensional data lie close to a low-dimensional submanifold and it is of interest to understand the geometry of these submanifolds. The homology groups of a manifold are important topological invariants that provide an algebraic summary of the manifold. These groups contain rich topological information, for instance, about the connected components, holes, tunnels and sometimes the dimension of the manifold. We consider the statistical problem of estimating the homology of a manifold from noisy samples under several different noise models. We derive upper and lower bounds on the minimax risk for this problem. Our upper bounds are based on estimators which are constructed from a union of balls of appropriate radius around carefully selected points. In each case, we establish complementary lower bounds using Le Cam's lemma [15].

6.2.7. Linear-size approximations to the Vietoris-Rips filtration

Participant: Donald Sheehy.

The Vietoris-Rips filtration is a versatile tool in topological data analysis. Unfortunately, it is often too large to construct in full. We show how to construct an $O(n)$-size filtered simplicial complex on an n-point metric space such that the persistence diagram is a good approximation to that of the Vietoris-Rips filtration. The filtration can be constructed in $O(n \log n)$ time. The constants depend only on the doubling dimension of the metric space and the desired tightness of the approximation. For the first time, this makes it computationally tractable to approximate the persistence diagram of the Vietoris-Rips filtration across all scales for large data sets. Our approach uses a hierarchical net-tree to sparsify the filtration. We can either sparsify the data by throwing out points at larger scales to give a zigzag filtration, or sparsify the underlying graph by throwing out edges at larger scales to give a standard filtration. Both methods yield the same guarantees [34].

6.2.8. A multicover nerve for geometric inference

Participant: Donald Sheehy.

We show that filtering the barycentric decomposition of a Čech complex by the cardinality of the vertices captures precisely the topology of k-covered regions among a collection of balls for all values of k. Moreover, we relate this result to the Vietoris-Rips complex to get an approximation in terms of the persistent homology [33].

6.2.9. Computing well diagrams for vector fields on $\mathbb{R}^n$

Participant: Frédéric Chazal.

In collaboration with Primoz Skraba (Lubiana Univ.), Amit Patel (Rutgers Univ.)

Using topological degree theory, we present and prove correctness of a fast algorithm for computing the well diagram, a quantitative property, of a vector field on Euclidean space [17].

6.3. Data Structures and Robust Geometric Computation

6.3.1. Straight-line graph drawing on the torus

Participant: Olivier Devillers.

In collaboration with Luca Castelli Aleardi and Éric Fusy (LIX, Palaiseau).

We extend the notion of canonical orderings to cylindric triangulations. This allows us to extend the incremental straight-line drawing algorithm of de Fraysseix et al. to this setting. Our algorithm yields in linear time a crossing-free straight-line drawing of a cylindric triangulation T with n vertices on a regular grid $\mathbb{Z}/w\mathbb{Z} \times [0, h]$, with $w \leq 2n$ and $h \leq n(2d + 1)$, where d is the (graph-) distance between the two boundaries. As a by-product, we can also obtain in linear time a crossing-free straight-line drawing of a toroidal triangulation with n vertices on a periodic regular grid $\mathbb{Z}/w\mathbb{Z} \times \mathbb{Z}/h\mathbb{Z}$, with $w \leq 2n$ and $h \leq 1 + n(2c + 1)$, where c is the length of a shortest non-contractible cycle. Since $c \leq \sqrt{2n}$, the grid area is $O(n^{5/2})$ [24].

6.3.2. Qualitative symbolic perturbation

Participants: Olivier Devillers, Monique Teillaud.

In collaboration with Menelaos Karavelas (University of Crete).

In the literature, the generic way to address degeneracies in computational geometry is the *Symbolic Perturbation* paradigm: the input is made dependent of some parameter ε so that for ε positive and close to zero, the input is close to the original input, while at the same time, in non-degenerate position. A geometric predicate can usually be seen as the sign of some function of the input. In the symbolic perturbation paradigm, if the function evaluates to zero, the input is perturbed by a small positive ε, and the sign of the function evaluated at the perturbed input is used instead.

The usual way of using this approach is what we will call *Algebraic Symbolic Perturbation* framework. When the function to be evaluated is a polynomial of the input, its perturbed version is seen as a polynomial in ε, whose coefficients are polynomials in the input. These coefficients are evaluated by increasing degree in ε until a non-vanishing coefficient is found. The number of these coefficients can be quite large and expressing them in an easily and efficiently computable manner (e.g., factorized) may require quite some work.

We propose to address the handling of geometric degeneracies in a different way, namely by means of what we call the *Qualitative Symbolic Perturbation* framework. We no longer use a single perturbation that must remove all degeneracies, but rather a sequence of perturbations, such that the next perturbation is being used only if the previous ones have not removed the degeneracies. The new perturbation is considered as *symbolically smaller* than the previous ones. This approach allows us to use simple elementary perturbations whose effect can be analyzed and evaluated: (1) by geometric reasoning instead of algebraic development of the predicate polynomial in ε, and (2) independently of a specific algebraic formulation of the predicate.

We apply our framework to predicates used in the computation of Apollonius diagrams in 2D and 3D, as well as the computation of trapezoidal maps of circular arcs [57].

6.3.3. Covering spaces and Delaunay triangulations of the 2D flat torus

Participants: Mikhail Bogdanov, Monique Teillaud.

In collaboration with Gert Vegter (Johan Bernoulli Institute, Groningen University)

A previous algorithm was computing the Delaunay triangulation of the flat torus, by using a 9-sheeted covering space [64]. We propose a modification of the algorithm using only a 8-sheeted covering space, which allows to work with 8 periodic copies of the input points instead of 9. The main interest of our contribution is not only this result, but most of all the method itself: this new construction of covering spaces generalizes to Delaunay triangulations of surfaces of higher genus.

6.3.4. Hyperbolic Delaunay complexes and Voronoi diagrams made practical

Participants: Mikhail Bogdanov, Olivier Devillers, Monique Teillaud.

We study Delaunay complexes and Voronoi diagrams in the Poincaré ball, a confomal model of the hyperbolic space, in any dimension. We elaborate on our earlier work on the space of spheres [65], giving a detailed description of algorithms, and presenting a static and a dynamic variants. All proofs are based on geometric reasoning, they do not resort to any use of the analytic formula of the hyperbolic distance. We also study algebraic and arithmetic issues, observing that only rational computations are needed. This allows for an exact and efficient implementation in 2D. All degenerate cases are handled. The implementation will be submitted to the CGAL editorial board for future integration into the CGAL library [44].

6.3.5. The stability of Delaunay triangulations

Participants: Jean-Daniel Boissonnat, Ramsay Dyer.

In collaboration with Arijit Ghosh (Indian Statistical Institute, Kolkata, India)

We introduce a parametrized notion of genericity for Delaunay triangulations which, in particular, implies that the Delaunay simplices of δ-generic point sets are thick. Equipped with this notion, we study the stability of Delaunay triangulations under perturbations of the metric and of the vertex positions. We quantify the magnitude of the perturbations under which the Delaunay triangulation remains unchanged. We also present an algorithm that takes as input a discrete point set in $\mathbb{R}^m$, and performs a small perturbation that guarantees that the Delaunay triangulation of the resulting perturbed point set has quantifiable stability with respect to the metric and the point positions. There is also a guarantee on the quality of the simplices: they cannot be too flat. The algorithm provides an alternative tool to the weighting or refinement methods to remove poorly shaped simplices in Delaunay triangulations of arbitrary dimension, but in addition it provides a guarantee of stability for the resulting triangulation [21], [47].

6.3.6. Constructing intrinsic Delaunay triangulations of submanifolds

Participants: Jean-Daniel Boissonnat, Ramsay Dyer.

In collaboration with Arijit Ghosh (Indian Statistical Institute, Kolkata, India)

This work is the algorithmic counterpart of our previous paper [21]. We describe an algorithm to construct an intrinsic Delaunay triangulation of a smooth closed submanifold of Euclidean space. We also provide a counterexample to the results announced by Leibon and Letscher on Delaunay triangulations on Riemannian manifolds. In general the nerve of the intrinsic Voronoi diagram is not homeomorphic to the manifold. The density of the sample points alone cannot guarantee the existence of a Delaunay triangulation. To circumvent this issue, we use results established in our companion paper on the stability of Delaunay triangulations on δ-generic point sets. We establish sampling criteria which ensure that the intrinsic Delaunay complex coincides with the restricted Delaunay complex and also with the recently introduced tangential Delaunay complex. The algorithm generates a point set that meets the required criteria while the tangential complex is being constructed. In this way the computation of geodesic distances is avoided, the runtime is only linearly dependent on the ambient dimension, and the Delaunay complexes are guaranteed to be triangulations of the manifold [46].

6.3.7. Equating the witness and restricted Delaunay complexes

Participants: Jean-Daniel Boissonnat, Ramsay Dyer, Steve Oudot.

In collaboration with Arijit Ghosh (Indian Statistical Institute, Kolkata, India)

It is a well-known fact that the restricted Delaunay and witness complexes may differ when the landmark and witness sets are located on submanifolds of Rd of dimension 3 or more. Currently, the only known way of overcoming this issue consists of building some crude superset of the witness complex, and applying a greedy sliver exudation technique on this superset. Unfortunately, the construction time of the superset depends exponentially on the ambient dimension, which makes the witness complex based approach to manifold reconstruction impractical. This work provides an analysis of the reasons why the restricted Delaunay and witness complexes fail to include each other. From this, a new set of conditions naturally arises under which the two complexes are equal [37].

6.3.8. Simpler complexity analysis of random geometric structures

Participants: Olivier Devillers, Marc Glisse.

In collaboration with Xavier Goaoc (EPI VEGAS*).*

Average-case analysis of data-structures or algorithms is commonly used in computational geometry when the, more classical, worst-case analysis is deemed overly pessimistic. Since these analyses are often intricate, the models of random geometric data that can be handled are often simplistic and far from "realistic inputs". We present a new simple scheme for the analysis of geometric structures. While this scheme only produces results up to a polylog factor, it is much simpler to apply than the classical techniques and therefore succeeds in analyzing new input distributions related to smoothed complexity analysis.

We illustrate our method on two classical structures: convex hulls and Delaunay triangulations. Specifically, we give short and elementary proofs of the classical results that n points uniformly distributed in a ball in $\mathbb{R}^d$ have a convex hull and a Delaunay triangulation of respective expected complexities $\widetilde{\Theta}(n^{\frac{d-1}{d+1}})$ and $\widetilde{\Theta}(n)$. We then prove that if we start with n points well-spread on a sphere, e.g. an (ϵ,κ)-sample of that sphere, and perturb that sample by moving each point randomly and uniformly within distance at most δ of its initial position, then the expected complexity of the convex hull of the resulting point set is $\widetilde{\Theta}\left((\sqrt{n})^{1-\frac{1}{d}}\left(\frac{1}{\sqrt[4]{\delta}}\right)^{d-\frac{1}{d}}\right)$ [55].

6.3.9. Analysis of cone vertex walk in Poisson Delaunay triangulation

Participants: Olivier Devillers, Ross Hemsley.

In collaboration with Nicolas Broutin (EPI RAP*).*

Walking strategies are a standard tool for point location in a triangulation of size n. Although often claimed to be $\Theta(\sqrt{n})$ under random distribution hypotheses, this conjecture has only been formally proved by Devroye, Lemaire, and Moreau [*Comp Geom–Theor Appl*, vol. 29, 2004], in the case of the so called *straight walk* which has the very specific property that deciding whether a given (Delaunay) triangle belongs to the walk may be determined without looking at the other sites. We analyze a different walking strategy that follows vertex neighbour relations to move towards the query. We call this walk *cone vertex walk*. We prove that cone vertex walk visits $\Theta(\sqrt{n})$ vertices and can be constructed in $\Theta(\sqrt{n})$ time. We provide explicit bounds on the hidden constants [50].

6.3.10. The monotonicity of f-vectors of random polytopes

Participants: Olivier Devillers, Marc Glisse.

In collaboration with Xavier Goaoc and Guillaume Moroz (EPI VEGAS*) and Matthias Reitzner (Universität Osnabrück, Germany).*

Let K be a compact convex body in $\mathbb{R}^d$, let K_n be the convex hull of n points chosen uniformly and independently in K, and let $f_i(K_n)$ denote the number of i-dimensional faces of K_n.

We show that for planar convex sets, $E[f_0(K_n)]$ is increasing in n. In dimension $d \geq 3$, we prove that if $\lim_{n\to\infty} \frac{E[f_{d-1}(K_n)]}{An^c} = 1$ for some constants A and $c > 0$ then the function $n \mapsto E[f_{d-1}(K_n)]$ is increasing for n large enough. In particular, the number of facets of the convex hull of n random points distributed uniformly and independently in a smooth compact convex body is asymptotically increasing. Our proof relies on a *random sampling* argument [57].

6.3.11. Efficient Monte Carlo sampler for detecting parametric objects in large scenes

Participants: Florent Lafarge, Yannick Verdie.

Point processes have demonstrated efficiency and competitiveness when addressing object recognition problems in vision. However, simulating these mathematical models is a difficult task, especially on large scenes. Existing samplers suffer from average performances in terms of computation time and stability. We propose a new sampling procedure based on a Monte Carlo formalism. Our algorithm exploits Markovian properties of point processes to perform the sampling in parallel. This procedure is embedded into a data-driven mechanism such that the points are non-uniformly distributed in the scene. The performances of the sampler are analyzed through a set of experiments on various object recognition problems from large scenes, and through comparisons to the existing algorithms [35], [63].

6.4. Applications

6.4.1. Creating large-scale city models from 3D-point clouds: a robust approach with hybrid representation

Participant: Florent Lafarge.

We present a novel and robust method for modeling cities from 3D-point data. Our algorithm provides a more complete description than existing approaches by reconstructing simultaneously buildings, trees and topologically complex grounds. A major contribution of our work is the original way of modeling buildings which guarantees a high generalization level while having semantized and compact representations. Geometric 3D-primitives such as planes, cylinders, spheres or cones describe regular roof sections, and are combined with mesh-patches that represent irregular roof components. The various urban components interact through a non-convex energy minimization problem in which they are propagated under arrangement constraints over a planimetric map. Our approach is experimentally validated on complex buildings and large urban scenes of millions of points, and is compared to state-of-the-art methods [19].

6.4.2. The sticky geometry of the cosmic web

Participant: Monique Teillaud.

In collaboration with Johan Hidding, Rien van de Weygaert, Bernard J.T. Jones (Kapteyn Institute, Groningen University) and Gert Vegter (Johan Bernoulli Institute, Groningen University)

We highlight the application of Computational Geometry to our understanding of the formation and dynamics of the Cosmic Web. The emergence of this intricate and pervasive weblike structure of the Universe on Megaparsec scales can be approximated by a well-known equation from fluid mechanics, the Burgers' equation. The solution to this equation can be obtained from a geometrical formalism. We have extended and improved this method by invoking weighted Delaunay and Voronoi tessellations. The duality between these tessellations finds a remarkable and profound reflection in the description of physical systems in Eulerian and Lagrangian terms [28].

GRACE Team

5. New Results

5.1. Modular curves

F. Morain has been studying the theory and practice of modular curves associated with Weber's invariants. His paper ... is accepted for publication in *Acta Arithmetica*.

5.2. Computing discrete logarithms using codes

D. Augot and F. Morain have been working on the practical application of Reed–Solomon decoding to speed up discrete logarithm computations, following the work of Cheng and Wan. This work is available as a preprint [22], and a Magma implementation was written in support of the many experiments needed.

5.3. Interleaved codes and codes over rings

G. Quintin designed a decoding algorithm based on a lifting decoding scheme. He obtained a unique decoding algorithm with quasi-linear complexity in all parameters for Reed–Solomon codes over Galois rings. Using erasures, he improved the decoding radius with the same complexity. He then applied these techniques to interleaved linear codes over a finite field, and obtained a decoding algorithm that can recover more errors than half the minimum distance. This work has been presented at IEEE ISIT 2012 (Boston, USA).

5.4. Number fields codes

J.-F. Biasse and G. Quintin described an algorithm for list decoding algebraic number field codes in polynomial time in [24]. This is the first explicit procedure for decoding number field codes, whose construction were previously described by Lenstra [33] and Guruswami [32]. They rely on a new algorithm for computing the Hermite normal form of the basis of an $\mathcal{O}_K$-module due to Biasse and Fieker [31], where $\mathcal{O}_K$ is the ring of integers of a number field K. This work has been presented at IEEE ISIT 2012 (Boston, USA).

5.5. Point counting using p-adic methods

C. Gonçalvès designed a new algorithm to compute Zeta functions of cyclic covers of the projective line. This algorithm is a generalisation of the one for superelliptic curves provided by P. Gaudry and N. Gürel and has the same complexity. Moreover, optimal bounds for the precision have been proved. An alternative basis for computations has been studied and the resulting algorithm is faster, even if the asymptotic complexity is the same.

5.6. Codes and Cartier Operator

A. Couvreur proposed a new construction of codes from algebraic curves over a finite field in [25]. This class of codes is a natural geometric generalisation of classical Goppa codes. In particular, the nice equalities "$\Gamma(L, g^{q-1}) = \Gamma(L, g^q)$" satisfied by classical Goppa codes (for instance, see [30]) extend naturally to this larger class of codes. This article is to appear in *Proceeding of the American Mathematical Society*.

5.7. Quantum Codes

A. Couvreur, N. Delfosse and G. Zémor studied a construction of quantum LDPC codes proposed by McKay, Mitchison and Shokrollahi in a draft. This construction involves Cayley graphs of $GF(2)^n$. A general lower bound for the minimum distance of such codes has been found. In addition, a family of such codes whose parameters are proved to be $[[n, O(\sqrt{n}, O(\sqrt{N}))]]$ is exhibited. Notice that up to now, no construction of quantum LDPC codes is known to have a minimum distance better that $O(\sqrt{n})$. The obtained parameters beat many well–known constructions. This work has been presented at IEEE ISIT 2011 (St Petersburg, Russia), and a long version paper [26] has been submitted to an international journal.

5.8. Code-based McEliece like cryptology

A. Couvreur is working with P. Gaborit, V. Gauthier, A. Otmani, and J.-P. Tillich on distinguisher-based attacks on cryptosystems based on Generalised Reed–Solomon codes. Using the particular structure of the square of an evaluation code, they have been able to break some variants of McEliece's cryptosystem using Generalised Reed–Solomon codes, such as Wieschebrink's variant [34]. An article is in preparation.

5.9. Cyclic Codes

A. Zeh is working with A. Wachter-Zeh (University of Ulm and Institut de Recherche de Mathématique de Rennes) and Sergey Bezzateev (St. Petersburg State University of Aerospace Instrumentation) on a new bound for the minimum distance of q-ary cyclic codes [19], [18]. The connection to the BCH bound and the Hartmann–Tzeng (HT) bound was formulated explicitly. Furthermore, the bound was refined for several families of cyclic codes. We defined syndromes and formulated a Key Equation that allows an efficient decoding up to our bound with the Extended Euclidean Algorithm. It turned out that low-rate cyclic codes with small minimum distances are useful for our approach.

5.10. Iterative List Decoding

A. Zeh is working with J. S. R. Nielsen (Department of Mathematics, DTU) on an iterative list decoding algorithm for generalized Reed–Solomon codes. The method is parametrizable and allows variants of the usual list decoding approach. An article is in preparation.

GRAND-LARGE Project-Team

5. New Results

5.1. Communication avoiding algorithms for linear algebra

Participants: Laura Grigori, Amal Khabou, Mathias Jacquelin, Sophie Moufawad.

The focus of this research is on the design of efficient parallel algorithms for solving problems in numerical linear algebra, as solving very large sets of linear equations and large least squares problems, often with millions of rows and columns. These problems arise in many numerical simulations, and solving them is very time consuming.

Our research focuses on developing new algorithms for linear algebra problems, that minimize the required communication, in terms of both latency and bandwidth. We have introduced in 2008 two communication avoiding algorithms for computing the LU and QR factorizations, that we refer to as CALU and CAQR (joint work with J. Demmel and M. Hoemmen from U.C. Berkeley, J. Langou from C.U. Denver, and H. Xiang then at Inria) [18] [8]. Since then, we continue designing communication avoiding algorithm for other operations in both dense and sparse linear algebra. The communication avoiding algorithms are now studied by several other groups, including groups at Inria, and they start being implemented and being available in public libraries as ScaLAPACK.

During 2012, our research [43] has focused on the design of the LU decomposition with panel rank revealing pivoting (LU_PRRP), an LU factorization algorithm based on strong rank revealing QR panel factorization. LU_PRRP is more stable than Gaussian elimination with partial pivoting (GEPP), with a theoretical upper bound of the growth factor of $(1+\tau b)^{(n/b)-1}$, where b is the size of the panel used during the block factorization, τ is a parameter of the strong rank revealing QR factorization, and n is the number of columns of the matrix. For example, if the size of the panel is $b = 64$, and $\tau = 2$, then $(1+2b)^{(n/b)-1} = (1.079)^{n-1} \ll 2^{n-1}$, where 2^{n-1} is the upper bound of the growth factor of GEPP. Our extensive numerical experiments show that the new factorization scheme is as numerically stable as GEPP in practice, but it is more resistant to pathological cases. The LU_PRRP factorization does only $O(n^2 b)$ additional floating point operations compared to GEPP. We have also introduced CALU_PRRP, a communication avoiding version of LU_PRRP that minimizes communication. CALU_PRRP is based on tournament pivoting, with the selection of the pivots at each step of the tournament being performed via strong rank revealing QR factorization. CALU_PRRP is more stable than CALU, the communication avoiding version of GEPP, with a theoretical upper bound of the growth factor of $(1+\tau b)^{\frac{n}{b}(H+1)-1}$, where H is the height of the reduction tree used during tournament pivoting. The upper bound of the growth factor of CALU is $2^{n(H+1)-1}$. CALU_PRRP is also more stable in practice and is resistant to pathological cases on which GEPP and CALU fail.

Our work has also focused on designing algorithms that are optimal over multiple levels of memory hierarchy and parallelism. In [32] we present an algorithm for performing the LU factorization of dense matrices that is suitable for computer systems with two levels of parallelism. This algorithm is able to minimize both the volume of communication and the number of messages transferred at every level of the two-level hierarchy of parallelism. We present its implementation for a cluster of multicore processors based on MPI and Pthreads. We show that this implementation leads to a better performance than routines implementing the LU factorization in well-known numerical libraries. For matrices that are tall and skinny, that is they have many more rows than columns, our algorithm outperforms the corresponding algorithm from ScaLAPACK by a factor of 4.5 on a cluster of 32 nodes, each node having two quad-core Intel Xeon EMT64 processors.

5.2. Preconditioning techniques for solving large systems of equations

Participants: Laura Grigori, Riadh Fezzanni, Sophie Moufawad.

A different direction of research is related to preconditioning large sparse linear systems of equations. This research is performed in the context of ANR PETALh project (2011-2012), which follows the ANR PETAL project (2008-2009). It is conducted in collaboration with Frederic Nataf from University Paris 6.

Several highly used preconditioners are for example the incomplete LU factorizations and Schwarz based approaches as used in domain decomposition. Most of these preconditioners are known to have scalability problems. The number of iterations can increase significantly when the size of the problem increases or when the number of independent domains is increased. This is often due to the presence of several low frequency modes that hinder the convergence of the iterative method. To address this problem, we study a different class of preconditioners, called direction preserving or filtering preconditioners. These preconditioners have the property of being identical to the input matrix on a given filtering vector. A judicious choice of the vector allows to alleviate the effect of low frequency modes on the convergence.

We consider in particular two classes of preconditioners. The first preconditoner is an incomplete decomposition that satisfies the filtering property [13]. The nested preconditioner has the same property for a specific vector of all ones. However the construction is different and takes advantage of a nested structure of the input matrix. The previous research on these methods considered only matrices arising from the discretization of PDEs on structured grids, where the matrix has a block tridiagonal structure. This structure imposes a sequential computation of the preconditioner and it is not suitable for the more general case of unsructured grids. Hence, while very efficient, the usage of these preconditioners was very limited. At the beginning of this research we have obtained several theoretical results for these methods that demonstrate their numerical behavior and convergence properties for cases arising from the discretization of PDEs on structured grids [13]. But the main result is the development of a generalized method [10], [11] that has two important properties: it allows the filtering property to be satisfied for any input matrix; the matrix can be reordered such that its computation is highly parallel. Experimental results show that the method is very efficient for certain classes of matrices, and shows good scalability results in terms of both problem size and number of processors. In addition to finalizing this work, our research also focused on extending the block filtering factorization to include other approximation techniques that allowed us to introduce a parameter whose tuning permits to solve very difficult problems.

5.3. MIcrowave Data Analysis for petaScale computers

Participants: Laura Grigori, Mikolaj Szydlarski, Meisam Shariffy.

Generalized least square problems with non-diagonal weights arise frequently in an estimation of two dimensional images from data of cosmological as well as astro- or geo- physical observations. As the observational data sets keep growing at Moore's rate, with their volumes exceeding tens and hundreds billions of samples, the need for fast and efficiently parallelizable iterative solvers is generally recognized.

In this work [36] we propose a new iterative algorithm for solving generalized least square systems with weights given by a block-diagonal matrix with Toeplitz blocks. Such cases are physically well motivated and correspond to measurement noise being piece-wise stationary – a common occurrence in many actual observations. Our iterative algorithm is based on the conjugate gradient method and includes a parallel two-level preconditioner (2lvl-PCG) constructed from a limited number of sparse vectors estimated from the coefficients of the initial linear system.

Our prototypical application is the map-making problem in the Cosmic Microwave Background data analysis. We show experimentally that our parallel implementation of 2lvl-PCG outperforms by a factor of up to 6 the standard one-level PCG in terms of both the convergence rate and the time to solution on up to $12,228$ cores of NERSC's Cray XE6 (Hopper) system displaying nearly perfect strong and weak scaling behavior in this regime.

5.4. Innovative linear system solvers for hybrid multicore/GPU architectures

Participant: Marc Baboulin.

The advent of new processor architectures (e.g. multicore, GPUs) requires the rethinking of most of the scientific applications and innovative methods must be proposed in order to take full advantage of current supercomputers [14].

To accelerate linear algebra solvers on current parallel machines, we introduced in public domain libraries a class of solvers based on statistical techniques. A first application concerns the solution of a square linear systems $Ax = b$. We study a random transformation of A that enables us to avoid pivoting and then to reduce the amount of communication [16]. Numerical experiments show that this randomization can be performed at a very affordable computational price while providing us with a satisfying accuracy when compared to partial pivoting. This random transformation called Partial Random Butterfly Transformation (PRBT) is optimized in terms of data storage and flops count. In the solver that we developed, PRBT combined with LU factorization with no pivoting take advantage of the latest generation of hybrid multicore/GPU machines and outperform existing factorization routines from current parallel library MAGMA.

A second application is related to solving symmetric indefinite systems via LDL^T factorization for which there was no existing parallel implementation in the dense library ScaLAPACK. We developed an efficient and innovative parallel tiled algorithm for solving symmetric indefinite systems on multicore architectures [54]& [25]. This solver avoids pivoting by using a multiplicative preconditioning based on symmetric randomization. This randomization prevents the communication overhead due to pivoting, is computationally inexpensive and requires very little storage. Following randomization, a tiled LDLT factorization is used that reduces synchronization by using static or dynamic scheduling. We compare Gflop/s performance of our solver with other types of factorizations on a current multicore machine and we provide tests on accuracy using LAPACK test cases.

5.5. MILEPOST GCC: machine learning enabled self-tuning compiler

Participant: Grigori Fursin [correspondant].

Tuning compiler optimizations for rapidly evolving hardware makes porting and extending an optimizing compiler for each new platform extremely challenging. Iterative optimization is a popular approach to adapting programs to a new architecture automatically using feedback-directed compilation. However, the large number of evaluations required for each program has prevented iterative compilation from widespread take-up in production compilers. Machine learning has been proposed to tune optimizations across programs systematically but is currently limited to a few transformations, long training phases and critically lacks publicly released, stable tools.

Our approach is to develop a modular, extensible, self-tuning optimization infrastructure to automatically learn the best optimizations across multiple programs and architectures based on the correlation between program features, run-time behavior and optimizations. In this paper we describe MILEPOST GCC, the first publicly-available open-source machine learning-based compiler. It consists of an Interactive Compilation Interface (ICI) and plugins to extract program features and exchange optimization data with the cTuning.org open public repository. It automatically adapts the internal optimization heuristic at function-level granularity to improve execution time, code size and compilation time of a new program on a given architecture. Part of the MILEPOST technology together with low-level ICI-inspired plugin framework is now included in the mainline GCC.

We developed machine learning plugins based on probabilistic and transductive approaches to predict good combinations of optimizations. Our preliminary experimental results show that it is possible to automatically reduce the execution time of individual MiBench programs on various machines from GRID5000, some by more than a factor of 2, while also improving compilation time and code size. We also present a realistic multi-objective optimization scenario for Berkeley DB library using MILEPOST GCC and improve execution time by approximately 17%, while reducing compilation time and code size by 12% and 7% respectively on Intel Xeon processor.

5.6. Loop Transformations: Convexity, Pruning and Optimization

Participant: Cédric Bastoul.

High-level loop transformations are a key instrument in mapping computational kernels to effectively exploit resources in modern processor architectures. However, determining appropriate compositions of loop transformations to achieve this remains a significantly challenging task; current compilers may achieve significantly lower performance than hand-optimized programs. To address this fundamental challenge, we first present a convex characterization of all distinct, semantics-preserving, multidimensional affine transformations. We then bring together algebraic, algorithmic, and performance analysis results to design a tractable optimization algorithm over this highly expressive space. The framework has been implemented and validated experimentally on a representative set of benchmarks run on state-of-the-art multi-core platforms.

5.7. Non-self-stabilizing and self-stabilizing gathering in networks of mobile agents–the notion of speed

Participants: Joffroy Beauquier, Janna Burman, Julien Clment, Shay Kutten.

In the population protocol model, each agent is represented by a finite state machine. Agents are anonymous and supposed to move in an asynchronous way. When two agents come into range of each other (“meet”), they can exchange information. One of the vast variety of motivating examples to the population protocols model is ZebraNet. ZebraNet is a habitat monitoring application where sensors are attached to zebras and collect biometric data (e.g. heart rate, body temperature) and information about their behavior and migration patterns (via GPS). The population protocol model is, in some sense, related to cloud computing and to networks characterized by asynchrony,large scale, the possibility of failures, in the agents as well as in the communications, with the constraint that each agent is resource limited.

In order to extend the computation power and efficiency of the population protocol model, various extensions were suggested. Our contribution is an extension of the population protocol model that introduces the notion of “speed”, in order to capture the fact that the mobile agents move at different speeds and/or have different communication ranges and/or move according to different patterns and/or visit different places with different frequencies. Intuitively, fast agents which carry sensors with big communication ranges communicate with other agents more frequently than other agents do. This notion is formalized by allocating a cover time, cv, to each mobile agent v. cv is the minimum number of events in the whole system that occur before agent v meets every other agent at least once. As a fundamental example, we have considered the basic problem of gathering information that is distributed among anonymous mobile agents and where the number of agents is unknown. Each mobile agent owns a sensed input value and the goal is to communicate the values (as a multi-set, one value per mobile agent) to a fixed non-mobile base station (BS), with no duplicates or losses.

Gathering is a building block for many monitoring applications in networks of mobile agents. For example, a solution to this problem can solve a transaction commit/abort task in MANETs, if the input values of agents are votes (and the number of agents is known to BS). Moreover, the gathering problem can be viewed as a formulation of the routing problem in Disruption Tolerant Networks.

We gave different solutions to the gathering in the model of mobile agents with speed and we proved that one of them is optimal.

5.8. Making Population Protocols Self-stabilizing

Participants: Joffroy Beauquier, Janna Burman, Shay Kutten, Brigitte Rozoy.

As stated in the previous paragraph, the application domains of the population protocol model are asynchronous large scale networks, in which failures are possible and must be taken into account. This work concerns failures and namely the technique of self-stabilization for tolerating them.

Developing self-stabilizing solutions (and proving them) is considered to be more challenging and complicated than developing classical solutions, where a proper initialization of the variables can be assumed. This remark holds for a large variety of models and hence, to ease the task of the developers, some automatic techniques have been proposed to transform programs into self-stabilizing ones.

We have proposed such a transformer for algorithms in the population protocol model introduced for dealing with resource-limited mobile agents. The model we consider is a variation of the original one in that there is a non mobile agent, the base station, and that the communication characteristics (e.g. moving speed, communication radius) of the agents are considered through the notion of cover time.

The automatic transformer takes as an input an algorithm solving a static problem and outputs a self-stabilizing solution for the same problem. To the best of our knowledge, it is the first time that such a transformer for self-stabilization is presented in the framework of population protocols. We prove that the transformer we propose is correct and we make the complexity analysis of the stabilization time.

5.9. Self-stabilizing synchronization in population protocols with cover times

Participants: Joffroy Beauquier, Janna Burman, Shay Kutten, Brigitte Rozoy.

Synchronization is widely considered as an important service in distributed systems which may simplify protocol design. Phase clock is a general synchronization tool that provides a form of a logical time. We have developed a self-stabilizing phase clock algorithm suited to the model of population protocols with cover time. We have shown that a phase clock is impossible in the model with only constant-state agents. Hence, we assumed an existence of resource unlimited agent - the base station. The clock size and duration of each phase of the proposed phase clock tool are adjustable by the user. We provided application examples of this tool and demonstrate how it can simplify the design of protocols. In particular, it yields a solution to Group Mutual Exclusion problem.

5.10. Impossibility of consensus for population protocol with cover times

Participants: Joffroy Beauquier, Janna Burman.

We have extended the impossibility result for asynchronous consensus of Fischer, Lynch and Paterson (FLP) to the asynchronous model of population protocols with cover times. We noted that the proof of FLP does not apply. Indeed, the key lemma stating that two successive factors in an execution, involving disjoint subsets of agents, commute, is no longer true, because of the cover time property. Then we developed a completely different approach and we proved that there is no general solution to consensus for population protocols with cover times, even if there is a single possible crash. We noted that this impossibility result also applies to randomized asynchronous consensus, contrary to what happens in the classical message-passing or shared memory communication models, in which the problem is solvable inside some bounds on the number of faulty processes. Then, for circumventing these impossibility results, we introduced the phase clock oracle and the S oracle, and we shown how they allow to design solutions.

5.11. Routing and synchronization in large scale networks of very cheap mobile sensors

Participants: Joffroy Beauquier, Brigitte Rozoy.

In a next future, large networks of very cheap mobile sensors will be deployed for various applications, going from wild life preserving or environmental monitoring up to medical or industrial system control. Each sensor will cost only a few euros, allowing a large scale deployment. They will have only a few bit of memory, no identifier, weak capacities of computation and communication, no real time clock and will be prone to failures. Moreover such networks will be fundamentally dynamic. The goal of this subject is to develop the basic protocols and algorithms for rudimentary distributed systems for such networks. The studied problems are basic ones, like data collection, synchronization (phase clock, mutual exclusion, group mutual exclusion), fault tolerance (consensus), automatic transformers, always in a context of possible failures. A well known model has already been proposed for such networks, the population protocol model. In this model, each sensor is represented by a finite state machine. Sensors are anonymous and move in an asynchronous way. When two sensors come into range of each other ("meet"), they can exchange information. One of the vast variety of motivating examples for this model is ZebraNet. ZebraNet is a habitat monitoring application in

which sensors are attached to zebras in order to collect biometric data (e.g., heart rate, body temperature) and information about their behavior and migration patterns. Each pair of zebras meets from time to time. During such meetings (events), ZebraNet's agents (zebras' attached sensors) exchange data. Each agent stores its own sensor data as well as data of other sensors that were in range in the past. They upload data to a base station whenever it is nearby. It was shown that the set of applications that can be solved in the original model of population protocols is rather limited. Other models (such as some models of Delay/Disruption-Tolerant Networks - DTNs), where each node maintains links and connections even to nodes it may interact with only intermittently, do not seem to suit networks with small memory agents and a very large (and unknown) set of anonymous agents. That is why we enhance the model of population protocols by introducing a notion of "speed". We try to capture the fact that the mobile agents move at different speeds and/or have different communication ranges and/or move according to different patterns and/or visit different places with different frequencies. Intuitively, fast agents which carry sensors with large communication ranges communicate with other agents more frequently than other agents do. This notion is formalized by the notion of cover time for each agent. The cover time of an agent is the unknown number of events (pairwise meetings) in the whole system that occur (during any execution interval) before agent v meets every other agent at least once. The model we propose is somehow validated by some recent statistical results, obtained from empirical data sets regarding human or animal mobility. An important consequence of our approach is that the analytic complexity of the protocols designed in this model is possible, independently of any simulation or experimentation. For instance, we consider the fundamental problem of gathering different pieces of information, each sensed by a different anonymous mobile agent, and where the number of agents is unknown. The goal is to communicate the sensed values (as a multi-set, one value per mobile agent) to a base station, with no duplicates or losses. Gathering is a building block for many monitoring applications in networks of mobile agents. Moreover, the gathering problem can be viewed as a special case of the routing problem in DTNs, in which there is only one destination, the base station. Then we are able to compute the complexity of solutions we propose, as well as those of solutions used in experimental projects (like ZebraNet), and to compare them. The algorithms we present are self-stabilizing. Such algorithms have the important property of operating correctly regardless of their initial state (except for some bounded period). In practice, self-stabilizing algorithms adjust themselves automatically to any changes or corruptions of the network components (excluding the algorithm's code). These changes are assumed to cease for some sufficiently long period. Self-stabilization is considered for two reasons. First, mobile agents are generally fragile, subject to failures and hard to initialize. Second, systems of mobile agents are by essence dynamic, some agents leave the system while new ones are introduced. Self-stabilization is a well adapted framework for dealing with such situations.

5.12. Self-Stabilizing Control Infrastructure for HPC

Participants: Thomas Hérault, Camille Coti.

High performance computing platforms are becoming larger, leading to scalability and fault-tolerance issues for both applications and runtime environments (RTE) dedicated to run on such machines. After being deployed, usually following a spanning tree, a RTE needs to build its own communication infrastructure to manage and monitor the tasks of parallel applications. Previous works have demonstrated that the Binomial Graph topology (BMG) is a good candidate as a communication infrastructure for supporting scalable and fault-tolerant RTE.

In this work, we presented and analyzed a self-stabilizing algorithm to transform the underlying communication infrastructure provided by the launching service (usually a tree, due to its scalability during launch time) into a BMG, and maintain it in spite of failures. We demonstrated that this algorithm is scalable, tolerates transient failures, and adapts itself to topology changes.

The algorithms are scalable, in the sense that all process memory, number of established communication links, and size of messages are logarithmic with the number of elements in the system. The number of synchronous rounds to build the system is also logarithmic, and the number of asynchronous rounds in the worst case is square logarithmic with the number of elements in the system. Moreover, the salf-stabilizing property of the algorithms presented induce fault-tolerance and self-adaptivity. Performance evaluation based on simulations

predicts a fast convergence time (1/33s for 64K nodes), exhibiting the promising properties of such self-stabilizing approach.

We pursue this work by implementing and evaluating the algorithms in the STCI runtime environment to validate the theoretical results.

5.13. Large Scale Peer to Peer Performance Evaluations

Participant: Serge Petiton.

5.13.1. Large Scale Grid Computing

Recent progress has made possible to construct high performance distributed computing environments, such as computational grids and cluster of clusters, which provide access to large scale heterogeneous computational resources. Exploration of novel algorithms and evaluation of performance is a strategic research for the future of computational grid scientific computing for many important applications [82]. We adapted [63] an explicit restarted Lanczos algorithm on a world-wide heterogeneous grid platform. This method computes one or few eigenpairs of a large sparse real symmetric matrix. We take the specificities of computational resources into account and deal with communications over the Internet by means of techniques such as out-of-core and data persistence. We also show that a restarted algorithm and the combination of several paradigms of parallelism are interesting in this context. We perform many experimentations using several parameters related to the Lanczos method and the configuration of the platform. Depending on the number of computed Ritz eigenpairs, the results underline how critical the choice of the dimension of the working subspace is. Moreover, the size of platform has to be scaled to the order of the eigenproblem because of communications over the Internet.

5.13.2. High Performance Cluster Computing

Grid computing focuses on making use of a very large amount of resources from a large-scale computing environment. It intends to deliver high-performance computing over distributed platforms for computation and data-intensive applications. We propose [93] an effective parallel hybrid asynchronous method to solve large sparse linear systems by the use of a Grid Computing platform Grid5000. This hybrid method combines a parallel GMRES(m) (Generalized Minimum RESidual) algorithm with the Least Square method that needs some eigenvalues obtained from a parallel Arnoldi algorithm. All of these algorithms run on the different processors of the platform Grid5000. Grid5000, a 5000 CPUs nation-wide infrastructure for research in Grid computing, is designed to provide a scientific tool for computing. We discuss the performances of this hybrid method deployed on Grid5000, and compare these performances with those on the IBM SP series supercomputers.

5.13.3. Large Scale Power aware Computing

Energy conservation is a dynamic topic of research in High Performance Computing and Cluster Computing. Power-aware computing for heterogeneous world-wide Grid is a new track of research. We have studied and evaluated the impact of the heterogeneity of the computing nodes of a Grid platform on the energy consumption. We propose to take advantage of the slack-time caused by the heterogeneity in order to save energy with no significant loss of performance by using Dynamic Voltage Scaling (DVS) in a distributed eigensolver [64]. We show that using DVS only during the slack-time does not penalize the performances but it does not provide significant energy savings. If DVS is applied to all the execution, we get important global and local energy savings (respectively up to 9% and 20%) without a significant rise of the wall-clock times.

5.14. High Performance Linear Algebra on the Grid

Participants: Thomas Hérault, Camille Coti.

Previous studies have reported that common dense linear algebra operations do not achieve speed up by using multiple geographical sites of a computational grid. Because such operations are the building blocks of most scientific applications, conventional supercomputers are still strongly predominant in high-performance computing and the use of grids for speeding up large-scale scientific problems is limited to applications exhibiting parallelism at a higher level.

In this work, we have identified two performance bottlenecks in the distributed memory algorithms implemented in ScaLAPACK, a state-of-the-art dense linear algebra library. First, because ScaLAPACK assumes a homogeneous communication network, the implementations of ScaLAPACK algorithms lack locality in their communication pattern. Second, the number of messages sent in the ScaLAPACK algorithms is significantly greater than other algorithms that trade flops for communication.

This year, we presented a new approach for computing a QR factorization one of the main dense linear algebra kernels of tall and skinny matrices in a grid computing environment that overcomes these two bottlenecks. Our contribution is to articulate a recently proposed algorithm (Communication Avoiding QR) with a topology-aware middleware (QCG-OMPI) in order to confine intensive communications (ScaLAPACK calls) within the different geographical sites.

An experimental study conducted on the Grid5000 platform shows that the resulting performance increases linearly with the number of geographical sites on large-scale problems (and is in particular consistently higher than ScaLAPACKs).

5.15. Emulation of Volatile Systems

Participants: Thomas Largillier, Benjamin Quetier, Sylvain Peyronnet, Thomas Hérault, Franck Cappello.

In the process of developping grid applications, people need to often evaluate the robustness of their work. Two common approaches are simulation, where one can evaluate his software and predict behaviors under conditions usually unachievable in a laboratory experiment, and experimentation, where the actual application is launched on an actual grid. However simulation could ignore unpredictable behaviors due to the abstraction done and experimation does not guarantee a controlled and reproducible environment, and simulation often introduces a high level of abstraction that make the discovery and study of unexpected, but real, behaviors a rare event.

In this work, we proposed an emulation platform for parallel and distributed systems including grids where both the machines and the network are virtualized at a low level. The use of virtual machines allows us to test highly accurate failure injection since we can destroy virtual machines, and network virtualization provides low-level network emulation. Failure accuracy is a criteria that evaluates how realistic a fault is. The accuracy of our framework has been evaluated through a set of micro benchmarks and a very stable P2P system called Pastry.

We are in the process of developping a fault injection tool to work with the platform. it will be an extension of the work started in the tool Fail. The interest of this work is that using Xen virtual machines will allow to model strong adversaries since it is possible to have virtual machines with shared memory. These adversaries will be stronger since they will be able to use global fault injection strategies.

5.16. Exascale Systems

Participant: Franck Cappello.

Over the last 20 years, the open-source community has provided more and more software on which the world's high-performance computing systems depend for performance and productivity. The community has invested millions of dollars and years of effort to build key components. Although the investments in these separate software elements have been tremendously valuable, a great deal of productivity has also been lost because of the lack of planning, coordination, and key integration of technologies necessary to make them work together smoothly and efficiently, both within individual petascale systems and between different systems. A repository gatekeeper and an email discussion list can coordinate open-source development within a single project, but there is no global mechanism working across the community to identify critical holes in the overall software environment, spot opportunities for beneficial integration, or specify requirements for more careful coordination. It seems clear that this completely uncoordinated development model will not provide the software needed to support the unprecedented parallelism required for peta/exascale computation on millions of cores, or the flexibility required to exploit new hardware models and features, such as transactional

memory, speculative execution, and GPUs. We presented a rational promoting that the community must work together to prepare for the challenges of exascale computing, ultimately combing their efforts in a coordinated International Exascale Software Project.

Over the past few years resilience has became a major issue for high-performance computing (HPC) systems, in particular in the perspective of large petascale systems and future exascale systems. These systems will typically gather from half a million to several millions of central processing unit (CPU) cores running up to a billion threads. From the current knowledge and observations of existing large systems, it is anticipated that exascale systems will experience various kind of faults many times per day. It is also anticipated that the current approach for resilience, which relies on automatic or application level checkpoint/restart, will not work because the time for checkpointing and restarting will exceed the mean time to failure of a full system. This set of projections leaves the community of fault tolerance for HPC systems with a difficult challenge: finding new approaches, which are possibly radically disruptive, to run applications until their normal termination, despite the essentially unstable nature of exascale systems. Yet, the community has only five to six years to solve the problem. In order to start addressing this challenge, we synthesized the motivations, observations and research issues considered as determinant of several complimentary experts of HPC in applications, programming models, distributed systems and system management.

As a first step to adress the resilience challenge, we conducted a comprehensive study of the state of the art . The emergence of petascale systems and the promise of future exascale systems have reinvigorated the community interest in how to manage failures in such systems and ensure that large applications, lasting several hours or tens of hours, are completed successfully. Most of the existing results for several key mechanisms associated with fault tolerance in high-performance computing (HPC) platforms follow the rollback-recovery approach. Over the last decade, these mechanisms have received a lot of attention from the community with different levels of success. Unfortunately, despite their high degree of optimization, existing approaches do not fit well with the challenging evolutions of large-scale systems. There is room and even a need for new approaches. Opportunities may come from different origins: diskless checkpointing, algorithmic-based fault tolerance, proactive operation, speculative execution, software transactional memory, forward recovery, etc. We provided the following contributions: (1) we summarize and analyze the existing results concerning the failures in large-scale computers and point out the urgent need for drastic improvements or disruptive approaches for fault tolerance in these systems; (2) we sketch most of the known opportunities and analyze their associated limitations; (3) we extract and express the challenges that the HPC community will have to face for addressing the stringent issue of failures in HPC systems.

HIPERCOM Project-Team

6. New Results

6.1. Time Slot Assignment in Wireless Sensor Networks

Participants: Pascale Minet, Ridha Soua, Erwan Livolant.

6.1.1. NP-completeness of the Time Slot Assignment problem

In data gathering applications, wireless sensor networks (WSNs) collect data from sensor nodes towards a sink in a multi-hop convergecast structure. Assigning equal channel access to each node may lead to congestion and inefficient use of the bandwidth. That is why we focus on traffic-aware solutions. More precisely, we investigate the Time Slot Assignment problem, where nodes are assigned time slots to transmit their data to the sink, while minimizing the total number of slots. We considered the generalized h-hop Time Slot Assignment problem for any positive integer h, where any two nodes that are less than or equal to h-hop away are not scheduled simultaneously. We proved its NP-completeness.

6.1.2. Multichannel Slot Assignment

The throughput requirement of data gathering applications is difficult to meet with a single wireless channel. Furthermore, the considered channel may be temporarily jammed. That is why, we focus on a multichannel time slot assignment that minimizes the data gathering cycle. We first formalize the problem as a linear program and compute the optimal time needed for a raw data convergecast in various multichannel topologies (linear, multi-line, tree). These optimal times apply to nodes equipped with one or several radio interfaces. This work generalizes the results established by Incel. We then propose our algorithm called MODESA and prove its optimality in various multichannel topologies. We evaluate its performances in terms of number of slots, maximum buffer size and number of active/sleep switches per node. Furthermore, we present variants of MODESA achieving a load balancing between the channels used.

6.1.3. Multisink Multichannel Slot Assignment

We generalize this work, taking into account the existence of several sinks. We focus on the data gathering problem with differentiated traffic, each addressed to a specific sink in multichannel WSNs. In order to find a collision-free optimized multichannel time slot assignment that minimizes the data gathering cycle, we propose a centralized traffic-aware algorithm called MUSIKA. We formulate the problem as a linear program and compute the optimal time needed for a raw data convergecast in various multichannel topologies (linear, multi-line, tree). More generally, we run simulations on various network topologies to evaluate the performance of MUSIKA in terms of cycle length, maximum buffer size and slot reuse ratio for different use cases: redundant functional processing chains, different application functionalities per sink.

6.2. Multi-Sink Wireless Sensor deployment and energy analysis

Participants: Paul Mühlethaler, Nadjib Achir.

We propose a general framework for multi-sink Wireless Sensors networks (WNSs). This framework is devoted to computing the optimal deployment of sinks for a given maximum number of hops between nodes and sinks. This framework allows an estimation of the energy consumption to be computed. We consider the energy consumed due to reporting, forwarding and overhearing. In contrast to reporting and forwarding, the energy used in overhearing is difficult to estimate because it is dependent on the packet scheduling. We determine the upper-bound and lower-bound of overhearing. We also propose another estimation which can simulate non interfering parallel transmissions which is more tractable in large networks. We note that overhearing largely predominates in energy consumption. A large part of the optimizations and computations carried out in this paper are obtained using ILP formalization.

6.3. WSN Redeployment

Participants: Pascale Minet, Saoucene Mahfoudh Ridene, Ines Khoufi.

This is a joint work with Telecom SudParis: Anis Laouiti.

6.3.1. Centralized redeployment algorithm based on Virtual Forces

In many applications (e.g military, environment monitoring), wireless sensors are randomly deployed in a given area. Unfortunately, this deployment is not efficient enough to ensure full area coverage and total network connectivity. Hence, all the considered area must be covered by sensors ensuring that any event is detected in the sensing range of at least one sensor. In addition, the sensor network must be connected in terms of radio communication in order to forward the detected event to the sink(s). Thus, a redeployment algorithm has to be applied in order to achieve these two goals. In this context, we have proposed redeployment algorithms based on virtual forces. First, we have designed and simulated a centralized algorithm called CVFA. This algorithm is executed by a specific node which has global information of node positions.

6.3.2. Distributed redeployment algorithm based on Virtual Forces

Then, we proposed DVFA, Distributed Virtual forces Algorithm. Each node in the network executes DVFA and computes its new position based on information collected from its neighbors.
Performance evaluation shows that both CVFA and DVFA give very good coverage rate (between 98% and 100%) and ensure the connectivity between sensors.

6.3.3. Distributed redeployment algorithm based on Virtual Forces in the presence of obstacles

Moreover, in a real environment, obstacles such as trees, walls and buildings may exist and they may impact the deployment of wireless sensors. Obstacles can prohibit the network connectivity between nodes and create some uncovered holes or some accumulation of sensors in the same region. Consequently, an efficient wireless sensors deployment algorithm is required to ensure both coverage and network connectivity in the presence of obstacles. We have focused on this problem and enhanced our Distributed Virtual Force Algorithm (DVFA) to cope with obstacles. Simulation results show that DVFA gives very good performances even in the presence of obstacles.

6.4. Mesh Network Planning: Deployment and Canal Allocation

Participant: Nadjib Achir.

This is a joint work with University Paris XIII: A. Farsi, K. Boussetta.
We deal with the Wireless LAN planning problem. We study this problem and we propose to couple its two major issues: AP placement and channel assignment to treat them jointly. Here, we propose a novel fast and scalable three-phase heuristic algorithm (TPHA). Our proposal is able to resolve the defined multiobjective problem to provide (1) the efficient number of Access Points (APs) to be deployed, while (2) ensuring the coverage of all Test Points (TPs) and (3) maximizing their nominal data rate. To achieve the first objective, we propose an heuristic called MCL-ILP combining the quick decision making based on the Markovian CLustering algorithm and the exact solution provided by the Integer Linear Programming. Hence, a TPs-based Least Interfering Channel Search algorithm (TLICS) has been proposed for channel assignment to improve the throughput at TP locations. However, the Virtual Forces-based WLAN Planning Algorithm namely VFPA considers the results delivered by the two previous algorithms as an initial solution and tries to enhance it by adjusting the APs' positions and re-assigning their operating frequencies. Computational results exhibit that our proposal is highly beneficial to designing WLANs.

6.5. Routing in MANETs using slotted Aloha. End-to-end delays

Participants: Paul Mühlethaler, Iskander Banaouas.

This is a joint work with TREC: B. Blaszczyzyn.

Planar Poisson models with the Aloha medium access scheme have already proved to be very useful in studies of mobile ad-hoc networks (MANETs). However, it seems difficult to quantitatively study the performances of end-to-end routing in these models. In order to tackle this problem, in this paper we study a *linear stationary route embedded in an independent planar field of interfering nodes*. We consider this route as an idealization of a "typical" route in a MANET obtained by some routing mechanism. Such a decoupling allows us to obtain many numerically tractable expressions for local and mean end-to-end delays and the speed of packet progression, assuming slotted Aloha MAC and the Signal-to-Interference-and-Noise Ratio (SINR) capture condition, with the usual power-law path loss model and Rayleigh fading. These expressions show how the network performance depends on the tuning of Aloha and routing parameters and on the external noise level. In particular we show a need for a well-tuned lattice structure of fixed relaying nodes, which helps to relay packets on long random routes in the presence of a non-negligible noise. We also consider a *Poisson-line MANET model*, in which *all* nodes are located on roads forming a Poisson-line process. In this case our linear route is rigorously (in the sense of Palm theory) the typical route in this Poisson-line MANET.

6.6. Cognitive networks using a darwinian approach

Participant: Paul Mühlethaler.

This is a joint work with Alcatel Bell Labs: Philippe Jacquet.
We present a new approach for cognitive radio. In the usual approach the secondary network is in charge of monitoring the channel to determine whether or not the primary network is active in the area. If it is not, the secondary network is allowed to use the spectrum. In the new access scheme we propose, the primary network encompasses the techniques which allow it to capture the bandwidth even if the secondary network is transmitting in the area. The access scheme of the primary network preempts the secondary network activity. We present an access scheme which preempts the IEEE 802.11 decentralized scheme. This protocol is a generalized Carrier Sense Multiple Access scheme using active signaling. Instead of only sensing the carrier, this algorithm also transmits bursts of signal which may be sensed by the other nodes. If so, they give up the selection process. We show that this scheme preempts the IEEE 802.11 decentralized access scheme if the bursts transmitted by the node in the primary network are made up of special sequences which alternate between bursts of signal and periods of sensing. These sequences called (d, k) sequences encompass a minimum number d and a maximum number of k successive zeros during which the node senses the channel to find other possible concurrent transmissions. In practice we use $d = 0$ and k depends on the duration of the IEEE 802.11 interframe space and the duration of a signaling burst. We compute the number of $(0, k)$ sequences with respect to the length n of the sequence. We also show that (d, k) sequences (with $2d > k$) can be used if, by mistake, during the signaling phase one burst is not detected. We evaluate the number of such sequences.

6.7. Massive mobile dense wireless networks

Participants: Aline Carneiro Viana, Ana Cristina B. Kochem Vendramin, Kanchana Thilakarathna, Eduardo Mucceli.

routing protocols, analytical models, content distribution.

6.7.1. Scientific achievements

6.7.1.1. Social Relationship Classified

Understanding human mobility is of fundamental importance when designing new communication protocols that exploit opportunistic encounters among users. In particular, human behavior is characterized by an elevated rate of regularity, but random events are always possible in the routines of individuals as hardly predictable situations that deviate from the regular pattern and are unlikely to arise repeatedly in the future. These random events veil the ordinary patterns by introducing a significant amount of noise, thus making the process of knowledge discovery in social dataset a complex task. However, the ability to accurately identify random and social events in large datasets is essential to social analysis as well as to applications that rely

on a precise description of human routines, such as recommendation systems, forwarding strategies and ad-hoc message dissemination schemes focusing on coverage efficiency with a limited number of redundant messages. In such a context, we have proposed a strategy to analyze wireless network scenarios where mobile users interact in a rational manner, reflecting their interests and activity dynamics. Our strategy, named Random rElationship ClASsifier sTrategy (RECAST), allows to classify user relationships, separating random interactions from different kinds of social ties. The goal is achieved by observing how the real system differs from an equivalent one where entities decisions are completely random. We have evaluate the effectiveness of RECAST classification on datasets of real-world user contacts in diverse networking contexts. Our analysis unveils significant differences in the relationship dynamics of the datasets, proving that the evaluation of network protocols on a single dataset cannot lead to conclusions of general validity.

6.7.1.2. Social-aware Forwarding Protocol

Pervasiveness of computing devices, ubiquitous wireless communication, emergence of new applications, and cloud services are examples of current new emerging factors that emphasize the increasing need for adaptive networking solutions. The adaptation, most of the time, requires the design of more interdisciplinary approaches as those inspired by techniques coming from biology, social structures, games, and control systems. The approach we consider brings together solutions from different but complementary domains - i.e., networking, biology, and complex networks - aiming to deal with the problem of efficient data delivery in mobile and intermittently connected networks. For this, we have designed the Cultural Greedy Ant (CGrAnt) protocol to solve the problem of data delivery in mobile and intermittently connected networks referred as Delay Tolerant Networks (DTNs). CGrAnt is a hybrid Swarm Intelligence-based forwarding protocol designed to deal with the dynamic and complex environment of DTNs resulting from users mobility or varying conditions of wireless communications. CGrAnt is based on (1) Cultural Algorithms (CA) and Ant Colony Optimization (ACO) and (2) metrics which characterize opportunistic social connectivity between wireless users. CA and ACO are used to direct the network traffic, taking into account a set of social-aware metrics that may infer relevant structures in meeting regularities and mobility patterns of users. The most promising message forwarders are selected through a greedy transition rule based on local and global information captured from the DTN environment. Through simulation, we have analyzed the influence of ACO operators and CA's knowledge on CGrAnt performance. We have then compared the performance of CGrAnt with PROPHET and Epidemic protocols under varying networking parameters. Results have shown that CGrAnt achieves the highest delivery ratio and lowest byte redundancy.

6.7.1.3. Opportunistic Content Dissemination

Here, we focus on dissemination of content for delay tolerant applications/services, (i.e. content sharing, advertisement propagation, etc.) where users are geographically clustered into communities. Due to emerging security and privacy related issues, majority of users are becoming more reluctant to interact with strangers and are only willing to share information/content with the users who are previously identified as friends. In this environment, opportunistic communication will not be effective due to the lack of known friends within the communication range. Thus, we have proposed a novel architecture that addresses the issues of lack of trust, timeliness of delivery, loss of user control, and privacy-aware distributed mobile social networking by combining the advantages of distributed decentralized storage and opportunistic communications. We have formally defined a content replication problem in mobile social networks and show that it is computationally hard to solve optimally. Then, we have proposed a community based greedy heuristic algorithm with novel dynamic centrality metrics to replicate content in well-selected users, to maximize the content dissemination with limited number of replication. Using both real world and synthetic traces, we have shown that content replication can attain a large coverage gain and reduce the content delivery latency.

6.7.1.4. Data Offloading-aware Hotspot Deployment

With the steady growth of sales of smart-phones, the demand for services that generate mobile data traffic has grown tremendously. The growing use of traffic data generated from mobile devices overloads the network infrastructure, which is not always prepared to receive such demand. To tackle this problem, we are studying the mobile behavior and resource consumptions of people on a metropolitan area in a major city and turn it into a set of well located WiFi hotspots. For this, we have proposed a data offloading-aware hotspot deployment. It

is methodologically divided as (i) creation of a time dependent weighted graph to represent people's mobility, traffic and its relation with places/locations able to receive a hotspot, (ii) measurement of location's importance and selection of the best-ranked ones. Better positioned hotspots are likely to provide better coverage, and therefore, be able to offload more data.

6.7.2. Collaborations

- Professors Anelise Munaretto and Myriam Regattieri Delgado from Federal Technological University of Parana (UTFPR), Brazil,
- Professors Aruna Seneviratne and Henrik Petander from NICTA and School of EE&T, UNSW, Sydney, Australia,
- Pedro O.S. Vaz de Melo and Antonio A. F. Loureiro, Federal University of Minas Gerais, Brazil,
- Marco Fiore and Frederic Le Mouel from INSA Lyon, France,
- Katia Jaffrès-Runser, University of Toulouse, IRIT/ENSEEIHT, France.

6.8. New services and protocols

Participants: Aline Carneiro Viana, Guilherme Maia.

6.8.1. Scientific achievements

6.8.1.1. Network Discovery

Network discovery is a fundamental task in different scenarios of IEEE 802.15.4-based wireless personal area networks. Scenario examples are body sensor networks requiring health- and wellness-related patient monitoring or situations requiring opportunistic message propagation. Therefore, we have investigated optimized discovery of IEEE 802.15.4 static and mobile networks operating in multiple frequency bands and with different beacon intervals. We designed a linear programming model that allows finding two optimized strategies, named OPT and SWOPT, to deal with the asynchronous and multi-channel discovery problem. We have also proposed a simplified discovery solution, named SUBOPT, featuring a low-complexity algorithm requiring less memory usage. A cross validation between analytical, simulation, and experimental evaluation methods was performed. Our performance studies confirmed improvements achieved by our solutions in terms of first, average, and last discovery time as well as discovery ratio, when compared to IEEE 802.15.4 standard approach and the SWEEP approach known from the literature.

6.8.1.2. Distributed Data Storage

The deployment of large-scale Wireless Sensor Network (WSN) applications (e.g., environment sensing and military surveillance), which operate unattended for long periods of time and generate a considerable amount of data, poses several challenges. One of them is *how to retrieve the sensed data*. To tackle this issue, we have designed ProFlex, a distributed data storage protocol for large-scale heterogeneous wireless sensor networks (HWSNs) with mobile sinks. ProFlex guarantees robustness in data collection by intelligently managing data replication among selected storage nodes in the network. Contrarily to related protocols in the literature, ProFlex considers the resource constraints of sensor nodes and constructs multiple data replication structures, which are managed by more powerful nodes. Additionally, ProFlex takes advantage of the higher communication range of such powerful nodes and uses the long-range links to improve data distribution by storage nodes. When compared with related protocols, we have shown through simulation that Proflex has an acceptable performance under message loss scenarios, decreases the overhead of transmitted messages, and decreases the occurrence of the energy hole problem. Moreover, we have proposed an improvement that allows the protocol to leverage the inherent data correlation and redundancy of wireless sensor networks in order to decrease even further the protocol's overhead without affecting the quality of the data distribution by storage nodes.

6.8.2. Collaborations

- PhD Niels Karowski, Technische Universitat Berlin, Germany,
- Professor Adam Wolisz, Technische Universitat Berlin, Germany,
- Antonio A. F. Loureiro, Federal University of Minas Gerais, Brazil,

IN-SITU Project-Team

6. New Results

6.1. Interaction Techniques

Participants: Caroline Appert, Michel Beaudouin-Lafon, David Bonnet, Anastasia Bezerianos, Olivier Chapuis, Emilien Ghomi, Stéphane Huot, Can Liu, Wendy Mackay [correspondant], Mathieu Nancel, Cyprien Pindat, Emmanuel Pietriga, Theophanis Tsandilas, Julie Wagner.

We explore interaction techniques in a variety of contexts, including individual interaction techniques on mobile devices, the desktop, and very large wall-sized displays, using one or both hands. We also explore interaction with physical objects and across multiple devices, to create mixed or augmented reality systems. This year, we explored interaction techniques based on time (*EWE* and *Dwell-and-Spring*), bimanual interaction on mobile devices (*Bipad*) and interaction on very large wall displays (*Jelly Lenses*, *Looking-Around-Bezels*). We also developed interactive paper systems to support early, creative design (*Pen-based Mobile Assistants*, *Paper Tonnetz*, *Paper Substrates*). We also explored augmented reality systems, using tactile feedback (*TactileSnowboard Instructions*) and tangible interaction (*Mobile AR*, *Combinatorix*) to support learning.

EWE – Although basic interaction techniques, such as multiple clicks or spring-loaded widgets, take advantage of the temporal dimension, more advanced uses of rhythmic patterns have received little attention in HCI. Using temporal structures to convey information can be particularly useful in situations where the visual channel is overloaded or even not available. We introduce Rhythmic Interaction [24] which uses rhythm as an input technique (Figure 10). Two experiments demonstrate that (i) rhythmic patterns can be efficiently reproduced by novice users and recognized by computer algorithms, and (ii) rhythmic patterns can be memorized as efficiently as traditional shortcuts when associated with visual commands. Overall, these results demonstrate the potential of Rhythmic Interaction and richer repertoire of interaction techniques. (*Best Paper award, CHI'12*)

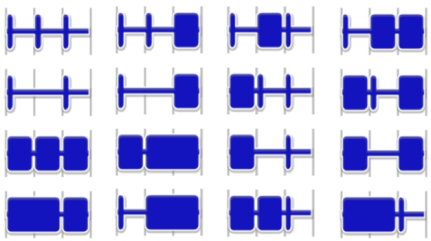

Figure 10. We defined 16 three-beat patterns: Each rectangle represents a tap, the thin gray lines show beats.

Dwell-and-Spring – Direct manipulation interfaces consist of incremental actions that should be reversible The challenge is how to provide users with an effective "undo". Actions such as manipulating geometrical shapes in a vector graphics editor, navigating a document using a scrollbar, or moving and resizing windows on the desktop rarely offer an undo mechanism. Users must manually revert to the previous state by recreating a similar sequence of direct manipulation actions, with a high level of associated motor and cognitive costs. We need a consistent mechanism that supports undo in multiple contexts. *Dwell-and-Spring* [19] uses a spring metaphor that lets users undo a variety of direct manipulations of the interface. A spring widget pops up whenever the user dwells during a press-drag-release interaction, giving her the opportunity to either cancel the

current manipulation (Figure 11) or undo the last one. The technique is generic and can easily be implemented on top of existing applications to complement the traditional undo command. A controlled experiment demonstrated that users can easily discover the technique and adopt it quickly when it is discovered.

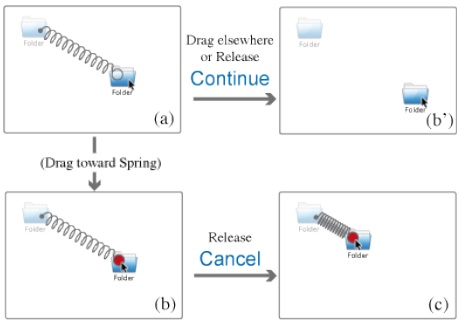

Figure 11. Cancel scenario: The user dwells while dragging an icon (a), which pops up a spring. She either (b) catches the spring's handle and releases the mouse button to cancel the current drag and drop, causing the spring to shrink smoothly (c) and returning the cursor and icon to their original locations, or she continues dragging the spring's handle any direction (b').

BiPad – Although bimanual interaction is common on large tabletops, it is rare on hand-held devices. We take advantage of the advanced multitouch input capabilities available on today's tablets to introduce new bimanual interactio techniques, under a variety of mobility conditions. We found that, when users hold a tablet, the primary function of the non-domininant hand is to provide support, which limits its potential movement. We studied how users "naturally" hold multi-touch tablets to identify comfortable holds, and then developed a set of 10 two-handed interaction techniques that accounts for the need to support the device while interacting with it. We introduced the *BiTouch* design space that extends Guiard's "Kinematic Chain Theory" [49] to account for the *support function* in bimanual interaction. We also designed and implemented the *BiPad* toolkit and set of interactions, which enables us to implement bimanual interaction on multitouch tablets (Figure 12). Finally, a controlled experiment demonstrated the benefits and trade-offs among specific techniques and offered insights for designing bimanual interaction on hand-held devices [31].

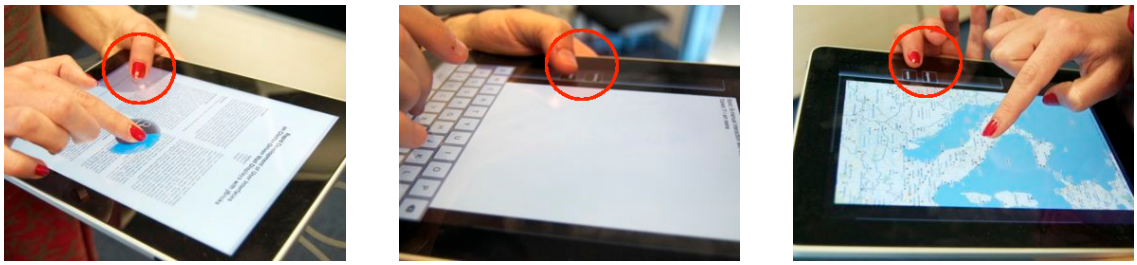

Figure 12. Bimanual interaction on a multitouch tablet with BiPad: (left) navigating in a document; (center) switching to uppercase while typing on a virtual keyboard; (right) zooming a map. The non-dominant hand is holding the device and could perform 'tapa', 'gestures' or 'chords' in order to augment dominant hand's interactions.

Jelly Lenses – Focus+context lens-based techniques smoothly integrate two levels of detail using spatial distortion to connect the magnified region and the context. Distortion guarantees visual continuity, but causes problems of interpretation and focus targeting, partly due to the fact that most techniques are based on statically-defined, regular lens shapes, that result in far-from-optimal magnification and distortion (Figure 13 left and center). JellyLenses [27] dynamically adapt to the shape of the objects of interest, providing detail-in-context visualizations of higher relevance by optimizing what regions fall into the focus, context and spatially-distorted transition regions (Figure 13 -right). A multi-scale visual search task experiment demonstrated that JellyLenses consistently perform better than regular fisheye lenses.

Figure 13. Magnifying the Lido in Venice. (left) a small fisheye magnifies one part of the island (Adriatic sea to Laguna Veneta), but requires extensive navigation to the whole island in detail; (center) a large fisheye magnifies a bigger part of the island, but severely distorts almost the entire image, hiding other islands; (right) a JellyLens automatically adapts its shape to the region of interest, with as much relevant information in the focus as (b) while better preserving context: surrounding islands are almost untouched from (a).

Looking behind Bezels – Using tiled monitors to build wall-sized displays has multiple advantages: higher pixel density, simpler setup and easier calibration. However, the resulting display walls suffer from the visual discontinuity caused by the bezels that frame each monitor. To avoid introducing distortion, the image has to be rendered as if some pixels were drawn behind the bezels. In turn, this raises the issue that a non-negligible part of the rendered image, that might contain important information, is visually occluded. We drew upon the analogy to french windows that is often used to describe this approach, and make the display really behave as if the visualization were observed through a french window [21]. We designed and evaluated two interaction techniques that let users reveal content hidden behind bezels. One enables users to offset the entire image through explicit touch gestures. The other adopts a more implicit approach: it makes the grid formed by bezels act like a true french window using head tracking to simulate motion parallax, adapting to users' physical movements in front of the display. The two techniques work for both single- and multiple-user contexts.

Pen-based Mobile Assistants – Digital pen technology allows easy transfer of pen data from paper to the computer. However, linking handwritten content with the digital world remains difficult as it requires the translation of unstructured and highly personal vocabularies into data structured so as to be understood and processed by a computer. Automatic recognition can help, but is not always reliable: it require active cooperation between users and recognition algorithms. We examined [30] the use of portable touch-screen devices in connection with pen and paper to help users direct and refine the interpretation of their strokes on paper. We explored four bimanual interaction techniques that combine touch and pen-writing, where user attention is divided between the original strokes on paper and their interpretation by the electronic device. We demonstrated these techniques through a mobile interface for writing music (Figure 14) that complements the automatic recognition with interactive user-driven interpretation. An experiment evaluated the four techniques and provided insights as to their strengths and limitations.

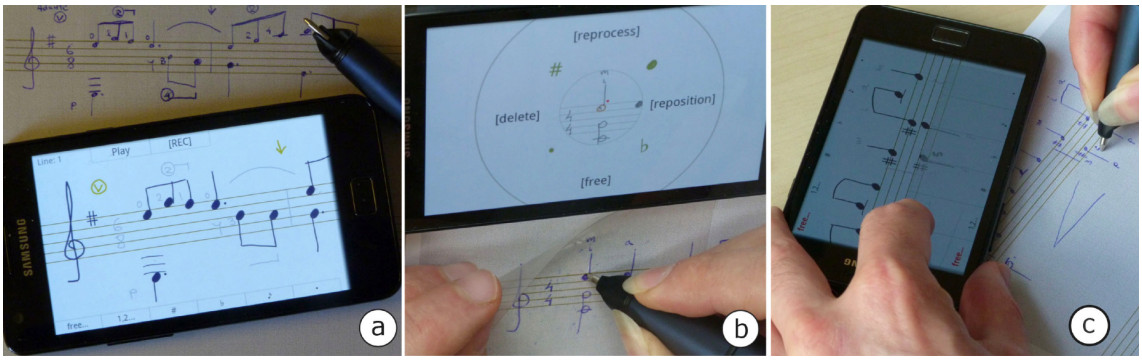

Figure 14. Writing music with a pen and a smartphone. (a) Handwritten score translated by the device. (b) Correcting the recognition of a note over a plastic sheet. (c) Guiding the interpretation of strokes with the left hand.

Paper Tonnetz – Tonnetz are space-based musical representations that lay out individual pitches in a regular structure. We investigated how properties of Tonnetz can be applied in the composition process, including how to represent pitch based on chords or scales and lay them out in a two-dimensional space (Figure 15). *PaperTonnetz*[20] is a tool that lets musicians explore and compose music with Tonnetz representations by making gestures on interactive paper, creating replayable patterns that represent pitch sequences and/or chords. An initial test in a public setting demonstrated significant differences between novice and experienced musicians and led to a revised version that explicitly supports discovering, improvising and assembling musical sequences in a Tonnetz.

Paper Substrates – Our goal is to design novel interactive paper interfaces that support the creative process. We ran a series of participatory design sessions with music composers to explore the concept of "paper substrates" [23]. Substrates are ordinary pieces of paper, printed with an Anoto dot pattern, that suppport a variety of advanced forms of interaction (Figure 15). Each substrate is strongly typed, such as a musical score or a graph, which faciliates interpretation by the computer. The composers were able to create, manipulate and combine layers of data, rearranging them in time and space as an integral part of the creative process. Moreover, the substrates approach fully supported an iterative process in which templates can evolve and be reused, resulting in highly personal and powerful interfaces. We found that paper substrates take on different roles, serving as data containers, data filters, and selectors. The design sessions resulted in several pen interactions and tangible manipulations of paper components to support these roles: drawing and modifying specialized data over formatted paper, exploring variations by superimposing handwritten data, defining programmable modules, aligning movable substrates, linking them together, overlaying them, and archiving them into physical folders.

Tactile Snowboard Instructions – Beginnning snowboarders have difficulty getting instructions and feedback on their performance if they are separated spatially from their coach. Snowboarders can learn correct technique by wearing a system with actuators (vibration motors) attached to the thighs and shoulders, which reminds them to shift their weight and to turn their upper body in the correct direction (Figure 16). A field study with amateur snowboarders demonstrated that these "tactile instructions" are effective for learning basic turns and offered recommendations for incorporating tactile instructions into sports training. *Best Paper award, Mobile HCI'12*

Mobile AR – We examined how new capabilities of hand-held devices, specifically higher resolution screens, camera and localization, can be used to create mobile *Augmented Reality (AR)* to help users learn and manage their interactions with everyday physical objects, such as door codes and home appliances. We explored AR-based mobile note-taking [50] to provide real-time on-screen feedback of physical objects that the user must manipulate, such as entering a door code. Here, the user uses the device to identify the required values of sliders

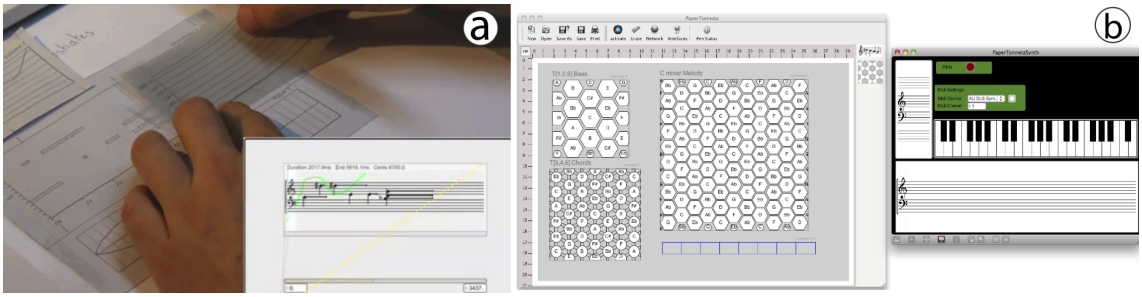

Figure 15. Paper-based interfaces for musical creation. (a) Paper substrates are interactive paper components for working with musical data. (b) PaperTonnetz main interface representing the virtual page with three Tonnetz and one sequencer (left). The Max/MSP patch to play and visualize created sequences (right).

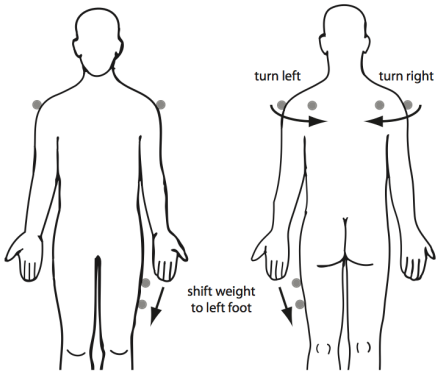

Figure 16. Two vibration motors are placed at each shoulder and laterally at the thigh that points forward during the ride. Arrows illustrate the direction of the stimuli on the skin, labels show the corresponding messages.

and buttons (Figure 17). A controlled experiment showed that mobile AR improved both speed and accuracy over traditional text or picture-based instructions. We also demonstrated that adding real-time feedback in the AR layer that shows the user's actions with respect to the physical controls further increases performance [25]. (*Honorable Mention award, CHI 2012*)

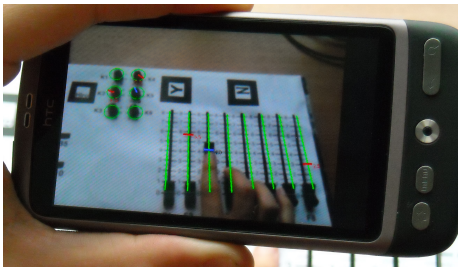

Figure 17. Mobile augmented reality for setting physical controls. Required values are displayed in red and turn blue when set correctly.

Combinatorix – We developed Combinatorix [28], a mixed tabletop system to help groups of students work collaboratively to solve probability problems. Users combine tangible objects in different orders and watch the effects of various constraints on the problem space (Figure 18). A second screen displays an abstract representation, such as a probability tree, to show how their actions influenced the total number of combinations. We followed an iterative participatory design process with college students taking a combinatorics class and demonstrated the benefits of using a tangible approach to facilitate learning abstract concepts.

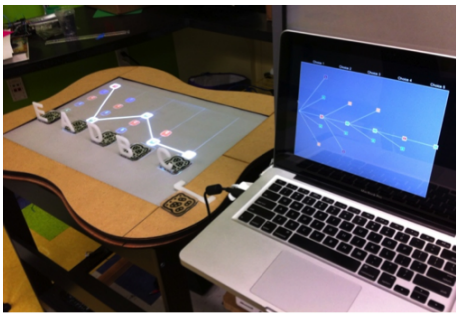

Figure 18. Combinatorix uses tangible objects on an interactive tabletop to control the tabletop display and associated screen, to help users explore and understand complex problems in combinatorial statistics.

6.2. Research Methods

Participants: Caroline Appert, Michel Beaudouin-Lafon, Anastasia Bezerianos, Olivier Chapuis, Jérémie Garcia, Stéphane Huot, Ilaria Liccardi, Wendy Mackay [correspondant], Emmanuel Pietriga.

Human-Computer Interaction is a multi-disciplinary field, with elements of computer science, software engineering, experimental psychology and anthropology. More recently, designers have joined the CHI community, offering an important perspective, but also a different fundamental research paradigm, which

differs from the value systems of engineering and the natural sciences. We explored the paradigm of *Research through Design* [34], which we differentiate from traditional epistemologies in the human sciences. We distinguish design from research-through-design: the end goal is not to produce an artifact, but rather to frame an alternative future and uncover unmet human needs, desires, emotions, and aspirations. We identified three research perspectives that have been adopted within the HCI community: *Projection* explores possible future states, *Place* specifies the context in which design artifacts presented to gather data, and *Point-of-View* identifies the philosophical perspective imposed by researchers. Our goal is to understand what it means to conduct research through design and how to value research-through-design contributions.

In addition to exploring general questions about research paradigms, we also explore more focused questions that apply new research methods to the design of multi-surface environments. Large interactive surfaces (like the WILD platform) are interesting collaborative settings that allow viewing of large amounts of visual information. Before we are ready to use these platforms in real visual analysis situations, where people can place themselves at different positions around the display, we must first understand how the perception of visual information is affected by perspective distortion introduced by varying viewing distances and angles. A deeper understanding of such distortion effects can help visualization researchers design effective visualizations for these spaces and implement interaction techniques to aid in extreme distortion situations. We conducted [16] two studies on the perception of visual variables that encode information, such as Angle, Area, and Length, and found that perception is impacted differently at different locations on the screen, depending on the vertical and horizontal positioning of this information. The visual variable of Length was the most accurately perceived across the display. Our second study examined the effect of perception when participants can move freely in such situations, compared to when they have a static viewpoint, and found that a far but static viewpoint was as accurate but less time consuming than one that included free motion. But we observed that in free motion participants often chose non-optimal walking strategies that can increased perception errors, thus we provide precise recommendations on where and how to move in such environments. This work is a first step towards understanding and predicting the impact of large display environments to people's understanding and tasks.

Annotations play an important role in visual analysis and record-keeping. We discuss the use of annotations on visualization dashboards citeelias:hal-00719221, collections of linked visualizations, focusing on business intelligence analysis through a user centered design process from expert analysts in this domain. The first contribution bridges the gap between expert analyst needs and designers, when it comes to visualization annotations. The second offers a novel approach to annotating of visualizations, "context aware annotations": We annotate data queries directly, rather than image/chart locations. Annotations are present irrespective of the visual data representations users select (different charts, numeric tabular views of their data, etc). We focus particularly on novel annotation aspects made possible by this approach, such as multi-visualization annotations, annotation transparency across charts and data hierarchy levels, and the possibility of recommending to users annotations done to similar data to enable annotation re-use. We also consider new challenges that arise from such our approach, such as what happens to annotations when the underlying data is changed, and provide recommendations and design solutions.

6.3. Engineering of interactive systems

Participants: Caroline Appert, Michel Beaudouin-Lafon [correspondant], Olivier Chapuis, Stéphane Huot, Wendy Mackay, Emmanuel Pietriga, Clément Pillias, Romain Primet.

INSITU continues to develop and apply toolkits to explore and implement interactive systems. Most of the projects listed in the *Interaction Techniques* section either build upon existing toolkits, e.g., *Jelly Lenses* to improve management of focus+context on a wall-sized display, or created new ones, e.g., *BiPad* to create various bimanual interaction techniques for hand-held tablets.

INSITU's primary testbed for exploring multi-surface interaction is the WILD Room [15] (Wall-sized Interaction with Large Datasets), a multisurface environment featuring a wall-sized display, a multitouch table, and various mobile devices. Our goal is to explore the next generation of interactive systems by distributing interaction across these diverse computing devices, enabling multiple users to easily and seamlessly create,

share, and manipulate digital content. Our research strategy is to design an extreme environment that pushes the limits of technology – both hardware and software. To ground the design process, we work with extreme users – scientists whose daily work both inspires and stress-tests the environment as they seek to understand exceptionally large and complex datasets. The WILD room, and the soon-to-be-built WILDER room are part of *DigiScope*, a 22.5 Meuro "Equipement d'Excellence" project led by INSITU.

INSITU's collaboration with the ALMA radio-telescope on the design and implementation of user interfaces for operations monitoring and control continued this year [26], and was eventually transferred to Inria Chile in July (see Section 7.4.2). The ALMA radio-telescope, currently under construction in northern Chile, is a very advanced instrument that presents numerous challenges. From a software perspective, one critical issue is the design of graphical user interfaces for operations monitoring and control that scale to the complexity of the system and to the massive amounts of data users are faced with. Early experience operating the telescope with only a few antennas showed that conventional, WIMP-based user interfaces are not adequate in this context. They consume too much screen real-estate, require many unnecessary interactions to access relevant information, and fail to provide operators and astronomers with a clear mental map of the instrument. They increase extraneous cognitive load, impeding tasks that call for quick diagnosis and action. To address this challenge, the ALMA software division adopted a user-centered design approach in collaboration with members of INSIUT. For the last two years, astronomers, operators, software engineers and human-computer interaction researchers from INSITU have been working on the design of better user interfaces based on state-of-the-art visualization techniques. This eventually led to the joint development of those interface components using various software toolkits, some of them developed at INSITU (Section 5.2).

MACS Project-Team

6. New Results

6.1. Asymptotic and multiscale modeling in biomechanics

6.1.1. Detailed validations of muscle model

Participants: Matthieu Caruel, Dominique Chapelle, Alexandre Imperiale, Philippe Moireau.

Until recently we had only considered simple isotropic passive laws of Mooney-Rivlin type in our muscle model, albeit with an overall behavior already highly non-isotropic due to the fiber-oriented active component. We have now implemented and calibrated a new visco-hyperelastic passive law of exponential and orthotropic type for the hyperelastic part, in better agreement with the models and data generally found in the literature. It should be noted that most experimental data available concern the passive behavior only, indeed. In addition, we implemented a new conservative numerical scheme for the time discretization of the contractile variables. Moreover an original boundary condition of contact type has been successfully applied on several detailed cardiac geometries to represent the interactions between the epicardium, pericardium and the surrounding structures.

Major advances in the understanding of heart contraction cycle can be achieved by testing papillary muscle preparations *in vitro*. Single papillary muscles have an essentially one-dimensional structure suitable for uniaxial mechanical testing, and therefore represent the simplest setup to test the robustness of a model of heart contraction against a vast set of experimental results available in the literature. In collaboration with Y. Lecarpentier (Institut du Coeur, Pitié-Salpêtrière Hospital, Paris) and R. Chabiniok (King's College London), we have further refined and calibrated the muscle mechanical model in order to quantitatively reproduce experimental data from rat cardiomyocytes. These results include the static stress-strain constitutive relation, kinetic response to isotonic loadings, and force-velocity relation see Fig.1 .

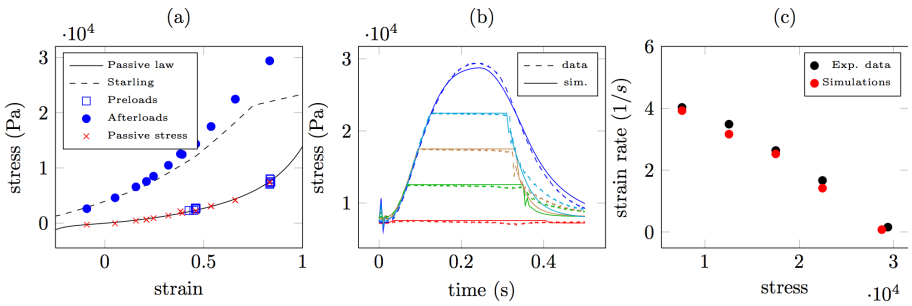

Figure 1. Results of the model compared with experimental data

6.1.2. Multi-scale mechanics of muscle contraction

Participant: Matthieu Caruel.

Muscles are an active tissue material capable of producing force. At the microscale, force is the result of complex interactions between two types of proteins, namely, actin and myosin, which work coherently in very large assemblies ($\sim 10^9$). The passive mechanical response of so-called striated muscles at fast time scales is dominated by long range interactions inducing cooperative behavior without breaking the detailed balance. This leads to such unusual material properties as negative equilibrium stiffness and drastically different behavior in force and displacement controlled loading conditions. Analysing experimental data strongly suggests that muscles are finely tuned to perform close to a critical point (see Fig.2). This work in collaboration with Jean-Marc Allain and Lev Truskinovsky (LMS, Ecole Polytechnique) is the subject of a paper submitted to Physical Review Letters (see [22]).

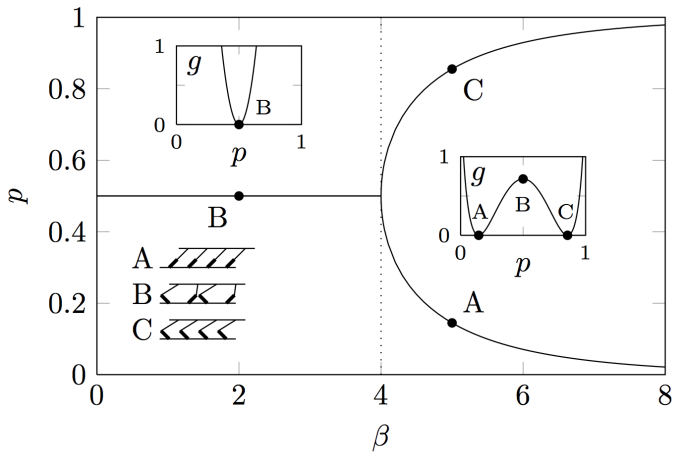

Figure 2. Bifurcation diagram of a model of coupled molecular motors. p is the fraction of motors in the stress generating configuration (post-power-stroke). β is a non dimensional parameter representing the intensity of thermal fluctuations ($\beta \to 0$ represents infinitely strong thermal forces). For $\beta < 4$, the system lives in a mixed configuration (B): the free energy is convex with a minimum at $p = 1/2$ (see the left inset showing the energy landscape g). For $\beta > 4$ the system is organized in two distinct populations (A and C) corresponding to the 2 minima of a non-convex energy landscape (see the right inset). One population is mainly pre-power-stroke (A) while the other is post-power-stroke (C). This is a signature of mechanical cooperativity.

6.1.3. Asymptotic analysis applied to cardiac electrophysiology modeling

Participants: Dominique Chapelle, Annabelle Collin, Jean-Frédéric Gerbeau [(REO team)].

Computational electrophysiology is a very active field with tremendous potential in medical applications, albeit leads to highly intensive simulations. We here propose a surface-based electrophysiology formulation, motivated by the modeling of thin structures such as cardiac atria, which greatly reduces the size of the computational models. Moreover, our model is specifically devised to retain the key features associated with the anisotropy in the diffusion effects induced by the fiber architecture, with rapid variations across the thickness which cannot be adequately represented by naive averaging strategies. Our proposed model relies on a detailed asymptotic analysis in which we identify a limit model and establish strong convergence results. We also provide detailed numerical assessments which confirm an excellent accuracy of the surface-based model – compared with the reference 3D model – including in the representation of a complex phenomenon, namely, spiral waves, see Figure 3 . This work was submitted for publication in "M3AS: Mathematical Models and Methods in Applied Sciences".

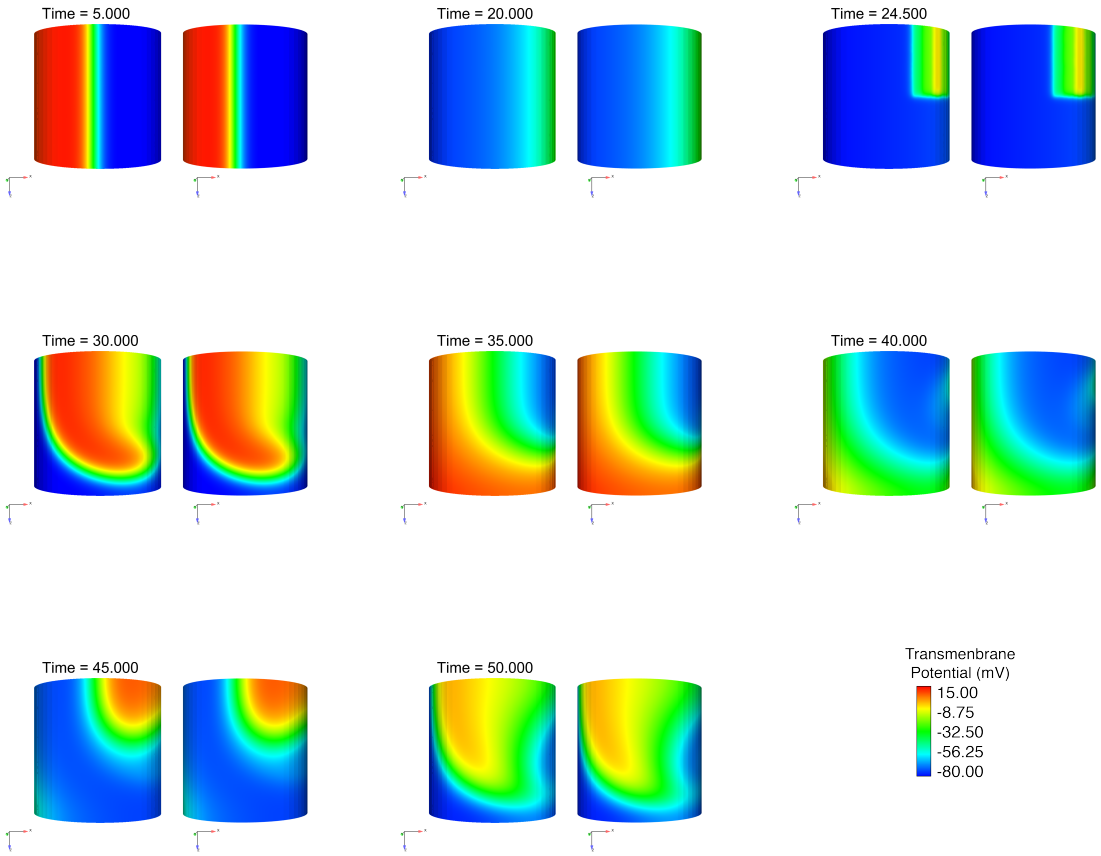

Figure 3. Spiral wave on cylinder – Comparison of asymptotic surface model (left), 3D model (center) and naive 2D model (right) on the midsurface at 8 consecutive times

6.1.4. Cardiac atria electrophysiology surface-based modeling and assessment of physiological simulations

Participants: Dominique Chapelle, Annabelle Collin, Jean-Frédéric Gerbeau [(REO team)].

We aim at validating the 2D (namely, surface-based) electrophysiology model designed for thin cardiac structures with strongly heterogeneous anisotropy, presented in Paragraph 6.1.3 with a real model of the atria. We produced a surface mesh representing the mid-surface of the two atria. We used the bibliography to identify and prescribe the fibers directions at the endocardium and epicardium. Figure 4 displays the simulation results obtained with the surface-based model.

6.1.5. Strong convergence results in the asymptotic behavior of the 3D-shell model

Participants: Dominique Chapelle, Annabelle Collin.

The objective of this work is to establish the strong convergence for the asymptotic analysis of the so-called 3D-shell model presented in [2]. We apply similar methods to those used in the work on "Asymptotic analysis applied to cardiac electrophysiology modeling".

6.2. Estimation in biomechanics

6.2.1. Exponential convergence of an observer based on partial field measurements for the wave equation

Participants: Dominique Chapelle, Philippe Moireau.

We analyze an observer strategy based on partial—that is, in a subdomain—measurements of the solution of a wave equation, in order to compensate for uncertain initial conditions. We prove the exponential convergence of this observer under a nonstandard observability condition, whereas using measurements of the time derivative of the solution would lead to a standard observability condition arising in stabilization and exact controllability. Nevertheless, we directly relate our specific observability condition to the classical geometric control condition. Finally, we provide some numerical illustrations of the effectiveness of the approach. This work in collaboration with M. de Buhan (Univ. Paris V) and N. Cîndea (Univ. Clermont-Ferrand) is published in [13].

6.2.2. Sequential identification of boundary support parameters in a fluid-structure vascular model using patient image data

Participants: Dominique Chapelle, Philippe Moireau.

This work [17] is in collaboration with C. Bertoglio and J.-F. Gerbeau (REO team) and N. Xiao, C.A. Figueroa and C.A. Taylor (Stanford University), where we propose a complete methodological chain for the identification of the corresponding boundary support parameters, using patient image data. We consider distance maps of model to image contours as the discrepancy driving the data assimilation approach, which then relies on a combination of (1) state estimation based on the so-called SDF filtering method, designed within the realm of Luenberger observers and well-adapted to handling measurements provided by image sequences, and (2) parameter estimation based on a reduced-order UKF filtering method which has no need for tangent operator computations and features natural parallelism to a high degree. Implementation issues are discussed, and we show that the resulting computational effectiveness of the complete estimation chain is comparable to that of a direct simulation. Furthermore, we demonstrate the use of this framework in a realistic application case involving hemodynamics in the thoracic aorta. The estimation of the boundary support parameters proves successful, in particular in that direct modeling simulations based on the estimated parameters are more accurate than with a previous manual expert calibration. This paves the way for complete patient-specific fluid-structure vascular modeling in which all types of available measurements could be used to estimate additional uncertain parameters of biophysical and clinical relevance.

This work published in BMMB (impact factor 3.192) can be considered as the first trial of data assimilation using real data in hemodynamics.

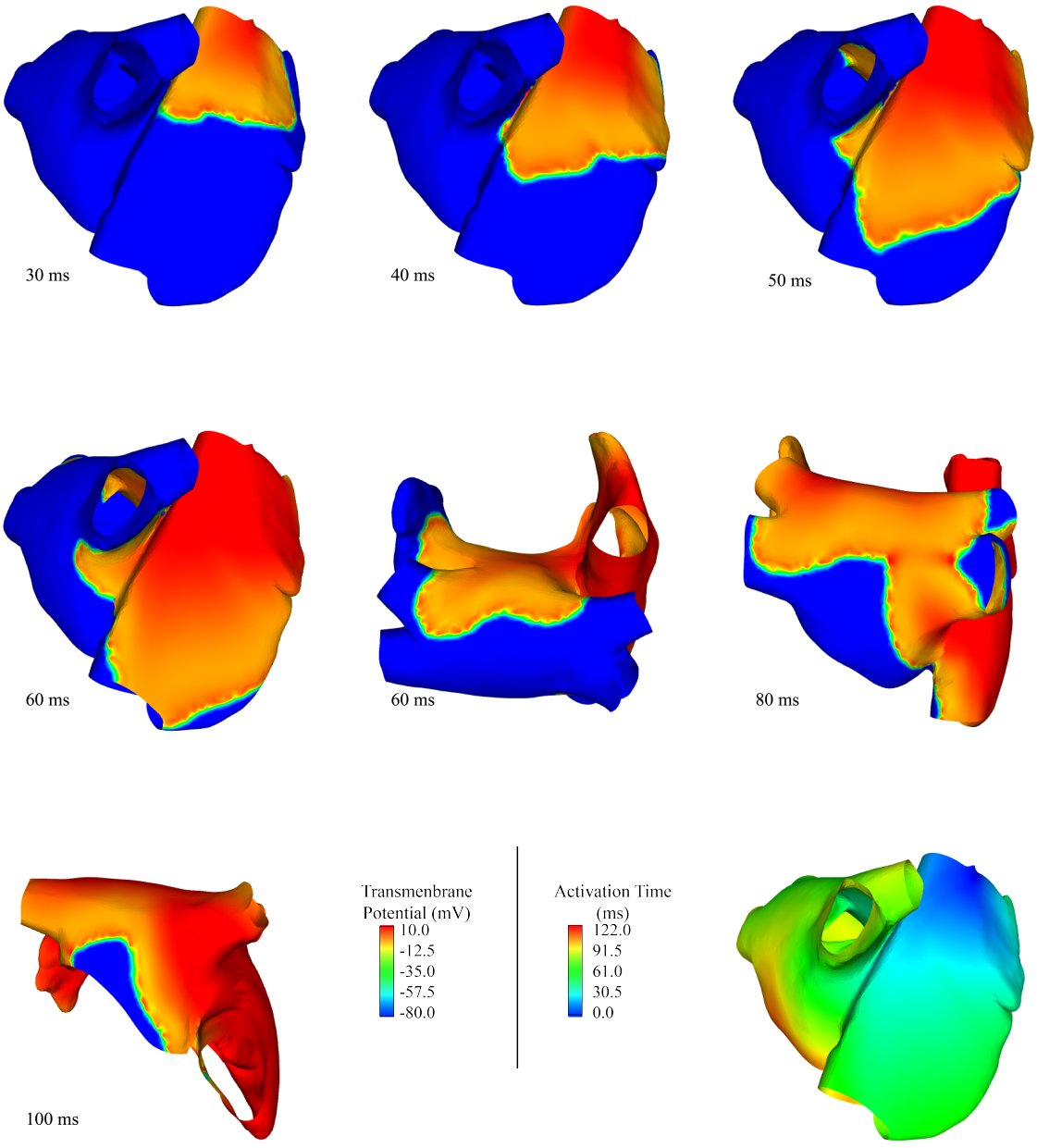

Figure 4. Simulation of atrial depolarization

6.2.3. Filtering strategies using image data

Participants: Alexandre Imperiale, Philippe Moireau, Alexandre Routier.

Some progress has been achieved concerning the Luenberger filtering procedure – also known as nudging – for the cardiovascular system in several directions. We have studied the impact of data interpolation (in time and space) on the method performance (a paper on this subject is being prepared) and, during Alexandre Routier end-of-curriculum internship from INSA Rouen, we have adapted the formalism of currents (inspired by a collaboration with S. Durrleman). This formalism in an elegant way to represent geometric objects (endo- and epicardium surfaces for example) as operators on a test vector space defined on the ambient space. From this key idea the main work was to define a numerical tractable norm on the space of surfaces and derive it with respect to the Lagrangian displacement of the solid domain in order to incorporate such a representation of surfaces into our filtering technique. Among other advantages this new observer requires significantly less prior efforts in terms of image processing.

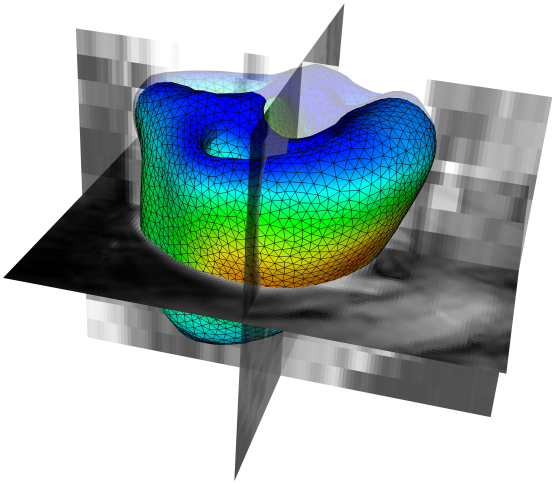

Figure 5. Results of the model compared with experimental data

6.2.4. Formulation of observers for parabolic equations

Participants: Karine Mauffrey, Philippe Moireau.

We are currently working on optimal filtering using observers for a class of evolution PDEs including heat-like equations. As for the optimal control issue,the optimal filtering issue is related to the resolution of a differential Riccati equation. In [25] or [29], the link between the optimal filtering formulations and the derived Riccati equation is done by finite dimension arguments. There exist also other results on the linear quadratic optimal control that are based on infinite dimensional considerations (see, for example, [26] and [28]). A work in progress consists in presenting a direct approach for the optimal filtering issue, using infinite dimension considerations only. Then we should be able to introduce reduced-rank considerations to be able to stabilize only the low frequencies part of the parabolic system, and therefore offer a discretization strategy. This discretization will the be analyzed in details.

6.3. Other topics

6.3.1. Sail modeling

Participants: Dominique Chapelle, Daniele Trimarchi.

This is a collaboration with Marina Vidrascu (REO team) and Stephen Turnock and Dominic Taunton (Southampton University), as part of the recently completed PhD of Daniele Trimarchi. We propose a method of modelling sail type structures which captures the wrinkling behaviour of such structures. The method is validated through experimental and analytical test cases, particularly in terms of wrinkling prediction. An enhanced wrinkling index is proposed as a valuable measure characterizing the global wrinkling development on the deformed structure. The method is based on a pseudo-dynamic finite element procedure involving non-linear MITC shell elements. The major advantage compared to membrane models generally used for this type of analysis is that no ad hoc wrinkling model is required to control the stability of the structure. We demonstrate our approach to analyse the behaviour of various structures with spherical and cylindrical shapes, characteristic of downwind sails over a rather wide range of shape and constitutive parameters. In all cases convergence is reached and the overall flying shape is most adequately represented, which shows that our approach is a most valuable alternative to standard techniques to provide deeper insight into the physical behaviour. Limitations appear only in some very special instances in which local wrinkling-related instabilities are extremely high and would require specific additional treatments, out of the scope of the present study. This work has been published in [20].

6.3.2. PODs for parameter-dependent problems and estimation

Participants: Dominique Chapelle, Philippe Moireau.

This work – submitted to M2AN [24] – is derived from the latest part of Asven Gariah's PhD, jointly supervised by Jacques Sainte-Marie (Bang team) and D. Chapelle, and defended in late 2011. We address the issue of parameter variations in POD approximations of time-dependent problems, without any specific restriction on the form of parameter dependence. Considering a parabolic model problem, we propose a POD construction strategy allowing us to obtain some *a priori* error estimates controlled by the POD remainder – in the construction procedure – and some parameter-wise interpolation errors for the model solutions. We provide a thorough numerical assessment of this strategy with the FitzHugh-Nagumo 1D model. Finally, we give detailed illustrations of the approach in two parameter estimation applications, the first in a variational estimation framework with the FitzHugh-Nagumo model, and the second with a beating heart mechanical model for which we employ a sequential estimation method to characterize model parameters using real image data in a clinical case.

MAXPLUS Project-Team

6. New Results

6.1. Théorie spectrale max-plus et géométrie métrique/Max-plus spectral theory and metric geometry

6.1.1. Introduction

Participants: Marianne Akian, Stéphane Gaubert, Cormac Walsh.

Étant donné un noyau $a : S \times S \to \mathbb{R} \cup \{-\infty\}$, on peut lui associer le problème spectral max-plus

$$\sup_{y \in S} a(x,y) + u(y) = \lambda + u(x), \quad \forall x \in S, \tag{1}$$

dans lequel on cherche le vecteur propre $u : S \to \mathbb{R} \cup \{-\infty\}$ et la valeur propre correspondante $\lambda \in \mathbb{R} \cup \{-\infty\}$. Comme nous l'avons rappelé dans les §3.2 et 3.3 , le problème spectral (9) intervient en contrôle ergodique: l'ensemble S est l'espace des états, et l'application $a(x,y)$ fournit le gain associé à la transition $x \to y$. Le cas où S est fini est classique, l'on a alors un résultat précis de représentation de l'espace propre, à l'aide d'un certain graphe, dit graphe critique. Des résultats existent également lorsque S est compact et que le noyau vérifie certaines propriétés de régularité.

Dans [64], nous avons considéré le cas où S est non compact. Lorsque $\lambda = 0$, l'espace propre est analogue à l'espace des fonctions harmoniques défini en théorie (classique ou probabiliste) du potentiel. En introduisant l'analogue max-plus de la frontière de Martin, nous avons obtenu un analogue de la formule de représentation de Poisson des fonctions harmoniques : toute solution u de (9) peut être représentée sous la forme :

$$u = \sup_{w \in \mathcal{M}_m} w + \mu_u(w) \ , \tag{2}$$

où $\mathcal{M}_m \subset (\mathbb{R} \cup \{-\infty\})^S$ est l'analogue max-plus de la frontière de Martin minimale (l'ensemble des fonctions harmoniques extrémales normalisées), et où μ_u joue le rôle de la mesure spectrale. Nous avons montré aussi que les éléments de l'espace de Martin minimal peuvent être caractérisés comme les limites de "quasi-géodésiques". La frontière de Martin max-plus généralise dans une certaine mesure la frontière d'un espace métrique construite à partir des horo-fonctions (fonctions de Busemann généralisées), ou horo-frontière. Ces résultats inspirent les travaux des sections suivantes, qui portent sur des cas remarquables d'espaces métriques (§6.1.4) ou sur des applications en théorie des jeux (§6.1.2).

English version

Let the kernel $a : S \times S \to \mathbb{R} \cup \{-\infty\}$ be given. One may associate the max-plus spectral equation (9), where the eigenvector $u : S \to \mathbb{R} \cup \{-\infty\}$ and the eigenvalue $\lambda \in \mathbb{R} \cup \{-\infty\}$ are unknown. As we recalled in §3.2 and refmonotone, this spectral problem arises in ergodic optimal control: the set S is the *state space*, and the map $a(x,y)$ is the *transition reward*. The case when S is finite is classical, a precise spectral theorem is known, with a characterisation of the eigenspace in terms of a critical graph. Some results have been shown when S is compact, assuming that the kernel a satisfies some regularity properties.

In [64], we considered the case where S is non-compact. When $\lambda = 0$, the eigenspace is analoguous to the set of harmonic functions defined in classical or probabilistic potential theory. By introducing a max-plus analogue of the classical Martin boundary, we obtained an analogue of the Poisson representation of harmonic functions, showing that any solution u of (9) may be represented as in (10) where $\mathcal{M}_m \subset (\mathbb{R} \cup \{-\infty\})^S$ is a max-plus analogue of the minimal Martin boundary (the set of normalised extremal harmonic functions), and μ_u plays the role of the spectral measure. We also showed that the elements of the minimal Martin boundary can be characterised as limits of certain "almost-geodesics". The max-plus Martin boundary generalises to some extent the boundary of metric spaces defined in terms of horofunctions (generalised Busemann functions), or horoboundary. These results have inspired the work of the next sections, which deal either with remarkable examples of metric spaces (§6.1.4) or applications to zero-sum games (§6.1.2).

6.1.2. Asymptotiques d'itérées d'applications contractantes au sens large et jeux à somme nulle en horizon long/Asymptotics of iterates of nonexpansive mappings and zero-sum games

Participants: Jérôme Bolte, Stéphane Gaubert, Guillaume Vigeral.

Dans [116], on a établi des résultats de type Denjoy-Wolff pour l'étude asymptotique de la valeur d'un jeu répété, lorsque l'horizon tend vers l'infini. On s'intéresse pour cela plus généralement au "taux de fuite" $\rho(f) = \lim_{k\to\infty} d(x, f^k(x))/k$ où f est une application contractante au sens large pour une "métrique" d sur un espace X. Dans le cas des jeux, X est l'espace des fonctions continues sur l'ensemble des états, f est l'opérateur de Shapley, la métrique d est la norme sup (ou une métrique faible, non-symmétrique, comme $d(x, y) = \max_i (x_i - y_i)$), et $\rho(f)$ représente le maximum du paiement moyen quand l'état initial varie. On a montré, que si l'espace X est de courbure négative en un sens faible (Busemann), alors il existe une horofonction h telle que $h \circ f \geq h + \rho(f)$. Ceci entraîne par exemple, lorsque l'espace d'état est compact, l'existence d'un état dont la valeur croît linéairement avec un taux $\rho(f)$, lorsque l'horizon croît. On a travaillé cette année à la généralisation de ce résultat au temps continu (semigroupes associés à des équations d'Hamilton-Jacobi-Isaacs).

Par ailleurs, dans un travail avec J. Bolte (eprint récent [43]), on s'est intéressé, dans le cas où X est de dimension finie, à l'existence de la limite $\lim_k f^k(x)/k$ (vecteur de paiement moyen). On montre que cette limite existe si l'application f est définissable dans une structure o-minimale. Ceci généralise des résultats de Bewley, Kohlberg, et Neyman, qui montraient que la limite existe si f est semi-algébrique. L'extension au cas o-minimal permet notamment de traiter des opérateurs de type "log-exp" apparaissant en contrôle sensible au risque. Ce travail traite aussi de la question de savoir si un jeu dont les fonctions de paiement et de transition sont définissables dans une structure o-minimale admet un opérateur de Shapley f définissable. Un contre exemple montre que f n'est pas forcément définissable dans la même structure, mais l'on montre qu'il en est ainsi dès que les probabilités de transition ont une structure séparable.

English version

In [116], we established Denjoy-Wolff type results for the asymptotic behaviour of the value of a zero-sum game, when the horizon tends to infinity. To this end, we consider more generally the "escape rate" $\rho(f) = \lim_{k\to\infty} d(x, f^k(x))/k$ where f is a nonexpansive self-map of a "metric" space (X, d). In the case of games, X is the space of continuous functions on the set of states, f is the Shapley operator, and d is the sup-norm (or a weak, non-symmetric, metric like $d(x, y) = \max_i (x_i - y_i)$), and $\rho(f)$ represents the "maximal mean payoff", the maximum being taken over all possible initial states. We showed that if the space X is of nonpositive curvature in a mild sense (Busemann), then, there exists a horofunction h such that $h \circ f \geq h + \rho(f)$. This implies in particular, when the space state is compact, the existence of an initial state from which the value grows linearly with a rate $\rho(f)$, as a function of the horizon. We worked this year on the generalisation of this result to the continuous time case (semigroups associated to Hamilton-Jacovi-Isaacs PDE).

Moreover, in a joint work with J. Bolte (recent eprint [43]), we considered the case in which X is finite dimensional, and studied the existence of the limit $\lim_k f^k(x)/k$ (mean payoff vector). We showed that this limit does exist as soon as the map f is definable in an o-minimal structure. This generalizes results of Bewley, Kohlberg, and Neyman, who showed that this limit exists if f is semi-algebraic. The extension to the case

of o-minimal structures allows one in particular to deal with log-exp type operators arising in risk sensitive control. This work also adresses the question of knowing whether a game with definable payment and transition functions has a Shapley operator that is definable in the same structure. We gave a counter example showing that this may not be the case, but showed that the Shapley operator is definable as soon as the transition probabilities have a separable structure.

6.1.3. Isométries de la géométrie de Hilbert/Isometries of the Hilbert geometry

Participants: Cormac Walsh, Bas Lemmens [Kent University, UK].

L'un des intérêts de l'horo-frontière est de renseigner sur le groupe des isométries d'un espace métrique. En effet, ce groupe agit naturellement sur l'horo-frontière, et cette action peut parfois être mieux comprise que l'action du groupe sur l'espace d'origine.

Nous étudions le groupe des isométries pour la métrique de Hilbert. De La Harpe [181] a donné plusieurs conjectures relatives à ce groupe. Nous conjecturons que le groupe des isométries est exactement le groupe des transformations linéaires projectives à moins que le domaine ne soit une coupe d'un cône symmétrique non-Lorentzien. Nous avons démontré précédemment cette conjecture lorsque le domaine est un polytope [135].

Dans le cas général, on peut prouver, en utilisant les horo-fonctions, que si il existe une bijection entre deux cônes homogéne de degré -1, antitone, et d'inverse antitone, ces deux cônes sont symétriques. Nous essayons maintenant de montrer que toute isométrie de Hilbert sur un domaine convexe est la version projective d'un automorphisme linéaire du cône sur le domaine, ou d'une bijection du cône, homogéne de degré -1, qui est antitone et d'inverse antitone. Ce résultat pemettrait de compléter la preuve de la conjecture proposée plus haut.

L'état actuel de l'étude de ce problème est résumé dans un article de Walsh [41] (chapître d'un "handbook on the Hilbert geometry" à paraître).

English version

One use for the horofunction boundary is to study the group of isometries of a metric space. This is because this group has a well defined action on the horoboundary and it is likely that in many cases this action will be easier to understand than the action on the space itself.

We have been investigating the isometries of the Hilbert geometry. De La Harpe [181] has previously made several conjectures about the isometry group of this space. We conjecture that the isometry group is exactly the group of projective linear transformations unless the domain on which the geometry is defined is a cross section of a non-Lorentzian symmetric cone. We have previously proved that this conjecture is true in the case of a polytope domain [135].

In the general case, we can now prove, using horofunctions, that if a bijection between cones is homogeneous of degree -1, order inverting, and has an order inverting inverse, then both cones are symmetric. We are working on showing that every Hilbert isometry on a convex domain arises by considering projectively either a linear automorphism on the cone over the domain, or a homogeneous -1, order inverting bijection on this cone with order inverting inverse. Establishing this result would complete our proof of the above conjecture.

The current state of knowledge about this problem has been summarized in a paper by Walsh [41] that will appear as a chapter in a forthcoming handbook on the Hilbert geometry.

6.1.4. Espace de Teichmüller/Teichmüller space

Participant: Cormac Walsh.

L'espace de Teichmüller d'une surface est un espace métrique composé des structures conformes de cette surface. On peut le voir comme l'ensemble des classes d'équivalence des métriques riemanniennes de cette surface, où deux métriques sont équivalentes si il existe une application conforme homotope à l'identité qui envoie l'une des métriques sur l'autre.

Il existe plusieurs métriques naturelles sur l'espace de Teichmüller. Nous avons travaillé précédemment sur la métrique Lipschitz de Thurston et avons prouvé [174] que l'horo-frontière de cet espace métrique était la frontière de Thurston.

Néanmoins, la métrique la plus utilisées ur l'espace de Teichmüller est la métrique de Teichmüller. L'horo-frontière de cet espace métrique n'est autre que la frontière déja introduite dans la littérature sous le nom de frontière de Gardiner–Masur. Nous étudions cette frontière, en particulier nous donnons explicitement ses points de Busemann [55].

Par la suite, nous avons l'intention d'utiliser cette propriété afin d'étudier les sous-groupes du groupe modulaire, qui est le groupe des isométries de la métrique de Teichmüller.

English version

An interesting metric space is the Teichmüller space of a surface. This is the space of conformal structures on the surface. One may think of it as the space of equivalence classes of Riemannian metrics on the surface, where two such metrics are regarded as being equivalent if there is a conformal map on the surface taking one to the other that is homotopic to the identity.

There are several natural metrics on Teichmüller space. Previously, we have worked with Thurston's stretch metric and have shown [174] that the horofunction boundary with this metric is just the usual Thurston boundary.

However, the most commonly used metric on Teichmüller space is Teichmüller's metric. The horofunction boundary of this metric space turns out to be the same as a previously defined boundary, called the Gardiner–Masur boundary. We have been investigating this boundary. In particular, we have managed to work out explicitly its Busemann points [55].

In future work, we intend to apply this knowledge to study subgroups of the mapping class group, which is the isometry group of the Teichmüller metric.

6.1.5. Consensus non-commutatif et contraction d'opérateurs de Kraus/Noncommutative consensus and contraction of Kraus maps

Participants: Stéphane Gaubert, Zheng Qu.

Dans un travail récent [44], on s'est intéressé à la vitesse de convergence vers l'équilibre d'une itération de la forme $x^{k+1} = T(x^k)$, $x^k \in X$, où T est une application linéaire préservant un cône dans un espace de Banach X, telle que $T(e) = e$, pour un certain vecteur e dans l'interieur du cône. On s'intéresse aussi à l'itération dans l'espace dual, $y^{k+1} = T^*(y^k)$, $y^k \in X^*$, lorsque $\langle y^0, e\rangle = 1$.

Le cas classique est celui où $T(x) = Px$ est un opérateur de Markov. L'itération primale traduit alors la convergence vers le "consensus", et l'itération duale traduit la convergence de la distribution de probabilité en temps k vers l'état stationnaire. Dans ce cas, le taux de contraction (en un coup) $\kappa(P)$ d'une itération primale, pour la semi-norme de Hilbert $\|z\|_H := \max_i z_i - \min_j z_j$, ainsi que le taux de contraction d'une itération duale, pour la métrique en variation totale, coïncident et sont caractérisés par une formule dûe à Doeblin et Dobrushin (coefficient d'ergodicité),

$$\kappa(P) := 1 - \min_{i,j} \sum_{s=1}^{n} \min\left(P_{is}, P_{js}\right).$$

On a donné ici une généralisation de cette formule au cas d'opérateurs abstraits, qui s'applique en particulier aux opérateurs de Kraus qui interviennent en information quantique. Ces derniers opérent sur l'espace des matrices symmétriques, et sont de la forme

$$T(x) = \sum_k a_k x a_k^* \qquad \text{avec} \qquad \sum_k a_k a_k^* = I \ .$$

English version

In a recent work [44], we studied the speed of convergence to equilibrium of an iteration of the form $x^{k+1} = T(x^k)$, $x^k \in X$, where T is a linear map preserving a cone in a Banach space X, such that $T(e) = e$, for some vector e in the interior of the cone. We also considered the iteration in the dual space X^*, $y^{k+1} = T^*(y^k)$, $y^k \in X^*$, where $\langle y^0, e\rangle = 1$.

The classical application arises when $T(x) = Px$ is a Markov operator. Then, the primal iteration represents the dynamics of consensus, whereas the dual iteration represents the evolution of the probability distribution as a function of time. Then, the (one-shot) contraction rate $\kappa(P)$ of the primal iteration, with respect to Hilbert's seminorm $\|z\|_H := \max_i z_i - \min_j z_j$, and the contraction rate of the dual iteration, with respect to the total variation metric, coincide, and are characterized by a formula of Doeblin and Dobrushin (ergodicity coefficient),

$$\kappa(P) := 1 - \min_{i,j} \sum_{s=1}^{n} \min\left(P_{is}, P_{js}\right).$$

We gave here a generalization of this formula to an abstract operators on a cone. This covers in particular the Kraus maps arising in quantum information theory. The latter maps act on the space of symmetric matrices. They can be written as

$$T(x) = \sum_k a_k x a_k^* \qquad \text{with} \qquad \sum_k a_k a_k^* = I \ .$$

6.2. Algèbre linéaire max-plus et convexité abstraite/Max-plus linear algebra and abstract convex analysis

6.2.1. Convexité max-plus ou tropicale/Max-plus or tropical convexity

Participants: Xavier Allamigeon, Stéphane Gaubert, Eric Goubault [CEA], Ricardo Katz [Conicet, Argentine].

On étudie les analogues max-plus ou tropicaux des ensembles convexes. Ceux-ci sont utiles en particulier pour représenter de manière effective les ensembles d'états accessibles de systèmes à événements discrets [9], ils sont aussi apparus récemment en géométrie tropicale, dans toute une série de travaux à la suite de Sturmfels et Develin [98]. Les polyèdres max-plus peuvent aussi être vus comme des limites de déformations de polyèdres classiques, sur lesquels ils donnent un éclairage de nature combinatoire. Toutes ces motivations ont inspiré la recherche d'analogues des résultats fondamentaux d'analyse convexe classique: séparation, projection, points extrémaux, à la suite en particulier de [8].

Dans un travail de X. Allamigeon, S. Gaubert, et E. Goubault [68], [16], on a mis en évidence un critère combinatoire pour la caractérisation des sommets des polyèdres tropicalement convexes. Celui-ci s'exprime à l'aide d'hypergraphes orientés, et de leurs composantes fortement connexes. Ce critère possède la propriété d'être vérifiable en un temps presque linéaire en la taille de l'hypergraphe.

On en déduit un analogue tropical de la méthode de la double description [16] (méthode très utilisée sur les polyèdres classiques, et dûe à Motzkin *et al.* [147]). Cet algorithme permet de calculer les sommets d'un polyèdre défini de façon externe (intersection de demi-espaces ou d'hyperplans tropicaux). Grâce au critère combinatoire précédent, l'algorithme améliore de plusieurs ordres de grandeur les techniques connues jusqu'alors. Ceci est confirmé par de nombreuses expérimentations. Ce travail est motivé par des applications à l'analyse statique [67] et aux systèmes à événements discrets [101], dans lesquelles la manipulation de tels polyèdres est le goulot d'étranglement.

Dans un travail de X. Allamigeon, S. Gaubert, et R. Katz [69], on étend le théorème de McMullen au cas tropical: ce dernier caractérise le nombre maximal de points extrêmes d'un polyèdre, en fonction du nombre d'inégalités qui le définissent et de sa dimension. Nous montrons que la même borne est valide dans le cas tropical (à une modification triviale près). Cependant, le calcul de la borne optimale est encore ouvert dans ce cas.

Il est connu qu'un polyèdre tropical peut être représenté comme l'enveloppe convexe d'un ensemble minimal de points et rayons, donnés par ses sommets et ses rayons extrêmes [112]. Dans un travail réalisé par X. Allamigeon et R. Katz [48], et effectué en partie lors d'une visite de R. Katz à Inria (juillet 2011), on étudie la question duale de la caractérisation des représentations minimales par demi-espaces. On montre qu'un polyèdre tropical possède *essentiellement* une unique représentation minimale par demi-espaces, lorsque leurs apex appartiennent au polyèdre. On montre que les apex de ces demi-espaces non-redondants correspondent à certains sommets du complexe tropical introduit par Develin et Sturmfels [98]. On introduit également un critère combinatoire pour l'élimination de demi-espaces redondants à l'aide d'hypergraphes orientés.

Dans un travail en cours de X. Allamigeon, P. Benchimol, S. Gaubert et R. Katz (débuté lors d'une visite de ce dernier à Inria en novembre 2012), nous étudions la tropicalisation des représentations par demi-espaces des polyèdres convexes sur le corps des séries de Puiseux. Nous démontrons ainsi une conjecture de Develin et Yu [99]. Celle-ci assure qu'étant donné un polytope tropical pur, il existe un polytope *lift* sur les séries de Puiseux, dont les demi-espaces associés aux faces se "tropicalisent" en une représentation par demi-espaces du polytope tropical initial.

English version

We study the max-plus or tropical analogues of convex sets. These have been used in particular to represent effectively the accessible sets of certain discrete event systems [9]. They also appeared in tropical geometry, following the work of Sturmfels and Develin [98]. Max-plus polyhedra can be thought of as limits of deformations of classical polyhedra, on which they give a combinatorial insight. These motivations have inspired the investigation of analogues of basic results of classical convex analysis: separation, projection, representation by extreme points, following [8].

In a work of X. Allamigeon, S. Gaubert, and E. Goubault [16], we introduce a combinatorial criterion for the characterization of the vertices of tropically convex polyhedra. It is expressed in terms of directed hypergraphs and their strongly connected components. This criterion can be verified in almost linear time in the size of the hypergraph.

This allows to develop a tropical analogue of the double description method [16] (this method is widely used for classical convex polyhedra, and is due to Motzkin *et al.* [147]). This algorithm is able to determine all the vertices of a polyhedron defined externally (intersection of tropical half-spaces of hyperplanes). Thanks to the combinatorial criterion mentioned above, the algorithm improves the existing methods by several orders of magnitude. This is confirmed by several experiments. This is motivated by applications to static analysis [67] and discrete event systems [101], in which computing such polyhedra turns out to be the bottleneck.

In a work of X. Allamigeon, S. Gaubert, and R. Katz [69], we extend the McMullen upper bound theorem to the tropical case. This theorem characterises the maximal number of extreme points of a polyhedron, as a function of the number of inequalities defining it, and of the dimension. We show that the same bound is valid in the tropical case (up to a trivial modification). However, computing the optimal bound is an open problem in this case.

It is well-known that a tropical polyhedron can be represented as the convex hull of a minimal set of points and rays, provided by its vertices and extreme rays [112]. In a work of X. Allamigeon and R. Katz [48], partly done during the visit of R. Katz at Inria (July 2011), the dual problem of characterizing the minimal representations by half-spaces is studied. We show that a tropical polyhedron admits *essentially* a unique minimal external representation by half-spaces, provided that their apices belong to the polyhedron. We prove that the apices of these half-spaces correspond to certain vertices of the tropical complex introduced by Develin and Sturmfels [98]. We also establish a combinatorial criterion allowing to eliminate redundant half-spaces using directed hypergraphs.

In an ongoing work of X. Allamigeon, P. Benchimol, S. Gaubert and R. Katz (started during a visit of the latter at Inria in Novembre 2012), we study the tropicalization of the representation by half-spaces of convex polyhedra over the field of Puiseux series. In particular, we prove a conjecture of Develin and Yu [99]. It states that, given a pure tropical polytope, there exists a lifting polytope over Puiseux series, such that the facet-defining half-spaces are "tropicalized" into a representation by half-spaces of the initial polytope.

6.2.2. Convexes max-plus et jeux avec paiements ergodiques/Max-plus convex sets and mean payoff games

Participants: Marianne Akian, Xavier Allamigeon, Stéphane Gaubert, Alexander Guterman [Moscow State University], Ricardo Katz [Conicet, Argentine], Sergei Sergeev [Birmingham, UK].

Dans un travail d'Akian, Gaubert et Guterman [15], on a montré un résultat d'équivalence entre les jeux ergodiques à somme nulle et les systèmes d'inégalités max-plus linéaires: décider la non-vacuité d'un polyèdre tropical est équivalent à vérifier si un jeu déterministe à somme nulle a un paiement moyen par unité de temps positif ou nul. Plus généralement, la même question pour un jeu stochastique à somme nulle est équivalente à vérifier si un convexe tropical (non-polyédral, i.e., défini par un système infini d'inégalités) est vide. Ces résultats sont démontrés à l'aide de techniques de théorie de Perron-Frobenius non-linéaire. Ils sont ensuite appliqués à l'étude de l'indépendance linéaire dans le semi-anneau tropical.

Le résultat de [15] a eu plusieurs retombées.

Dans un travail de Gaubert et Sergeev [24], on réduit le problème spectral tropical de type faisceaux, $Ax = \lambda Bx$, à un jeu paramétrique (ce qui permet de calculer le spectre en temps pseudo-polynômial).

Enfin, dans un travail de Gaubert, Katz, et Sergeev [22], on développe un algorithme de programmation linéaire tropicale (pseudo-polynômial) basé sur cette correspondance avec les jeux répétés.

English version

In a work by Akian, Gaubert and Guterman [15], we showed the equivalence mean payoff games and max-plus linear inequalities: testing whether a tropical polyhedron is non-empty is equivalent to checking whether a mean payoff deterministic game is winning. More generally, checking whether a mean payoff stochastic game is winning is equivalent to checking the non-emptyness of a tropical convex set defined by an infinite family of inequalities. These results are established using techniques of non-linear Perron-Frobenius theory. Then, they are applied to the study of linear independence over the tropical semiring.

The equivalence established in [15] had several consequences.

In a work of Gaubert and Sergeev [24], the tropical spectral problem for matrix pencils, $Ax = \lambda Bx$, is reduced to a parametric game (which allows one to compute the spectrum in pseudo-polynomial time).

Finally, in a work of Gaubert, Katz, and Sergeev [22], a (pseudo-polynomial) tropical linear programming algorithm is developed, based on the same correspondence with mean payoff games.

6.3. Algèbre max-plus, déformations et asymptotiques /Max-plus algebra, deformations and asymptotic analysis

6.3.1. Introduction

Comme indiqué dans le §3.7 , l'algèbre max-plus est la limite d'une déformation de l'algèbre classique, ou plutôt du semi-corps des réels positifs. Elle peut aussi fournir des estimations de ces déformations, puisque

$$\max(a,b) \leq \epsilon \log\left(e^{a/\epsilon} + e^{b/\epsilon}\right) \leq \epsilon \log(2) + \max(a,b) \ . \tag{3}$$

L'utilisation de ces propriétés a déjà conduit dans le passé aux travaux sur les perturbations de valeurs propres [60], [59], [58], ou sur les grandes déviations [1], [62]. Dans les travaux qui suivent, nous exploitons ces propriétés dans des contextes reliés ou similaires à ceux de nos travaux précédents.

English version

As detailled in §3.7 , max-plus algebra is the limit of a deformation of classical algebra, or more precisely of the semi-field of usual real positive numbers. It can also give estimations for these deformations using for instance (11). By using these properties, we already obtained some works on singular perturbations of matrix eigenvalues [60], [59], [58], or on large deviations [1], [62]. In the works described below, we are exploiting again these properties in contexts that are related or similar to those of our earlier works.

6.3.2. Aspects tropicaux des algorithmes de scaling matriciel/Tropical aspects of matrix scaling problems

Participants: Marianne Akian, Stéphane Gaubert, Meisam Sharify Najafabadi [LRI, Paris Sud].

Une partie du travail de thèse de M. Sharify [167] portait sur les méthodes de mise à l'échelle pour améliorer la précision du calcul de valeurs propres. En appliquant les techniques de [58], [59], on montrait notamment que l'ordre de grandeur des valeurs propres d'un faisceau matriciel est donné (sous des conditions de non-dégénerescence) par les valeurs propres tropicales, qui peuvent être calculées de manière robuste, et fournissent ainsi une mise à l'échelle pour calculer les valeurs propres classiques.

Nous avons poursuivi ce travail dans [47], qui a été présenté dans [36]. On calcule cette fois l'ordre de grandeur des valeurs propres d'un polynôme matriciel au moyen des racines tropicales du polynôme obtenu en appliquant une norme donnée aux coefficients. Les racines dépendent de la norme choisie, et la norme de Frobenius est optimale en un certain sens. On obtient des bornes générales pour les ratios entre modules des valeurs propres et racines tropicales qui généralisent les bornes obtenues par Polya et Ostrowski dans le cas de polynômes scalaires. On raffine aussi ces bornes, en particulier lorsque les racines tropicales sont bien séparées les unes des autres.

English version

A part of the PhD work of M. Sharify [167] dealt with scaling methods to improve the accuracy of eigenvalue numerical computions. Applying the techniques of [58], [59], we shown in particular that the order of magnitude of the eigenvalues of a matrix pencil can be determined (under nondegeracy conditions) by computing tropical eigenvalues. The latter can always be computed accurately and provide a scaling which can be combined with standard numerical methods for matrix pencils.

We have pursued this work in [47], which has been presented in [36]. Now we compute the order of magnitude of the eigenvalues of a matrix polynomial by using the tropical roots of the polynomial the coefficients of which are obtained by applying a norm to the coefficients of the matrix polynomial. The tropical roots depend on the chosen norm, and the Frobenius turns out to be optimal in a certain sense. We obtain indeed general bounds on the ratios between the modulus of the eigenvalues of the matrix polynomial and the tropical roots which generalize the bounds of Polya and Ostrowski available for scalar polynomials. We also improve these bounds, in particular when the tropical roots are well separated.

6.3.3. Méthodes tropicales de localisation de valeurs propres de matrices/Tropical methods for the localisation of matrix eigenvalues

Participants: Marianne Akian, Stéphane Gaubert, Andrea Marchesini.

Lors de son stage de M2 dans l'équipe, Andrea Marchesini a obtenu des inégalités de type majorisation entre les valeurs propres d'une matrice et les valeurs propres tropicales de la matrice de ses modules. En particulier, la majoration est une généralisation de l'inégalité de Friedland [108] concernant le rayon spectral.

La thèse d'Andrea Marchesini s'inscrit dans le prolongement de son stage de M2 dans l'équipe et certains des travaux de la thèse de Meisam Sharify [167]. Le but est d'obtenir des inégalités de type majorisation permettant d'estimer a priori les valeurs propres de matrices ou de faisceaux de matrices, en faisant éventuellement intervenir des hypothèses de bon conditionnements. En particulier on recherche la localisation de ces valeurs propres en fonction de valeurs propres de matrices agrégées ou simplifiées. On cherchera aussi à obtenir le même type de localisation ou d'estimation dans le cas des vecteurs propres associés, par exemple en utilisant les techniques de compléments de Schur de [59] ou les idées de Murota [148].

L'idée est ensuite d'utiliser ces résultats de localisation pour améliorer la précision des algorithmes de calcul numérique de valeurs propres de matrices, en particulier en construisant des changements d'échelle exploitant les calculs tropicaux, à effectuer préalablement à l'appel d'algorithmes classiques comme QZ. Les travaux de Stéphane Gaubert et Meisam Sharify [115] ont montré l'intérêt de cette approche, notamment pour les problèmes de faisceaux quadratiques de valeurs propres issus de systèmes mécaniques pour lesquels on dispose de nombreux exemples pathologiques pour les algorithmes existants.

English version

During his M2 internship in the team, Andrea Marchesini has obtained majorization type inequalities between the eigenvalues of a matrix and the tropical eigenvalues of the matrix obtained by applying the modulus entrywise. In particular, the bound is a generalization of the inequality of Friedland [108] concerning the spectral radius.

The PhD thesis follows his M2 internship and some of the works of Meisam Sharify's PhD thesis [167]. The aim is to obtain majorization type inequalities allowing one to estimate the eigenvalues of matrices or matrix polynomials, using possibly assumptions on condition numbers. In particular, one may look for estimates of these eigenvalues using the eigenvalues of aggregated or simplified matrices. One may also try to find the same type of estimates for the associated eigenvectors, for instance by using techniques of Schur complements from [59] or ideas of Murota [148].

One would like to use these estimation results to improve the accuracy of eigenvalue numerical computions, in particular by constructing scaling methods using tropical techniques, which may be used before calling usual algorithms as QZ. The works of Stéphane Gaubert and Meisam Sharify [115] showed the interest of this approach, in particular for quadratic matrix polynomials issued from mechanical systems for which there exists several pathological examples for existing algorithms.

6.3.4. Mesures et applications maxitives/Maxitive measures and maps

Participants: Marianne Akian, Stéphane Gaubert, Paul Poncet.

La thèse de Paul Poncet [154] concernait essentiellement ce que l'on appelle l'analyse idempotente, c'est-à dire l'étude des espaces fonctionnels ou linéaires de dimension infinie sur l'algèbre tropicale, ou tout autre semi-anneau idempotent. Paul Poncet a développé pour cela un point de vue treillis continus comme dans [1], ou plus généralement domaines. Depuis la soutenance en novembre 2011, plusieurs articles issus du manuscrit de thèse sont en cours de publication ou de soumission, et d'autres travaux pousuivant ceux de la thèse sont en cours avec les membres de l'équipe.

La première partie de la thèse traitait des mesures maxitives, en particulier de l'existence d'une densité cardinale ou d'une densité d'une mesure par rapport à une autre (théorème de Radon-Nikodym), et de la régularité d'une mesure maxitive. Paul Poncet donnait en particulier une caractérisation des mesures maxitives régulières à valeurs dans un domaine, qui raffinait le théorème de décomposition des mesures maxitives établi dans [153]. Ces résultats font maintenant l'objet de [54].

La deuxième partie concernait les convexes dans les semi-treillis ou l'algèbre max-plus, pour lesquels Paul Poncet a pu établir des théorèmes de type Krein-Milman, réciproque de Milman, et représentation de Choquet. Un article concernant le cas des semi-treillis [53] a été soumis.

Enfin la troisième et dernière partie qui traitait des semi-groupes inverses dans une tentative d'unification de l'algèbre usuelle et de l'algèbre tropicale fait l'objet de l'article [31].

On sait que les résultats sur les convexes tropicaux de dimension infinie de [154], qui se déduisent soit des résultats correspondants sur les semi-treillis, soit de résultats de théorie des mesures maxitives de la première partie de la thèse [154], permettent de retrouver partiellement les résultats sur la frontière de Martin max-plus décrits dans la section 6.1.1 . Dans un travail commun nous essayons d'obtenir d'autres applications et extensions du théorème de représentation de Choquet tropical. En particulier on considère le cas d'ensembles ordonnés qui ne sont pas forcément des treillis tels que le cône des matrices symmetriques positives muni de l'ordre de Loewner.

English version

The PhD thesis work of Paul Poncet [154] concerned essentially what is called idempotent analysis, that is the study of infinite dimensional functional or linear spaces over tropical algebra, or any other idempotent semiring. For this aim, Paul Poncet developped the point of view of continuous lattices, as in [1], or more generally of domains. Since the defense of his thesis in November 2011, several papers derived from the thesis manuscript have been submitted and some are published or up to be published. Some other works pursuing the thesis work are done with team members.

The first part of the Paul Poncet's thesis concerned maxitive measures, in particular the existence of a cardinal density of a measure, or that of a density of a measure with respect to another (Radon-Nikodym theorem), and the regularity of a maxitive measure. Paul Poncet gave in particular a characterization of domain valued maxitive measures that are regular, which improved the decomposition theorem of maxitive measures stated in [153]. These results are now gathered in [54].

A second part concerned convex sets in lattices or max-plus algebra, for which Paul Poncet showed results such as a Krein-Milman type theorem, a Milman converse type theorem, and a Choquet representation type theorem. A manuscript concerning the case of semilattices [53] has been submitted.

The third and last part which studied inverse semigroups in an attempt to unify usual and tropical algebras is presented in [31].

We know that the results on infinite dimensional tropical convex sets of [154], which are deduced either from the corresponding results on semilattices, or from the results on maxitive measures of the first part of the thesis manuscript, allow one to recover at least partially the results on max-plus Martin boundaries described in Section 6.1.1 . In a joint work, we try to obtain other applications and extensions of the max-plus Choquet representation theorem. In particular, we consider the case of ordered sets that are not necessarily semilattices, such as the cone of nonnegative symmetric matrices endowed with the Loewner order.

6.4. Algorithmes/Algorithms

6.4.1. Méthodes multigrilles pour le contrôle stochastique et les jeux répétés à somme nulle/Multigrid methods for stochastic control and repeated zero sum games

Participants: Marianne Akian, Sylvie Detournay.

L'algorithme d'itération sur les politiques est bien connu pour résoudre efficacement les équations de la programmation dynamique associées à des problèmes de contrôle stochastique avec critère à horizon infini (Howard) ou ergodique (Howard, et Denardo et Fox). Récemment, il a été généralisé au cas de problèmes de jeux à deux joueurs et somme nulle dégénérés (avec paiements ergodiques et de type "multi-chaîne"), au moyen de techniques d'algèbre max-plus et de théorie du potentiel non linéaire [87]. Chaque itération de base de cet algorithme utilise la résolution d'un système d'équations linéaires dont l'opérateur est monotone, mais dont la taille peut être grande, soit parce qu'il provient d'une discrétisation fine d'une équation aux dérivées partielles, soit parce qu'il est associé à un problème discret de grande taille comme le graphe du Web.

Or, la méthode multigrille est l'une des rares méthodes permettant de résoudre, au moins dans les bons cas, des systèmes linéaires en un temps de l'ordre de la taille du système. De plus, alors que la méthode multigrille classique ne s'applique qu'à des discrétisations d'équations aux dérivées partielles elliptiques, la méthode multigrille algébrique (voir par exemple [164]) peut s'appliquer à tout système linéaire présentant des propriétés de monotonie (principe du maximum ou système avec M-matrice).

L'association entre méthodes multigrilles et itérations sur les politiques avait déjà été utilisée et étudiée dans le cas de problèmes de contrôle stochastique actualisé (voir par exemple [57], [65]), ainsi que dans le cas d'un algorithme d'itération sur les politiques simplifié pour le contrôle ergodique (voir par exemple [5]), mais pour lequel il n'existe pas de preuve de convergence. La méthode multigrille algébrique avait été récemment associée à des méthodes d'apprentissage (voir par exemple [180]). Nous l'avions aussi testée dans le cas de l'itération sur les politiques pour des problèmes de jeux à somme nulle actualisés au cours du stage de Shantanu Gangal en 2007.

La thèse de Sylvie Detournay a eu pour but de développer et d'étudier un algorithme associant une méthode d'itération sur les politiques du type de celle introduite par Cochet-Terrasson et Gaubert dans [87] et une méthode multigrille algèbrique, afin de résoudre des problèmes de jeux à somme nulle dégénérés, éventuellement posés directement sous forme discrète. Au cours de sa thèse, Sylvie Detournay a codé l'ensemble des algorithmes en C, en faisant appel éventuellement à des librairies existantes en particulier les méthodes multigrilles algèbriques d'Yvan Notay. L'ensemble des codes nouveaux est déposé sur le projet "pigames" de la gforge et sera disponible librement.

Une première partie de la thèse [11] qui a été publiée dans [14] concerne le cas non dégénéré (actualisé). Elle comprend en particulier des tests sur des discrétisations d'équations aux dérivées partielles d'Hamilton-Jacobi-Bellman ou d'Isaacs, ou d'inéquations variationnelles.

Le reste de la thèse concerne le cas de problèmes avec critère moyen en temps. Sylvie Detournay a en particulier implémenté et raffiné l'algorithme proposé par Cochet-Terrasson et Gaubert [87], en l'associant soit à des méthodes de résolution exacte de systèmes linéaires, soit à des méthodes multigrilles algébriques, en utilisant aussi des méthodes multigrilles multiplicatives pour le calcul de la mesure invariante de chaînes de Markov irréductibles, comme celles introduites par De Sterck. Ceci a permis l'obtention de résultats numériques dans le cas de discrétisations d'équations d'Isaacs associées à des jeux de poursuite déterministes ou aléatoires. Cela a aussi permis de tester de manière systématique l'algorithme sur des instances aléatoires de jeux de type Richman. Certains de ces résultats, ainsi que la présentation de l'algorithme (de manière plus concrète que dans [87], et avec les détails d'implémentation) et les preuves de sa convergence sont regroupés dans le manuscrit [45] écrit avec Jean Cochet-Terrasson et Stéphane Gaubert.

Ces travaux ont aussi conduit à l'introduction dans [11] d'une nouvelle méthode multigrille multiplicative pour le calcul de la mesure invariante de chaînes de Markov irréductibles, qui consiste en l'application de l'algorithme d'itération sur les politiques combiné aux méthodes multigrilles algèbriques au problème de contrôle optimal (à un joueur) avec critère moyen en temps obtenu par transformation log-exp du système linéaire initial. Cette méthode a été testée et comparée aux méthodes multigrilles multiplicatives existantes.

English version

Policy iteration is a powerful and well known algorithm to solve the dynamic programming equation associated to stochatic control (one player game) problems with infinite horizon criterion (Howard) or ergodic criterion (Howard and Denardo and Fox). It has recently been extended to degenerate two players problems (with ergodic payoff and in "multichain" cases) using ideas from max-plus algebra and nonlinear potential theory [87]. One basic iteration of the algorithm consists in solving a linear system the operator of which is monotone, but with a size which may be large since it comes from the discretization of a partial differential equation or since it is associated to a large size discrete problem arising from instance from the Web graph.

For the solution of large size linear systems, the state of art consists of multigrid methods which are often able to solve systems in linear time. Whereas multigrid methods can only be applied to systems that come from discretizations of elliptic partial differential equations, algebraic multigrid methods (see for instance [164]) can be applied to any linear system with monotonicity properties (discrete maximum principle or system with a M-matrix).

The association of multigrid methods with policy iteration has been used and studied in the case of discounted stochastic control problems (see for instance [57], [65]), or in the case of a simplified policy iteration algorithm for ergodic control (see for instance [5]), but for which no proof of convergence is known. Some recent work combines the algebraic multigrid method with learning methods [180]. We also tested it in the case of policy iterations for discounted zero-sum two-player games, during the internship of Shantanu Gangal in 2007.

The aim of the PhD thesis of Sylvie Detournay was to develop and study an algorithm for degenerate two player games (that may come from a discrete time and finite state space model) combining a policy iteration such as the one introduced in [87] and an algebraic multigrid method (AMG). During her thesis, Sylvie Detournay coded all algorithms in C, using eventually existing librairies in particular the algebraic multigrid

libray of Yvan Notay. All new algorithms belong to the gforge project "pigames" and will be distributed openly.

A first part of the thesis manuscript [11], which has published in [14], concerns the nondegenerate (discounted) case. It contains in particular some tests on discretisations of Hamilton-Jacobi-Bellman or Isaacs partial differential equations or variational inequalities.

The rest of the thesis concerns the case of problems with mean-payoff criteria. In particular, Sylvie Detournay has implemented and refined the algorithm proposed by Cochet-Terrasson and Gaubert [87], while associating it either to direct linear solvers, or to the AMG methods already used in the nondegenerate case, and using also multiplicative AMG methods for computing invariant measures of Markov chains, such as the one introduced by De Sterck. This allowed her to obtain numerical results in the case of discretisations of Isaacs equations associated to deterministic or stochastic pursuit games. This also allowed her to test systematically the algorithm on random instances of Richman type games. Some of these results, together with the presentation of the algorithm (in a more practical manner than in [87], with implementation details), and convergence proofs are gathered in the article [45] with Jean Cochet-Terrasson and Stéphane Gaubert.

These works also led to the introduction in [11] of a new multiplicative AMG method for computing invariant measures of irreducible Markov chains. This method consists of the application of the policy iteration algorithm combined with AMG method to the optimal control (or one player) problem with mean-payoff criteria obtained after a log-exp transformation of the initial linear system. It has been tested and compared with previous multiplicative AMG methods.

6.4.2. Algorithmique des polyèdres tropicaux/Algorithmics of tropical polyhedra

Participants: Xavier Allamigeon, Pascal Benchimol, Stéphane Gaubert, Eric Goubault [CEA], Michael Joswig [TU Darmstadt].

X. Allamigeon, S. Gaubert, et E. Goubault, ont développé dans [67], [16] plusieurs algorithmes permettant de manipuler des polyèdres tropicaux. Ceux-ci correspondent aux travaux décrits dans §6.2.1 . Ils permettent notamment de déterminer les sommets et rayons extrêmes d'un polyèdre tropical défini comme intersection de demi-espaces, ou inversement, de calculer une représentation externe à partir d'un ensemble de générateurs. Ces algorithmes sont implémentés la bibliothèque TPLib (voir §5.3).

Dans un travail en cours de X. Allamigeon, P. Benchimol, M. Joswig et S. Gaubert, nous nous intéressons aux problèmes de programmation linéaire tropicale. Nous définissons un analogue tropical de la méthode du simplexe. L'algorithme repose sur une technique de pivotage entièrement combinatoire entre deux points de base, se fondant sur la notion d'hypergraphes tangents.

English version

X. Allamigeon, S. Gaubert, and E. Goubault, have developed in [67], [16] algorithms allowing one to manipulate tropical polyhedra. They correspond to the contributions described in §6.2.1 . In particular, they can be used to determine the vertices and extreme rays of a tropical polyhedron defined as the intersection of half-spaces, or inversely, to compute an external description from a set of generators. These algorithms are implemented in the library TPLib (see §5.3).

In an ongoing work of X. Allamigeon, P. Benchimol, M. Joswig and S. Gaubert, we study the problems of tropical linear programming. We define a tropical analog of simplex algorithm. It relies on a pivoting technique between two basis points, which is entirely combinatorial, and which involves the notion of tangent hypergraphs.

6.4.3. Problèmes d'accessibilité dans les hypergraphes orientés et leur complexité/Reachability problems in directed hypergraphs and their complexity

Participant: Xavier Allamigeon.

Les hypergraphes orientés sont une généralisation des graphes orientés, dans lesquelles chaque arc relie un ensemble de sommets à un autre. Ils jouent un rôle important dans les travaux récents sur la convexité tropicale (voir §6.2.1), puisqu'ils offrent une représentation naturelle des cônes définis sur le sous-semi-anneau booléen $\mathbb{B} = \{-\infty, 0\}$.

Dans un travail de X. Allamigeon [17], on étudie la complexité de problèmes d'accessibilité sur les hypergraphes orientés. Nous introduisons un algorithme de complexité presque linéaire permettant de déterminer les composantes fortement connexes terminales (qui n'accèdent à aucune autre composante si ce n'est elles-mêmes) d'un hypergraphe.

Nous établissons également une borne inférieure sur-linéaire sur la taille de la réduction transitive de la relation d'accessibilité dans les hypergraphes. Cela indique que la relation d'accessibilité dans les hypergraphes orientés est combinatoirement plus complexe que celle des graphes orientés. Cela suggère aussi que des problèmes comme le calcul des composantes fortement connexes est plus difficile sur les hypergraphes que sur les graphes. Nous mettons d'ailleurs en évidence une réduction en temps linéaire du problème du calcul des ensembles minimaux dans une famille d'ensembles donnée, vers le problème du calcul de toutes les composantes fortement connexes d'un hypergraphe. Le problème du calcul des ensembles minimaux a été largement étudié dans la littérature [155], [176], [175], [156], [157], [158], [103], [74], et aucune algorithme en temps linéaire n'est connu à ce jour.

English version

Directed hypergraphs are a generalization of directed graphs, in which the tail and the head of the arcs are sets of vertices. It appears that they play an important role in the recent works on tropical convexity (see §6.2.1), since they offer a natural representation of cones defined over the boolean sub-semiring $\mathbb{B} = \{-\infty, 0\}$.

In a work of X. Allamigeon [17], we study the complexity of reachability problems on directed hypergraphs. We introduce an almost linear-time algorithm allowing to determine the terminal strongly connected components (a component is said to be *terminal* when no other component is reachable from it).

We also establish a super-linear lower bound over the size of the transitive reduction of the reachability relation in directed hypergraphs. This indicates that the reachability relation is combinatorially more complex in directed hypergraphs than in directed graphs. This also suggests that reachability problems such as computing all strongly connected components are likely to be harder in hypergraphs than in graphs. Besides, we show that the minimal set problem can be reduced in linear time to the problem of computing all strongly connected components in hypergraphs. The former problem consists in finding all minimal sets among a given family of sets. It has been well studied in the literature [155], [176], [175], [156], [157], [158], [103], [74], and no linear time algorithm is known.

6.4.4. Approximation max-plus de fonctions valeurs et équations de Riccati généralisées/Max-plus approximation of value functions and generalized Riccati equations

Participants: Stéphane Gaubert, Zheng Qu, Shanjian Tang [Fudan University, Shanghai].

La thèse de Zheng Qu, supervisée par S. Gaubert et S. Tang, porte sur le développement de méthodes tropicales en programmation dynamique approchée.

Les méthodes d'approximation max-plus conduisent à approcher la fonction valeur d'un problème de contrôle ou de jeux par un supremum d'un nombre fini de formes quadratiques, voir notamment [114]. On s'intéresse ici à l'analyse théorique (complexité) ainsi qu'à l'amélioration de ces méthodes. Dans certains cas, ces formes quadratiques sont propagées par des flots d'équations de Riccati généralisées. Afin d'effectuer des analyses d'erreur, on exploite les propriétés de contraction du flot de Riccati pour certaines métriques connues sur le cône des matrices positives, et en particulier pour la métrique de Thompson. Celle-ci n'est rien d'autre que $d_T(A, B) = \| \log \mathrm{spec}\,(A^{-1}B) \|_\infty$, où spec désigne la suite des valeurs propres d'une matrice, et log s'entend composante par composante.

Ceci nous a amené à étudier le problème général du calcul du taux de contraction d'un flot monotone sur un cône, pour la métrique de Thompson. En effet, les propriétés de contraction de l'équation de Riccati standard sont connues (résultats de Bougerol pour la métrique Riemanienne invariante, et de Wojtowski pour la métrique de Thompson), mais les techniques de preuve employées dans ce cadre (semigroupes de matrices symplectiques) ne s'étendent pas aux équations généralisées.

On donne dans [51] une formule explicite générale pour le taux de contraction pour la métrique de Thompson d'un flot monotone, faisant seulement intervenir le générateur du flot et sa dérivée. On a notamment appliqué ce résultat à une équation de Riccati généralisée associé à des problèmes de contrôle stochastique avec critère quadratique, dans lesquels la dynamique comporte un terme bilinéaire en le contrôle et le bruit. On a montré dans ce cas que la métrique de Thompson est la seule métrique de Finsler invariante pour laquelle le flot est nonexpansif, et l'on a caractérisé la constante de contraction locale.

Une application de ces résultats à l'analyse d'une méthode de réduction de la malédiction de la dimension, dûe à McEneaney, a été réalisée récemment par Z. Qu.

English version

The PhD work of Zheng Qu is supervised by S. Gaubert and S. Tang, it aims in particular at developing tropical methods in approximate dynamic programming.

The max-plus methods lead to approach the value function of an optimal control or zero-sum game problem by a supremum of a finite number of quadratic forms, see in particular [114]. We are interested here in the theoretical analysis (complexity) of this class of methods, as well as of their improvement. In certain cases, the quadratic forms are propagated by the flows of generalized Riccati equations. In order to perform an error analysis, we need to use some contraction properties of the Riccati flow, for certain known metrics on the space of positive matrices, like Thompson's metric. The latter is nothing but $d_T(A,B) = \|\log \mathrm{spec}\,(A^{-1}B)\|_\infty$, where spec denotes the sequence of eigenvalues of a matrix, and $\log$ is understood entrywise.

This led us to study the general problem of computing the contraction rate of an order-preserving flow on a cone, with respect to Thompson's metric. Indeed, the contraction properties of the standard Riccati flow are known (theorem of Bougerol for the invariant Riemanian metric, of Wojtowski for the Thompson's metric), but the proof of these properties (based on symplectic semigroups) does not carry over to generalized Riccati equations.

We gave in [51] a general explicit formula for the contraction rate with respect to Thompson's metric of an order-preserving flow, involving only the generator of the flow and its derivative. We applied in particular this result to a generalized Riccati equation, associated to stochastic optimal control problems with a quadratic cost and a bilinear dynamics (presence of a bilinear term between the control and the noise). We showed that in this case, the Thompson's metric is the only invariant Finsler metric in which the generalized Riccati flow is nonexpansive, and we characterized the local contraction rate of this flow.

Z. Qu applied recently these results to the analysis of a method of reduction of the curse of dimensionality, introduced by McEneaney.

6.5. Applications

6.5.1. Introduction

Nous présentons maintenant plusieurs travaux de nature appliquée, touchant à des domaines variés, dans lesquels nous exploitons certaines des techniques mathématiques présentées précédemment, et particulièrement celles qui relèvent de la théorie de Perron-Frobenius non-linéaire et de la convexité tropicale. Ces applications utilisent aussi des techniques d'algèbre linéaire ou d'optimisation convexe.

English version

In this section, we describe several applied works in which we use some of the theoretical tools developed by the team, including non-linear Perron-Frobenius theory and tropical convexity. Some of these applications also make an intensive use of linear algebraic and convex programming methods.

6.5.2. Propriétés des valeurs propres de Perron et de Floquet, et application en chronothérapeutique/Properties of Perron and Floquet eigenvalue, with an application to chronotherapeutics

Participants: Frédérique Billy [Projet BANG, Inria], Jean Clairambault [Projet BANG, Inria], Olivier Fercoq, Stéphane Gaubert, Thomas Lepoutre [Projet BANG puis DRACULA, Inria].

On s'intéresse à des modèles de systèmes dynamiques monotones structurés en âge représentant la croissance de populations de cellules (saines ou tumorales), à la suite de travaux de Clairambault et Perthame. Il s'agit de comprendre l'influence du contrôle circardien sur la croissance des cellules. Dans le cas stationnaire, le taux de croissance est représenté par une valeur propre de Perron. Dans le cas périodique, il s'agit d'une valeur propre de Floquet. Les travaux [40], [18], [77] portent sur l'identification de ces modèles ainsi que sur un problème de contrôle thérapeutique, consistant à minimiser le taux de croissance des cellules tumorales sous une contrainte de non-toxicité du traitement (maintien d'une population de cellules saines). Ce travail s'appuie en particulier sur un algorithme d'optimisation de la valeur propre de Perron d'une matrice développé par Fercoq dans un autre contexte [106].

English version

We study monotone dynamical systems representing the growth of cells (healthy or tumoral), following a work of Clairambault and Perthame. The goal is to understand how the circadian control influences the growth of cells. In the case of stationnary monotone systems, this growth is measured by the Perron root. In the time periodic case, this Perron root is replaced by a Floquet multiplier.

The works [40], [18], [77] deal with the identification of these models, together with a therapeutic control problem, consisting in minimizing the growth rate of tumoral cells, under a non-toxicity constraint (preserving the population of healthy cells). This works relies in particular on a fast algorithm to optimize the Perron eigenvalue of a matrix, developed by Fercoq in a different context [106].

6.5.3. Équations aux dérivées partielles en dynamique des populations/Partial differential equations arising in population dynamics

Participants: Sepideh Mirrahimi, Stéphane Gaubert.

Nous étudions la limite en temps long de dynamiques des populations structurées. Il s'agit de l'étude asymptotique de l'équation suivante

$$\partial_t n_\varepsilon - \varepsilon \Delta n_\varepsilon = \frac{n_\varepsilon}{\varepsilon} R(x, I_\varepsilon), \quad I_\varepsilon(t) := \int \psi(x) n_\varepsilon(x,t) dx. \tag{4}$$

Il est connu qu'asymptotiquement, lorsque le taux de mutation est petit et en temps long, la solution de cette équation se concentre en une masse de Dirac en un point de maximum de $R(\cdot, I_M)$, avec $I_M = \lim_{t\to\infty} \lim_{\varepsilon\to 0} I(t)$. Un tel point s'appelle ESS (Evolutionary stable strategy) en dynamiques adaptatives. On s'intéresse à savoir, dans le cas où le problème admet plusieurs ESS (qui correspondent à des points de maximum de R), vers quel ESS la densité va converger en temps grand. Nous essayons de répondre à cette question en supposant que le taux de mutations est important (comme dans le cas des cellules cancéreuses). Nous voudrions déterminer la limite suivante: $\lim_{\varepsilon\to 0} \lim_{t\to\infty} n_\varepsilon(x,t)$. Une conjecture est que la limite est une masse de Dirac en un point x_M où x_M est le point de maximum de $R(\cdot, I_M)$ au voisinage duquel $R(x, I_M)$ est plus plat (une fonction F dépendant de la hessienne de R est maximisée en x_M). Celle-ci est motivée d'une part par un travail de M. Akian, R. Bapat et S. Gaubert, montrant à l'aide d'outils de théorie spectrale max-plus qu'une propriété analogue est vraie en dimension finie (convergence du vecteur propre de Perron de matrices dont les coefficients sont de la forme $\exp(A_{ij}/\varepsilon)$), et d'autre part par des travaux reliés en théorie de KAM faible (les points de maximum de R correspondent à l'ensemble d'Aubry projeté). L'objectif est donc ici de déterminer quel vecteur propre du problème ergodique est sélectionné à la limite visqueuse.

Nous avons déjà identifié la limite lorsque le taux de mutations tend vers 0 ($\varepsilon \to 0$) en partant de la solution stationnaire de (12). Il nous reste à démontrer que la solution de (12) converge en temps long vers la solution stationnaire. L'analogue discret de ce problème est également une question ouverte à laquelle on s'intéresse.

English version

We study the long-time asymptotic behaviour of structured population models. We consider specially the PDE (12). It is known that asymptotically, when the mutation rate is small, and the time horizon is large, the solution of this equation concentrates to a Dirac mass at a maximum point of $R(\cdot, I_M)$, with $I_M = \lim_{t\to\infty} \lim_{\varepsilon\to 0} I(t)$. Such a limit point is called ESS (Evolutionary stable strategy) in the field of adaptative dynamics. We are interested to know, when there are several ESS (corresponding to several points of maximum of R), to which ESS the density will converge as the horizon tends to infinity. We are studying this question in particular when the mutation rate is large (as in the case of tumor cells), leading to compute the following limit: $\lim_{\varepsilon\to 0} \lim_{t\to\infty} n_\varepsilon(x,t)$. We made a conjecture that the limit is a Dirac mass at a point x_M where among the points of maximum of $R(\cdot, I_M)$, x_M is the one at which $R(x, I_M)$ is the "flatest" (an auxiliary function F depending on the Hessian of R is maximized at point x_M). This is motivated on the one hand by a previous work of M. Akian, R. Bapat and S. Gaubert, showing, through max-plus spectral theory, that an analogous property does hold in finite dimension (convergence of the Perron eigenvector of matrices with coefficients $\exp(A_{ij}/\varepsilon)$), and on the other hand, by related works in weak KAM theory (the points of maximum of R correspond to the projected Aubry set); these works determine the eigenvector of the ergodic problem which is selected by the viscous limit.

We already identified the limit when the mutation rate tends to 0 ($\varepsilon \to 0$), starting from the stationnary solution of (12). We still need to show that the solution of (12) does converge in large time to the stationnary solution. Even the discrete analogue of this problem is an open issue, which we are studying.

6.5.4. Analyse statique de programmes et itération sur les politiques/Static analysis of computer programs and policy iteration

Participants: Assale Adjé [LSV, ENS Cachan], Stéphane Gaubert, Eric Goubault [CEA].

On applique ici des méthodes de théorie des jeux et d'optimisation (analyse convexe abstraite, programmation convexe et non convexe) aux problèmes de point fixe intervenant en analyse statique de programme. On a introduit dans [13] un nouveau domaine en analyse statique, qui étend au cas non-linéaire le domaine des "gabarits" introduit par Manna, Sankaranarayanan, and Sipma [166]. Ce domaine permet de représenter des ensembles accessibles non-convexes (définis par un nombre fini d'inégalités prises dans un dictionnaire). Ceci permet d'intégrer en particulier des informations liées à l'existence de fonctions de Lyapunov, qui sont souvent connues dans les applications issues de l'ingénierie. Nous avons montré dans [13] que des invariants (expérimentalement précis) pouvaient être obtenus en couplant l'itération sur les politiques avec des relaxations de Shor (relaxations SDP de problèmes quadratiques non-convexes), ce qui fournit des abstractions précises de certains programmes numériques (ex: filtres avec seuils).

Un problème important consiste à déterminer le plus petit point fixe (l'algorithme de [13] fournit un point fixe, qui peut ne pas être minimal). Ce problème est abordé dans [26], où l'approche de [13] est comparée avec une approche duale développée par Gawlitza et Seidl.

English version

We apply methods from game theory and optimization (generalized duality, convex and non convex programming) to the fixed point problems arising in static analysis of programs by abstract interpretation. We introduced in [13] a new domain in static analysis, which extends to nonlinear cases the "templates" introduced by Manna, Sankaranarayanan, and Sipma [166]. This domain allows one to represent accessible sets that are non convex. These are defined by finitely many inequalities taken from a dictionnary. This allows one to use in particular the information provided by Lyapunov functions, which are often known in applications arising from engineering. We showed in [13] that experimentally accurate invariants can be obtained by coupling

policy iteration with Shor relaxation (SDP relaxation of convex programming problems). This yields accurate abstractions of some numerical programs, like linear filters with thresholds.

An important problem consists in determining the smallest fixed point (the algorithm of [13] yields a possibly non minimal fixed point). This problem is addressed in [26], in which the approach of [13] is compared with a dual approach developed by Gawlitza and Seidl.

6.5.5. Optimisation du référencement sur la toile/Optimization of web referencing

Participants: Marianne Akian, Mustapha Bouhtou [Orange Labs], Olivier Fercoq, Stéphane Gaubert.

La thèse d'O. Fercoq [12], co-encadrée par M. Akian, M. Bouhtou, et S. Gaubert, financée par un CRE d'Orange Labs, avait pour but d'appliquer des méthodes d'optimisation et de théorie des jeux à l'optimisation de services en lignes. On a tout d'abord étudié le problème de l'optimisation du référencement, que l'on formalise en se donnant par exemple un ensemble d'hyperliens et de ressources obligatoires, dont la nature et la position sur le site web sont déterminées à l'avance par le concepteur. Cet ensemble forme en quelque sorte le squelette du site web. On se donne aussi un ensemble d'hyperliens ou de ressources facultatives, pour lesquels le concepteur du site a certains degrés de liberté (le lien ou le contenu peut être mis sur une page plutôt qu'une autre, voire être omis).

Dans [20], on aborde le problème de l'optimisation du "Pagerank" dans ce cadre, en appliquant des techniques de décision Markovienne classiques et sous-contraintes. Le problème peut en effet se ramener à un problème de contrôle ergodique ou de contrôle ergodique sous contraintes (ergodiques), selon que les contraintes sur les hyperliens sont locales à chaque page ou font intervenir plusieurs pages. On traite à la fois le cas relaxé où les probabilités de passage d'une page à une autre peuvent être des réels positifs quelconques (on peut par exemple supposer que cette probabilité dépend de la position et des caractères utilisés pour l'hyperlien correspondant) et le cas discret où ces probabilités sont uniformes parmis celles qui sont strictement positives (comme dans la modélisation classique conduisant au calcul du Pagerank). On montre que cette famille de problèmes correspondent à des problèmes de programmation dynamique avec un nombre exponentiel de contrôles, mais où les polytopes des mesures de probabilités de transition admettent des oracles de séparation polynômiaux. On obtient de la sorte des résultats de complexité, ainsi que, sous certaines hypothèses, des algorithmes adaptés à des instances de grande taille, couplant programmation dynamique et relaxation Lagrangienne. Ces algorithmes ont été testés sur un fragment du graphe du web.

Un critère de référencement classique, alternatif au pagerank, est donné par le vecteur propre de Perron, comme dans le cas de l'algorithme "HITS" de Kleinberg. O. Fercoq a abordé le problème associé d'optimisation du référencement, qui se révèle plus difficile que celui du pagerank, en raison de l'absence de propriété de convexité. Cependant, il a développé un algorithme rapide et creux (basé sur des propriétés de rang 1 d'opérateurs intervenant dans le calcul de dérivées du critère) permettant de calculer un optimum local du référencement [106].

O. Fercoq a aussi donné un algorithme analogue pour optimiser le score "HOTS" de Tomlin [38]. Cependant, la convergence de l'algorithme original de HOTS n'avait jamais été prouvée. Dans [50], O. Fercoq a identifié le taux de convergence de l'algorithme et de plusieurs de ses variantes grâce à des techniques d'applications contractantes au sens large et aux propriétés des problèmes de flot d'entropie maximale dans un réseau.

La thèse de Fercoq comprend aussi un algorithme de classement permettant de déterminer les pages de Spam parmi un ensemble de pages douteuses, supposant connues un autre ensemble de pages repertoriées comme spam [33]. Cet algorithme exploite les techniques développées pour l'optimisation du PageRank [33].

English version

The goal of the PhD work [12] of O. Fercoq, cosupervised by M. Akian, M. Bouhtou, and S. Gaubert, and supported by a research contract (CRE) of Orange Labs, was to apply optimization and game theory methods to the optimization of online services. We started by investigating the problem of the optimization of referencing, which we modelled by considering a family of compulsory hyperlinks and resources (fixed in advance by the website designer, these constitute the "skeletton" of the website) and also a family of facultative hyperlink or resources (some links may be ommited or some other links may be added).

In [20], we are approaching the problem of the pagerank optimization in this framework, by applying usual and constrained Markov decision processes techniques. This problem can indeed be reduced to an ergodic control problem without or with (ergodic) constraints, depending on the fact that hyperlinks constraints are local to each web page or depend on several web pages. We study the relaxed problem where the transition probabilities from one page to another may be any positive real (one may assume for instance that this probability depends on the position and type used for the corresponding hyperlink), as well as the discrete problem where these probabilities are uniform among the positive ones (as in the usual modelisation leading to the Pagerank). We show that these problems can be reduced to dynamic programming problems with exponentially many discrete actions, in which however the polytopes of transition probability measures admit polynomial time separation oracles. We derive from this approach polynomial time complexity results, as well as under some additional assumption, scalable algorithms (adapted to large web graphs), coupling dynamic programming and Lagrange relaxation. The latter have been tested on a real subgraph of the web.

A classical alternative ranking relies on the Perron eigenvector, as in the case of the algorithm "HITS" by Kleinberg. O. Fercoq treated the associated optimisation problem, which turns out to be harder than in the pagerank case, due to the lack of convexity properties. However, he developed a fast (sparse) algorithm, exploiting the rank 1 properties of operators appearing when computing the derivative of the objective function, allowing one to compute a local optimum [106].

O. Fercoq also developed a similar method to optimize Tomlin's "HOTS" score [38]. However, the convergence of the original HOTS algorithm was not proved. In [50], O. Fercoq has computed the convergence rate of the algorithm and of several of its variants, using techniques of nonexpansive mappings and properties of problems of flow with maximal entropy in a network.

The PhD thesis of Fercoq also comprises a ranking algorithm allowing one to detect spam pages [33] among dubious pages, starting from a seed (set of pages which are surely known to be spam). This algorithm relies on the Pagerank optimization techniques of [33].

6.5.6. Gestion du revenu appliquée à la tarification de services données/Yield management applied to pricing of data services

Participants: Mustapha Bouhtou [Orange Labs], Jean-Baptiste Dumont, Stéphane Gaubert.

Le travail de thèse CIFRE de J-B. Dumont, supervisée par M. Bouhtou et S. Gaubert, porte sur la tarification de services data et la gestion des ressources dans les réseaux mobiles. Celle-ci est abordée à l'aide de techniques de contrôle et d'optimisation stochastique. Dumont a développé un modèle de tarification, permettant d'analyser des mécanismes incitant les clients à reporter leur demande en dehors des periodes les plus chargées.

English version

The CIFRE PhD work of J-B. Dumont is jointly supervised by M. Bouhtou and S. Gaubert. It deals with the pricing of data services and resource allocation in mobile networks. This is addressed through stochastic control and stochastic optimization techniques. Dumont developed a model of pricing, in order to analyse incitations for customers to move their demand from loaded to less loaded time periods.

6.5.7. Vérification de systèmes temps-réels/Verification of real-time systems

Participants: Xavier Allamigeon, Uli Fahrenberg [IRISA], Stéphane Gaubert, Ricardo Katz [Conicet], Axel Legay [IRISA], Søren Ravn [Aalborg University].

Dans [140], Lu, Madsen, Milata, Ravn, Fahrenberg et Larsen ont montré que les polyèdres tropicaux peuvent être utilisés dans le cadre de l'analyse d'accessibilité d'automates temporisés. En effet, les polyèdres tropicaux expriment naturellement des invariants non-convexes, qui sont en fait des disjonctions d'invariants fournis par des DBM (*difference bound matrices*). A ce titre, les polyèdres tropicaux devraient permettre de réduire le nombre de disjonctions réalisées pendant l'analyse d'automates temporisés. Une limitation importante de cette approche est cependant que les polyèdres tropicaux sont topologiquement fermés, et qu'ils ne peuvent donc pas exprimer de contraintes d'inégalités strictes. Ces dernières sont néanmoins fondamentales dans l'analyse de systèmes temps-réels.

Nous avons donc développé une généralisation des polyèdres tropicaux permettant d'exprimer des contraintes mixtes, *i.e.* strictes ou larges. Notre approche repose sur l'utilisation d'inégalités tropicales linéaires à coefficients dans un (quotient du) semi-anneau de germes affines. Afin de réaliser des opérations sur cette nouvelle classe de polyèdres tropicaux, nous avons défini deux nouveaux algorithmes. Le premier est un analogue tropical de l'élimination de Fourier-Motzkin. Celle-ci s'applique plus généralement à des systèmes d'inégalités linéaires sur des semi-anneaux idempotents et totalement ordonnés. Le second algorithme permet de tester si un système de contraintes mixtes admet une solution. Nous montrons en effet que ce problème est équivalent en temps polynomial à la résolution d'un problème de jeux déterministes à somme nulle. Ces deux contributions nous permettent de définir les primitives requises pour l'analyse d'accessibilité d'automates temporisés.

Un autre problème important survenant dans cette application est l'élimination rapide de vecteurs linéairement dépendants (au sens tropical). Pendant son stage à Inria et au CEA (avril-juillet 2012) supervisé par X. Allamigeon, S. Gaubert et E. Goubault, S. Ravn a implémenté un algorithme dont la complexité est reliée à la taille du résultat (*output-sentive complexity*). Il a également implémenté une interface entre la bibliothèque TPLib et l'outil VerifyTAPN (https://launchpad.net/verifytapn).

English version

Lu, Madsen, Milata, Ravn, Fahrenberg and Larsen have shown in [140] that tropical polyhedra can be applied to the reachability analysis of timed automata. Indeed, tropical polyhedra naturally express non-convex invariants, which correspond to disjunctions of invariants provided by DBM (*difference bound matrices*). Consequently, tropical polyhedra should allow to reduce the number of disjunctions arising during the analysis of timed automata. An important limitation of this approach is that tropical polyhedra are topologically closed, and thus they cannot express strict inequality constraints. However, such constraints plays an important role in the analysis of real-time systems.

As a result, we have developed a generalization of tropical polyhedra, in order to express mixed constraints, *i.e.* strict or loose ones. Our approach relies on tropical linear inequalities with coefficients in a (quotient of) the semiring of affine germs. In order to perform operations on this new class of polyhedra, we have introduced two new algorithms. The first one is a tropical analog of Fourier-Moztkin elimination. In fact, it applies more generally to systems of linear inequalities over totally ordered and idempotent semirings. The second algorithm allows to test the feasability of a mixed constraint system. We indeed show that this problem is polynomial-time equivalent to solving mean payoff games. These two contributions allow to define the primitives required by the reachability analysis of timed automata.

Another important problem arising in this application is the fast elimination of linearly dependent vectors (in the tropical sense). During its internship at Inria and CEA (April-July 2012) supervised by X. Allamigeon, S. Gaubert and E. Goubault, S. Ravn has implemented an output-sensitive algorithm to eliminate such vectors. He has also implemented an interface between the library TPLib and the model-checker VerifyTAPN (https://launchpad.net/verifytapn).

MEXICO Project-Team

6. New Results

6.1. Avoiding shared clocks in networks of timed automata

Networks of timed automata (NTA) are widely used to model distributed real-time systems. Quite often in the literature, the automata are allowed to share clocks, i.e. the transitions of one automaton may be guarded by a condition on the value of clocks reset by another automaton. This is a problem when one considers implementing such model in a distributed architecture, since reading clocks a priori requires communications which are not explicitly described in the model.

In [58], we focus on the following question: given an NTA $A_1 \| A_2$ where A_2 reads some clocks reset by A_1, does there exist an NTA $A'_1 \| A'_2$ without shared clocks with the same behavior as the initial NTA? For this, we allow the automata to exchange information during synchronizations only, in particular by copying the value of their neighbor's clocks.

We discuss a formalization of the problem and give a criterion using the notion of contextual timed transition system, which represents the behavior of A_2 when in parallel with A_1. Finally, we effectively build $A'_1 \| A'_2$ when it exists.

6.2. Model checking languages over infinite alphabets

In [61], we consider data words, i.e, strings where each position carries both a label from a finite alphabet and some values from an infinite domain. The latter can be used to represent an unbounded number of process identifiers so that data words are suitable to model the behavior of a concurrent program with dynamic process creation. A variety of formalisms, including logic and automata, have been studied in the literature to specify sets of data words in the context of verification. Most of them focus on the satisfiability problem of very restricted logics, as the general problem is undecidable.

Here, we consider the dual approach of restricting the domain of data words instead of pruning the logic. This allows us to tackle the model-checking problem with respect to monadic second-order (MSO) properties. As model checking is undecidable for nearly all known automata models (including the model presented in the first part of the talk), we introduce data pushdown automata (DPA). DPA come with multiple pushdown stacks (where the access to stacks is bounded by a number of phase switches) and are enriched with parameters that can be instantiated with data values. DPA can model interesting protocols like a leader election protocol with an unknown number of processes. While satisfiability for MSO logic is undecidable (even for weaker fragments such as first-order logic), we show that one can decide if all words generated by a DPA satisfy a given formula from the full MSO logic.

6.3. Construction of Hanf sentences

A classical result by Hanf from the 60s states that first-order formulas over structures of bounded degree are equivalent to boolean combinations of statements of the form: "pattern P occurs at least n times". Hanf's theorem has many model-theoretic applications, in particular in automata theory and database query answering.

However, until recently, no elementary construction was known. In [49], we present the first algorithm that computes a Hanf normal in elementary time. More precisely, our algorithm is triply exponential, which we also show to be optimal.

6.4. A probabilistic Kleene theorem

In [63], we establish a Kleene theorem for (Rabin) probabilistic automata over finite words. Probabilistic automata generalize deterministic finite automata and assign to a word an acceptance probability. For convenient specification of probabilistic queries, we provide probabilistic expressions with probabilistic choice, guarded choice, concatenation, and a star operator. Our expressions are closer to language-theoretic operations than previous calculi for probabilistic systems, which were rather motivated by system modeling than query answering. We prove that probabilistic expressions and probabilistic automata are expressively equivalent. Our result extends to two-way probabilistic automata with pebbles and corresponding expressions.

6.5. A temporal logic for frequency properties

In, [62], we introduce fLTL, a quantitative extension of the widely used specification language LTL that allows us to express relative frequencies by a generalization of temporal operators. This facilitates the specification of requirements such as the deadlines in a real-time system must be met in at least 95% of all cases. For our novel logic, we establish an undecidability result regarding the satisfiability problem but identify a decidable fragment which strictly increases the expressiveness of LTL by allowing, e.g., to express non-context-free properties.

6.6. Adding pebbles to weighted automata: Easy specification & efficient evaluation

In [67], we extend weighted automata and weighted rational expressions with 2-way moves and (reusable) pebbles. We show with examples from natural language modeling and quantitative model-checking that weighted expressions and automata with pebbles are more expressive and allow much more natural and intuitive specifications than classical ones. We extend Kleene-Schützenberger theorem showing that weighted expressions and automata with pebbles have the same expressive power. We focus on an efficient translation from expressions to automata. We also prove that the evaluation problem for weighted automata can be done very efficiently if the number of (reusable) pebbles is low.

6.7. MSO decidability of multi-pushdown systems via split-width

Multi-threaded programs with recursion are naturally modeled as multi-pushdown systems. The behaviors are represented as multiply nested words (MNWs), which are words enriched with additional binary relations for each stack matching a push operation with the corresponding pop operation. Any MNW can be decomposed by two basic and natural operations: shuffle of two sequences of factors and merge of consecutive factors of a sequence. We say that the split-width of an MNW is k if it admits a decomposition where the number of factors in each sequence is at most k. The MSO theory of MNWs with split-width k is decidable. In [66], we introduce two very general classes of MNWs that strictly generalize known decidable classes and prove their MSO decidability via their split-width and obtain comparable or better bounds of tree-width of known classes.

6.8. Contextual Petri nets

Contextual nets (c-nets) are an extension of Petri nets that – unlike ordinary Petri nets – faithfully models concurrent read accesses to shared resources. This is not only interesting from a semantic but also from an algorithmic point of view, as the analysis of such nets can better exploit the fact that concurrent reads are independent and concurrent. In particular, the unfolding of a contextual net may be up to exponentially smaller in certain situations.

In previous work carried out in the Mexico project, we established theoretical foundations [6] and efficient algorithms for constructing c-net unfoldings [42]. More recently, we have investigated verification techniques based on c-nets. These exploit the advantages mentioned above to obtain results more efficiently. The results have been published in the Concur 2012 conference [70]. In parallel, the development of the Cunf tool has continued, see 5.1.2 . We are currently exploring how the technique can be combined with that of merged processes [107] for further speed-ups, and its applications in diagnosis.

6.9. Expressivity and Complexity of Timed Models

In [68], we show how to reliably compute fast-growing functions with timed-arc Petri nets and data nets. This construction provides ordinal-recursive lower bounds on the complexity of the main decidable properties (safety, termination, regular simulation, etc.) of these models. Since these new lower bounds match the upper bounds that one can derive from wqo theory, they precisely characterise the computational power of these so-called "enriched" nets. In [50], we characterize the importance of resources (like counters, channels, or alphabets) when measuring the expressiveness of Well-Structured Transition Systems (WSTS). We establish, for usual classes of well partial orders, the equivalence between the existence of order reflections (non-monotonic order embeddings) and the simulations with respect to coverability languages. We show that the non-existence of order reflections can be proved by the computation of order types. This allows us to extend the current classification of WSTS, in particular solving some open problems, and to unify the existing proofs.

6.10. Concurrent Games on Infinite State Systems

In [65], we propose to study concurrent games on a new extension of Vector Addition Systems with States, where inhibition conditions are added for modeling purposes. Games are a well-suited framework to solve control problems, and concurrent semantics reflect realistic situations where the environment can always produce a move before the controller, although it is never required to do so. This is in contrast with previous works, which focused mainly on turn-based semantics. Moreover, we consider asymmetric games, where environment and controller do not have the same capabilities, although they both have restricted power. In this setting, we investigate reachability and safety objectives, which are not dual to each other anymore, and we prove that (i) reachability games are undecidable for finite targets, (ii) they are 2-EXPTIME-complete for upward-closed targets and (iii) safety games are co-NP-complete for finite, upward-closed and semi-linear targets. Moreover, for the decidable cases, we build a finite representation of the corresponding controllers.

6.11. Rare Event Analysis for Markovian Systems

Model checking real time properties on probabilistic systems requires computing transient probabilities on continuous time Markov chains. Beyond numerical analysis ability, a probabilistic framing can only be obtained using simulation. This statistical approach fails when directly applied to the estimation of very small probabilities. In [60], combining the uniformization technique and extending our previous results, we design a method which applies to continuous time Markov chains and formulas of a timed temporal logic. The corresponding algorithm has been implemented in our tool cosmos. We present experimentations on a relevant system, with drastic time reductions with respect to standard statistical model checking.

Statistical model-checking is an alternative verification technique applied on stochastic systems whose size is beyond numerical analysis ability. Given a model (most often a Markov chain) and a formula, it provides a confidence interval for the probability that the model satisfies the formula. One of the main limitations of the statistical approach is the computation time explosion triggered by the evaluation of very small probabilities. In order to solve this problem, we develop in [59] a new approach based on importance sampling and coupling. The corresponding algorithms have been implemented in our tool cosmos. We present experimentation on several relevant systems, with estimated time reductions reaching a factor of $10^{1}20$.

6.12. Conformance Relations for Labeled Event Structures

In [69], we have extended several well known conformance (ioco style) relations for sequential models, to the concurrent framework of labeled event structures. With the interleaving semantics, the relations we obtained boil down to the same relations defined for labeled transition systemss. By contrast, under the partial order semantics, the relations we obtain allow to distinguish explicitly implementations where concurrent actions are implemented concurrently, from those where they are interleaved, i.e. implemented sequentially. Therefore, these relations will be of interest when designing distributed systems, since the natural concurrency between actions that are performed in parallel by different processes can be taken into account. In particular, the fact of being unable to control or observe the order between actions taking place on different processes will not be considered as an impediment for testing.

A complete testing framework for concurrent systems has been developped. We studied what kind of systems are testable in such a framework and we have proposed sufficient conditions for obtaining a complete test suite. Finally, an algorithm to construct a test suite with such properties was proposed. These result are summarized in a paper that is being prepared for a journal submission.

OAK Team

6. New Results

6.1. Efficient XML and RDF data management

6.1.1. Efficient and safe management of XML and JSON data

We addressed the problem of detecting independence between XML queries and updates. Since the problem is undecidable for XQuery queries and updates, and is intractable even for restricted fragments, we adopted an approximating technique based on a schema-based static analysis. Our analysis turned to be precise and, at the same time, fast to run. Main result about this research line have been published in [6], while the complete study is reported in Federico Ulliana's PhD Thesis (defended in December 12) [5].

To address the problem of manipulating large XML documents via main-memory XQuery engines, largely used for their efficiency and easiness of integration in a programming environment, we developed partitioning techniques for both XQuery queries and updates. Our technique is based on a static analysis over queries and updates (no schema is used) able to infer information that is used to partition the input document, in a streaming fashion. Besides allowing existing main-memory system to scale up in terms of query/update input size, our technique also admits a MapReduce implementation. Main results have been published in [11], while the complete study is reported in Noor Malla's PhD Thesis (defended on September 21) [3].

We also tackled the problem of safe manipulation of JSON data. Some typed and MapReduce-based programming languages for manipulating JSON data have been recently proposed. However, the problem of inferring a schema for untyped JSON data was still open, and having a schema for manipulated data is fundamental for the afore mentioned programming languages. We started investigating technique able to deal with massive JSON data sets. To ensure efficiency, our technique is based on Map-Reduce, while to ensure precision and conciseness it adopts type rewriting rules able to: i) compact as much as possible intermediate inferred types, and ii) to avoid gross approximation when compacting types. Some preliminary results are quite encouraging, and appeared in [21].

6.1.2. Hybrid models for XML and RDF

Considerable energy is spent towards enriching XML data on the web with semantics through annotations. These annotations can range from simple metadata to complex semantic relationships between data items. Although the vision of supporting such annotations is spreading, it still lacks the infrastructure that will enable it. To this end we have proposed a framework enabling the storage and querying of annotated documents. We have introduced (i) the XR data model, in which annotated documents are XML documents described by RDF triples and (ii) the query language XRQ to interrogate annotated documents through their structure and their semantics. A prototype platform XRP for the management of annotated documents has also been developed, to show the relevance of our approach through experiments [9].

6.1.3. RDF query answering

A promising method for efficiently querying RDF data consists of translating SPARQL queries into efficient RDBMS-style operations. However, answering SPARQL queries requires handling *RDF reasoning*, which must be implemented outside the relational engines that do not support it. We have introduced the *database (DB) fragment of RDF*, going beyond the expressive power of previously studied RDF fragments. Within this fragment, we have devised novel *sound and complete techniques* for answering *Basic Graph Pattern (BGP)* queries, exploring the two established approaches for handling RDF semantics, namely reformulation and saturation. In particular, we have focused on handling database *updates* within each approach and proposed a method for incrementally maintaining the saturation; updates raise specific difficulties due to the rich RDF semantics. Our techniques have been designed to be deployed on top of any RDBMS(-style) engine, and we have experimentally studied their performance trade-offs [20], [14], [25].

6.1.4. Efficient and scalable Web Data Entity Resolution

We addressed the problem of detecting multiple heterogeneous representations of a real-world object (often referred to as record linkage, duplicate detection, or entity resolution) in two contexts, i.e., for hierarchical data and for data where relationships between entities form a graph.

Concerning XML entity resolution, we contributed to a novel algorithm that uses a Bayesian network to determine the probability of two XML elements being duplicates. The probability is based both on content and on structure information given by the hierarchical XML model. To efficiently evaluate the Bayesian network to find duplicates, we devised two pruning techniques. Whereas the first is lossless in terms of not loosing any true duplicates, the second pruning heuristic trades off runtime for a somewhat lower accuracy of the duplicate detection result. An experimental evaluation shows that the proposed solutions are capable of outperforming other state-of-the art XML duplicate detection methods [8].

As for duplicate detection in entity graphs, we defined a general framework for algorithms tackling this problem. The general process consists of three steps, namely retrieval, classification, and update. We further proposed an algorithm complying to the framework that leverages an off-the-shelf relational database to store and to efficiently query information (both data and relationships) relevant for duplicate classification. We further extended our framework and algorithm to allow for parallel and batched processing. Our experimental validation on data of up to two orders of magnitude larger than data considered by other state-of-the-art algorithms showed that the proposed methods allow to scale duplicate detection in entity graphs to large volumes of data [7].

6.1.5. Warehousing RDF data

Data warehousing (DW) research has lead to a set of tools and techniques for efficiently analyzing large amounts of multi-dimensional data. As more data gets produced and shared in RDF, analytic concepts and tools for analyzing such irregular, graph-shaped, semantic-rich data are needed. We have introduced *the first all-RDF model for warehousing RDF graphs*. Notably, we have defined *RDF analytical schemas*, themselves full RDF graphs, and *RDF analytical queries*, corresponding to the relational DW star/snowflake schemas and cubes. We have shown how *RDF OLAP operations* can be performed on our RDF cubes. We have also performed experiments validating the practical interest of our approach.

6.2. Cloud-based Data Management

We investigate architectures for storing Web data (in particular, XML documents and RDF graphs) based on commercial cloud platforms. In particular, we have developed the AMADA platform, which operates in a Software as a Service (SaaS) approach, allowing users to upload, index, store, and query large volumes of Web data. Since cloud users support monetary costs directly connected to their consumption of cloud resources, we focus on indexing content in the cloud. We study the applicability of several indexing strategies, and show that they lead not only to reducing query evaluation time, but also, importantly, to reducing the monetary costs associated with the exploitation of the cloud-based warehouse [10], [12], [13].

6.3. Data Transformation Management

When developing data transformations – a task omnipresent in applications like data integration, data migration, data cleaning, or scientific data processing – developers quickly face the need to verify the semantic correctness of the transformation. Declarative specifications of data transformations, e.g. SQL or ETL tools, increase developer productivity but usually provide limited or no means for inspection or debugging. In this situation, developers today have no choice but to manually analyze the transformation and, in case of an error, to (repeatedly) fix and test the transformation.

The above observations call for a more systematic management of a data transformation. Within Oak, we have so far focused on the first phase of the process described above, namely the analysis phase. Leveraging results obtained in previous years (by us and others), we solidified the theory of why-not provenance. Analogously to a distinction between different types of why-provenance, we defined three types of why-not provenance. For each of the three types, we surveyed the semantics employed by different approaches, e.g., set vs. bag semantics or existential vs. universal quantification. We also identified cases of implication and equivalence between why-not provenance of different types. We have leveraged this theoretical work during the design of a novel algorithm that has the potential to overcome usability and efficiency limitations of previous algorithms after further optimization, implementation, and validation in the future. Furthermore, we implemented different approaches for why-provenance and why-not provenance and included them in the Nautilus Analyzer, a system prototype for declarative query debugging. We demonstrated this prototype at CIKM 2012 [15].

PARIETAL Project-Team

6. New Results

6.1. Randomized cluster-based predictive model

Participants: Gaël Varoquaux [Correspondant], Bertrand Thirion, Alexandre Gramfort.

Functional neuroimaging can measure the brain's response to an external stimulus. It is used to perform brain mapping: identifying from these observations the brain regions involved. This problem can be cast into a linear supervised learning task where the neuroimaging data are used as predictors for the stimulus. Brain mapping is then seen as a support recovery problem. On functional MRI (fMRI) data, this problem is particularly challenging as i) the number of samples is small due to limited acquisition time and ii) the variables are strongly correlated. We propose to overcome these difficulties using sparse regression models over new variables obtained by clustering of the original variables. The use of randomization techniques, e.g. bootstrap samples, and clustering of the variables improves the recovery properties of sparse methods. We demonstrate the benefit of our approach on an extensive simulation study as well as two fMRI datasets.

More details can be found in [32].

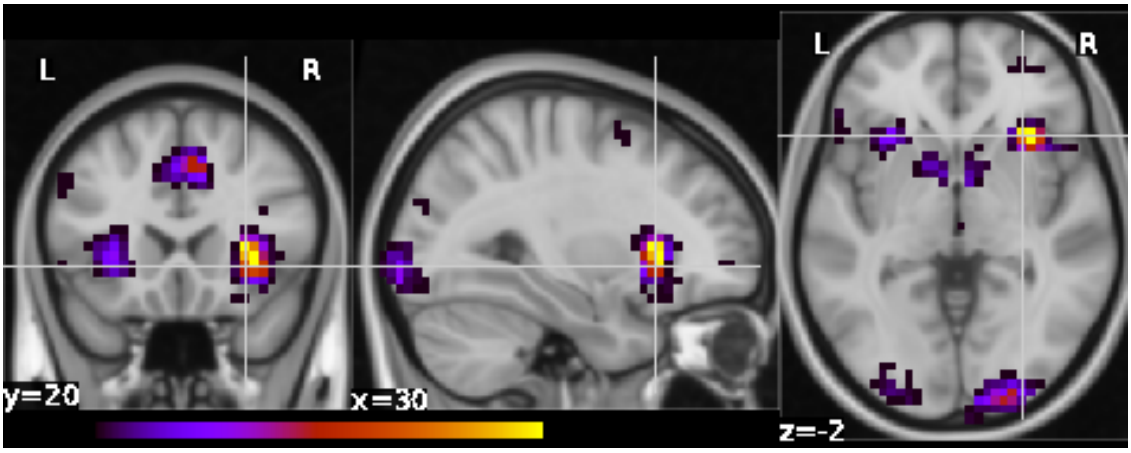

Figure 3. The randomized cluster-based predictive model can be used to predict the behavior of the subject, such as the gain in a gambling task [33]. More importantly, the support of the resulting classifier is indeed sparse and provides a reliable definition of the truly involved regions.

6.2. Random Projections for Outlier Detection

Participants: Gaël Varoquaux, Bertrand Thirion, Jean-Baptiste Poline, Virgile Fritsch [Correspondant].

Medical imaging datasets often contain deviant observations, the so-called outliers, due to acquisition or preprocessing artifacts or resulting from large intrinsic inter-subject variability. These can undermine the statistical procedures used in group studies as the latter assume that the cohorts are composed of homogeneous samples with anatomical or functional features clustered around a central mode. The effects of outlying subjects can be mitigated by detecting and removing them with explicit statistical control. With the emergence of large medical imaging databases, exhaustive data screening is no longer possible, and automated outlier detection methods are currently gaining interest. The datasets used in medical imaging are often high-dimensional and strongly correlated. The outlier detection procedure should therefore rely on high-dimensional statistical multivariate models. However, state-of-the-art procedures are not well-suited for such high-dimensional settings. In this work, we introduce regularization in the Minimum Covariance Determinant framework and investigate different regularization schemes. We carry out extensive simulations to provide backing for practical choices in absence of ground truth knowledge. We demonstrate on functional neuroimaging datasets that outlier detection can be performed with small sample sizes and improves group studies.

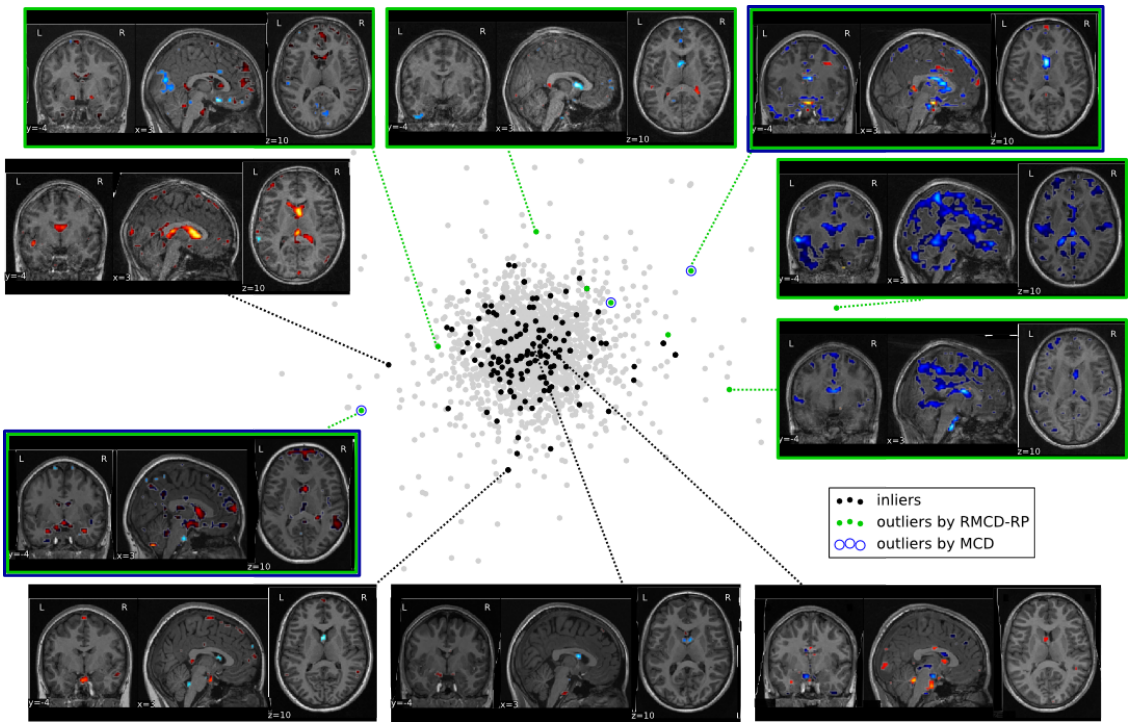

Figure 4. A large set of images can be mined for structures using the regularized MCD framework, which reveals both standard and unusual patterns in these images.

More details can be found in [11].

6.3. Registration of brain images based on Currents

Participants: Pierre Fillard, Bertrand Thirion, Viviana Siless [correspondant].

We present an extension of the diffeomorphic Geometric Demons algorithm which combines the iconic registration with geometric constraints. Our algorithm works in the log-domain space, so that one can efficiently compute the deformation field of the geometry. We represent the shape of objects of interest

in the space of currents which is sensitive to both location and geometric structure of objects. Currents provide a distance between geometric structures that can be defined without specifying explicit point-to-point correspondences. We demonstrate this framework by registering simultaneously T1 images and 65 fiber bundles consistently extracted in 12 subjects and compare it against non-linear T1, tensor, and multi-modal T1+ Fractional Anisotropy (FA) registration algorithms. Results show the superiority of the Log-domain Geometric Demons over their purely iconic counterparts.

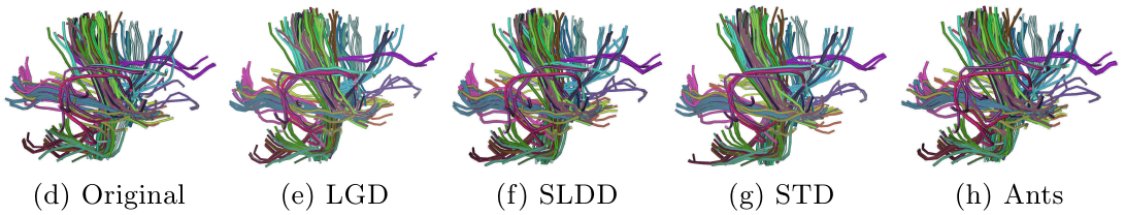

Figure 5. Comparison of the fiber registration through various algorithms. We display a moving and a reference fiber for 29 selected bundles. The proposed approach (SLDD) outperforms state-of-the art alternatives that do not take into account the fiber geometry explicitly.

More details can be found in [31].

6.4. Structured Sparsity for brain mapping

Participants: Gaël Varoquaux [Correspondant], Bertrand Thirion, Alexandre Gramfort.

Reverse inference, or *brain reading*, is a recent paradigm for analyzing functional magnetic resonance imaging (fMRI) data, based on pattern recognition and statistical learning. This approach aims at decoding brain activity by predicting some cognitive variables related to brain activation maps. Reverse inference takes into account the multivariate information between voxels and is currently the only way to assess how precisely some cognitive information is encoded by the activity of neural populations within the whole brain. However, it relies on a prediction function that is plagued by the curse of dimensionality, since there are far more features than samples, i.e., more voxels than fMRI volumes. To address this problem, different methods have been proposed, such as, among others, univariate feature selection, feature agglomeration and regularization techniques. In this work, we consider a sparse hierarchical structured regularization. Specifically, the penalization we use is constructed from a tree that is obtained by spatially-constrained agglomerative clustering. This approach encodes the spatial structure of the data at different scales into the regularization, which makes the overall prediction procedure more robust to inter-subject variability. The regularization used induces the selection of spatially coherent predictive brain regions simultaneously at different scales. We test our algorithm on real data acquired to study the mental representation of objects, and we show that the proposed algorithm not only delineates meaningful brain regions but yields as well better prediction accuracy than reference methods.

More details can be found in [15].

6.5. A Novel Sparse Graphical Approach for Multimodal Brain Connectivity Inference

Participants: Bertrand Thirion, Jean-Baptiste Poline, Gaël Varoquaux [Correspondant], Bernard Ng.

Despite the clear potential benefits of combining fMRI and diffusion MRI in learning the neural pathways that underlie brain functions, little methodological progress has been made in this direction. In this work, we propose a novel multimodal integration approach based on sparse Gaussian graphical model for estimating brain connectivity. Casting functional connectivity estimation as a sparse inverse covariance learning problem, we adapt the level of sparse penalization on each connection based on its anatomical capacity for functional interactions. Functional connections with little anatomical support are thus more heavily penalized. For validation, we showed on real data collected from a cohort of 60 subjects that additionally modeling anatomical capacity significantly increases subject consistency in the detected connection patterns. Moreover, we demonstrated that incorporating a connectivity prior learned with our multimodal connectivity estimation approach improves activation detection.

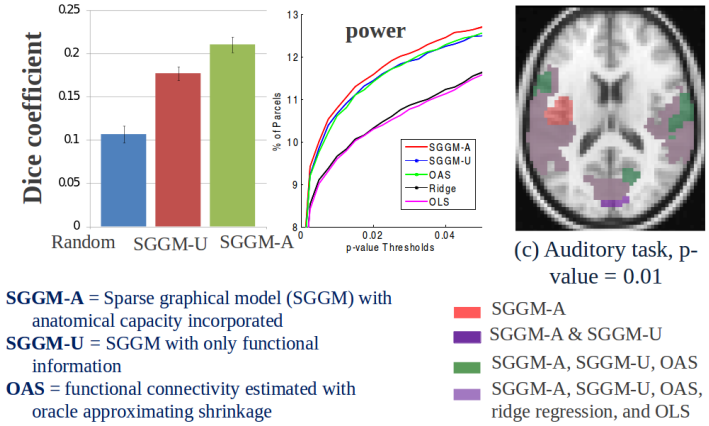

Figure 6. The information conveyed by anatomical connectivity improves the estimation of functional connectivity, as it makes it more reproducible. It also enhances the power of fMRI activation detection studies when used as a prior on these activation maps.

More details can be found in [26].

6.6. Transfer learning for met-analyses of functional neuroimaging datasets

Participants: Bertrand Thirion, Jean-Baptiste Poline, Gaël Varoquaux, Yannick Schwartz [Correspondant].

Typical cohorts in brain imaging studies are not large enough for systematic testing of all the information contained in the images. To build testable working hypotheses, investigators thus rely on analysis of previous work, sometimes formalized in a so-called meta-analysis. In brain imaging, this approach underlies the specification of regions of interest (ROIs) that are usually selected on the basis of the coordinates of previously detected effects. In this work, we propose to use a database of images, rather than coordinates, and frame the problem as transfer learning: learning a discriminant model on a reference task to apply it to a different but related new task. To facilitate statistical analysis of small cohorts, we use a sparse discriminant model that selects predictive voxels on the reference task and thus provides a principled procedure to define ROIs. The benefits of our approach are twofold. First it uses the reference database for prediction, i.e. to provide potential biomarkers in a clinical setting. Second it increases statistical power on the new task. We demonstrate on a set of 18 pairs of functional MRI experimental conditions that our approach gives good prediction. In addition, on a specific transfer situation involving different scanners at different locations, we show that voxel selection based on transfer learning leads to higher detection power on small cohorts.

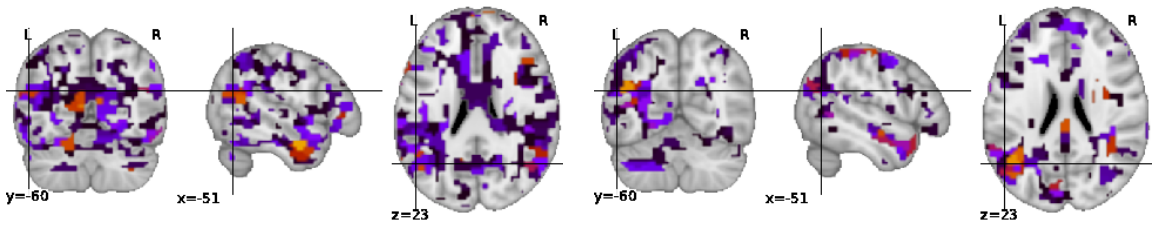

Figure 7. The brain regions that reliably predict that the subject is listening to Korean versus native (french) language (left) are similar to those that can be used to predict that the subject is listening an unintelligible language (jabberwoky) as opposed to their native (french) language (right).

More details can be found in [29] and [30].

6.7. Learning to rank medical images

Participants: Bertrand Thirion, Gaël Varoquaux, Alexandre Gramfort, Fabian Pedregosa [Correspondant].

Medical images can be used to predict a clinical score coding for the severity of a disease, a pain level or the complexity of a cognitive task. In all these cases, the predicted variable has a natural order. While a standard classifier discards this information, we would like to take it into account in order to improve prediction performance. A standard linear regression does model such information, however the linearity assumption is likely not be satisfied when predicting from pixel intensities in an image. In this work we address these modeling challenges with a supervised learning procedure where the model aims to order or rank images. We use a linear model for its robustness in high dimension and its possible interpretation. We show on simulations and two fMRI datasets that this approach is able to predict the correct ordering on pairs of images, yielding higher prediction accuracy than standard regression and multi-class classification techniques.

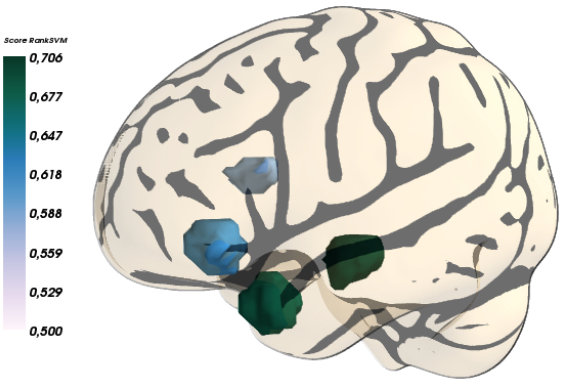

Figure 8. Based on a ranking procedure, the information present in different regions of interest of the brain volume can be used to predict a cognitive feature, in that case the level of complexity of sentences heared by the subject.

More details can be found in [27] and [28].

6.8. Decoding four letter words from brain activations

Participants: Bertrand Thirion, Alexandre Gramfort [Correspondant].

Word reading involves multiple cognitive processes. To infer which word is being visualized, the brain first processes the visual percept, deciphers the letters, bigrams, and activates different words based on context or prior expectation like word frequency. In this contribution, we use supervised machine learning techniques to decode the first step of this processing stream using functional Magnetic Resonance Images (fMRI). We build a decoder that predicts the visual percept formed by four letter words, allowing us to identify words that were not present in the training data. To do so, we cast the learning problem as multiple classification problems after describing words with multiple binary attributes. This work goes beyond the identification or reconstruction of single letters or simple geometrical shapes and addresses a challenging estimation problem, that is the prediction of multiple variables from a single observation, hence facing the problem of learning multiple predictors from correlated inputs.

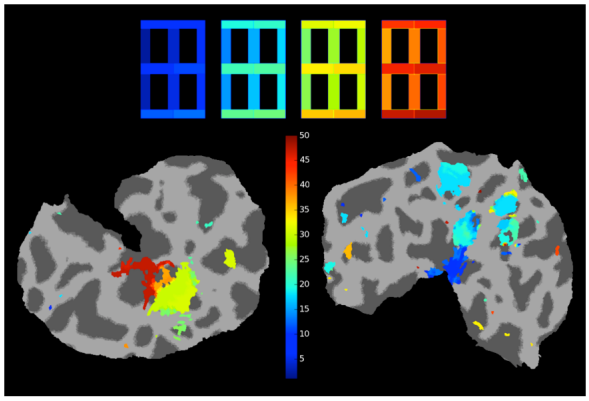

Figure 9. The bars of a word presented on a fixed visual brain activate specific domains of the visual field, and thus can be decoded through this marked. This makes it possible to identify a four letters word presented on a screen.

More details can be found in [22].

PARSIFAL Project-Team

6. New Results

6.1. Recovering Proof Structures in the Sequent Calculus

Participants: Kaustuv Chaudhuri, Stefan Hetzl, Dale Miller.

The *sequent calculus* is often criticized as a proof syntax because it contains a lot of noise. It records the precise minute sequence of operations that was used to construct a proof, even when the order of some proof steps in the sequence is irrelevant and when some of the steps are unnecessary or involve detours. These features lead to several technical problems: for example, cut-elimination in the classical sequent calculus LK, as originally developed by Gentzen, is not confluent, and hence proof composition in LK is not associative. Many people choose to discard the sequent calculus when attempting to design a better proof syntax with the desired properties.

In recent years, there has been a project at Parsifal to recover some of these alternative proof syntaxes by imposing a certain abstraction over sequent proofs. The earliest example of this was in [37], where we showed a class of sequent proofs that were isomorphic to proof nets for multiplicative linear logic. In 2012, we were able to obtain a similar result for first-order classical logic, wherein we defined a class of sequent proofs that are isomorphic to expansion trees, a generalization of Herbrand disjunctions that is in some sense a minimalistic notion of proof for classical logic. This result was published at the CSL 2012 conference [22] and a journal version is in preparation.

Our technique for recovering these dramatically different proof structures directly in the sequent calculus involves the use of *maximal multi-focusing* which gives a syntactic characterization of those sequent proofs that: (1) have a "don't care" ordering of proof steps where the order does not matter, and (2) groups larger logical steps, called *actions*, into a maximally parallel form where only important orderings of actions are recorded. This technique was pioneered at Parsifal, and we have barely scratched the surface of its applications.

6.2. Compact Proof Certificates By Bounded Contractions

Participant: Kaustuv Chaudhuri.

An important engineering question in the ProofCert project is that of communicating, manipulating, and storing formal proof certificates. A fully detailed proof certificate, especially one generated by proof search, can be very large. Using such proofs would require a high bandwidth interface between the proof producer and consumer, which limits the scalability of the *ensemble of proving systems* approach. It is therefore natural to ask if there are more compact formats for proof certificates. The ideal format would have a tunable level of detail, so that the size of the certificates can be tailored to the application domain.

Suppose the proof consumer is equipped with some proof search capabilities. What then needs to be transmitted to the consumer to guarantee that it can check a proof within desired complexity bounds? It turns out that there is a systematic and general answer to this problem: use *focusing* and record only the "decision" rules of focusing in the proof certificate. From a high level perspective, this answer is equivalent to designing a proof system where the contraction rules are carefully bounded.

A proposal along these lines was published at the CPP 2012 conference [21]. In fact, this paper solves a harder than necessary problem by building proof certificates for linear logic, where unconstrained proof search has very high complexity even in the propositional fragment. The proposed solution is a spectrum of certificates that trades off the size of the certificate for the complexity of checking the certificate. At one end we have a very compact certificate that essentially amounts to a maximum depth of the proof, but reconstructing a proof with only a depth bound tends to be infeasible as the search space grows super-exponentially with the depth. Certificates at other end of the spectrum contain information about all the contractions in the proof; these certificates can be checked deterministically, in time proportional to the size of the certificate. Moreover, there is a simple abstraction mechanism between different levels of detail in this spectrum that allows for a *proof elaborator* to alter the level of detail in the certificate.

6.3. A Two-level Approach to Reasoning about Computation

Participant: Dale Miller.

In a paper that appeared in the J. of Automated Reasoning, Gacek, Miller, and Nadathur [12] described the foundations and architecture of a new interactive theorem prover capable of reasoning with rich collections of inductive and coinductive relations. This prover, called Abella, also contains the "generic" quantifier ∇ that provides a direct and elegant treatment of term-level binding.

A novel aspect of Abella is that it can define provability in various simple logics and can also reason about provability in such logics. The current system includes a *specification logic* that is a (restricted) intuitionistic logic programming language (a sublanguage of λProlog). The main logic of Abella is then the second logic, called the *reasoning logic*, and it is capable of reasoning about provability in the specification language.

This approach to reasoning about computation has interesting applications. For example, the reasoning logic is aware of the fact that the cut and substitution rules can be eliminated in the specification logic. As a consequence, the notoriously difficult "substitution lemmas" that occur repeated in the study of operational semantics are proved essentially for free (that is, they are an immediate consequence of cut-elimination).

In [17], Accattoli showed that when one reasons about the *untyped*λ-calculus, the specification logic is often not needed. In particular, Accattoli reinterpreted the formalization by G. Huet of the meta-theory of λ-calculus residuals in Abella and showed that the resulting meta-theory had a much more elegant and natural specification than the one presented early by Huet in Coq. While the use of two-levels of logic was not important for this particular (untyped) example, other aspects of Abella—relation specifications, ∇-quantification, and strong induction principles—were critical for improving the expressivity of this prover.

6.4. A Non-local Method for Robustness Analysis of Floating Point Programs

Participants: Dale Miller, Ivan Gazeau.

Programs that must deal with floating point programs and their associate errors can have erratic behavior. In particular, a program that yields outputs that depend continuously on their inputs (in an idealized arithmetic setting) can behave non-continuously when using floating point arithmetic. There are few tools for reasoning about program correctness in a setting that allows for such discontinuous operators.

In [23], Gazeau, Miller, and Palamidessi provide an approach to reason about some programs that are not continuous. In that paper, they introduce the notion of "robustness", which intuitively means that if the input to the program changes less than a fixed small amount then the output changes only slightly. This notion is useful in the analysis of rounding error for floating point programs because it helps to establish bounds on output errors introduced by both measurement errors and by floating point computation. Compositional methods often do not work since key constructs—like the conditional and the while-loop—are not robust. The authors proposed a method for proving the robustness of a while-loop. This method is non-local in the sense that instead of breaking the analysis down to single lines of code, it checks certain global properties of its structure. This paper shows that both the CORDIC computation of the cosine and Dijkstra's shortest path algorithm are robust.

6.5. Herbrand Confluence

Participants: Stefan Hetzl, Lutz Straßburger.

It is well-known that cut-elimination in the sequent calculus for classical first-order logic is in its most general form, is neither confluent nor strongly normalizing. But if one takes a coarser (and mathematically more realistic) look at cut-free proofs, one can analyze which witnesses they choose for which quantifiers, or in other words: one can only consider the Herbrand-disjunction of a cut-free proof. This yields a surprising confluence result for a natural class of proofs: all (possibly infinitely many) normal forms of the non-erasing cut reduction lead to the same Herbrand-disjunction. This result has been presented at CSL 2012 [25].

6.6. Semi-Star-Autonomous Categories

Participants: Willem Heijltjes, Lutz Straßburger.

A curious aspect of Girard's proof nets for multiplicative linear logic without units is that, despite being a canonical representation of proof, their categorical semantics is not obvious—this in contrast to the situation *with* units, where star-autonomous categories form a natural semantics, but no canonical proof nets are known.

In the middle of the past decade several proposals for a categorical semantics of proof nets, a notion of *semi-star-autonomous* categories, were investigated: by Robin Houston and Dominic Hughes, by Kosta Došen, and by François Lamarche and Lutz Straßburger.

The present effort by Willem Heijltjes and Lutz Straßburger completes the notion in such a way that proof nets constitute the *free* semi-star-autonomous category.

6.7. Foundations and applications of explicit substitutions

Participant: Beniamino Accattoli.

Starting from the study of Linear Logic proof nets, a new approach to explicit substitutions for ł-calculus has recently been introduced by Accattoli and D. Kesner [31]. This approach has been systematically explored by Accattoli and his co-authors.

The rewriting theory of these new explicit substitutions *at a distance* has been studied in [11] and [16]. In [11] Accattoli and Kesner study the preservation of λ-calculus strong normalization (PSN) when explicit substitutions are extended with permutative axioms allowing to swap constructors in the term, generalizing considerably the already difficult case of PSN with composition of substitutions. In [16] Accattoli developed an abstract technique for proving factorizations theorems for generic explicit substitution calculi. The factorization theorem for λ-calculus says that any reduction can be re-organized as an *head* reduction followed by a non-head reduction.

In [16] it is shown how to prove this theorem in an uniform way for many explicit subsitutions calculi. The technique emerged as a generalization of the proofs for explicit substitutions at a distance, which are simpler than usual explicit substitutions and thus lead to cleaner and more compact arguments, easier to generalize.

Applications of explicit substitutions at a distance have been studied in [19], [18], [20]. In [19] Accattoli and Dal Lago show that the length of the head reduction in calculi at a distance is a measure of time complexity. More precisely, they show that such a quantity is polynomially related (in both directions) to the cost of evaluating with Turing Machines. This result is an important step forward towards the solution of the long-standing open problem of finding a time cost model for ł-calculus.

In [20] Accattoli and Paolini apply substitutions at a distance in a call-by-value setting. They show that in this new framework there is a natural characterization of *solvability*, an important notion related to denotational semantics and the representation of partial recursive functions. In [26] (a work presented to a workshop and currently submitted to the post-proceedings of the workshop) Accattoli shows the tight relations between the framework in [20] and linear logic proof nets, providing a new characterization of the proof nets representing the call-by-value λ-calculus.

Finally, in [18] Accattoli and Kesner introduce a calculus generalizing many different extensions of λ-calculus with permutations, appeared in various contexts (studies about call-by-value, postponing of reductions, monadic languages, etc) and prove confluence and preservation of strong normalization, exploiting and extending their own results in [11].

6.8. Sequent Calculus with Calls to a Decision Procedure

Participants: Mahfuza Farooque, Stéphane Lengrand.

In the PSI project, we have designed a version of the focussed sequent calculus (for first-order classical logic) that can call external decision procedures. Since the last Activity Report, we have finished proving the essential meta-theory for it: soundness, invertibility of asynchronous rules, cut-elimination, the fact that polarities do not affect provability but only the shape of proofs, and finally completeness.

The first properties are the object of [27], while the latter ones have been obtained later in 2012.

A side-product of this meta-theory is a technical device that could be used to encode other techniques from automated reasoning like *connection tableaux*.

Secondly, we have encoded the SMT-solving algorithm DPLL(T) as the incremental construction of proof-trees in that sequent calculus [29], [28]. A very interesting aspect of the encodings is that the basic rules of DPLL(T) makes use of cuts on atoms in sequent calculus, while the advanced jrules (e.g. backjumping) makes use of general cuts. This sheds a new light on the computational speed-ups that those advanced rules provide.

We have done the encoding for two distinct presentations of DPLL(T) in the literature, and we have formalised the connection between those two descriptions [29].

6.9. Martin-Löf Identity Type in the Category of Small Categories

Participant: François Lamarche.

For the last five or six years there has been a surge of interest in finding models for the identity type in Martin-Löf type theory, and it has been clear for some time that there was a tight connection with path objects in abstract homotopy theory. A lot of proposals have been made, but there are very few semantics that fit the necessary requirements of having dependent products and also an identity type which is fully stable under substitution. The most famous model of the sort is the one proposed by Voevodsky, in his Univalent Foundations project, which uses for base category the category of simplicial sets and models dependent types by the means of Kan Fibrations. In [13] François Lamarche proposes another such model, where the base category is the category of small categories, and dependent types are modelled with Grothendieck bifibrations (maps between categories that are Grothendieck fibrations as well as their duals between the opposite categories). The full requirements of modelling Martin-Löf type theory are met. Calculations show that the model shows some amount of degeneracy “in dimensions above 2” for the associativity of equality (which should not be strict in any dimension), which is a great improvement over the models on strict groupoids and strict ω-groupoids. The construction that models the identity type is a concrete path functor for categories. It is showing itself to be very useful in homotopy theory.

POEMS Project-Team

6. New Results

6.1. Numerical methods for time domain wave propagation

6.1.1. Coupling Retarded Potentials and Discontinuous Galerkin Methods for time dependent wave propagation problems

Participant: Patrick Joly.

This topic is developed in collaboration with J. Rodriguez (Santiago de Compostela) in the framework of the contract ADNUMO with AIRBUS. The general objective was to use time-domain integral equations - or retarded potentials - as a tool for contructing transparent boundary conditions for wave problems in unbounded media, by coupling them to an inerior volumic method, namely the Discontinuous Galerkin (DG) method.

Since last year, our new goal is to extend the method proposed in a previous work for DG with central fluxes to the case of upwind fluxes, while preserving most of the good properties of the original method from both theoretical (stability via energy dissipation - instead of energy conservation) and practical points of view. We have designed a method that achieves this goal at the only prize of a small deterioration of the CFL condition. The method has been successfully implemented and the numerical results clearly emphasize the superiority of upwing fluxes for taking into account the convection terms in the linearized Euler equations in aeroacoustics, the privileged application.

At the same time, we have used similar ideas for treating physical boundary conditions involving differential (in time) impedance operators.

6.1.2. Solving the Homogeneous Isotropic Linear Elastodynamics Equations Using Potentials and Finite Elements.

Participants: Aliénor Burel, Marc Duruflé, Patrick Joly.

This topic is the subject of the first part oh th PhD thesis of A. Burel. Its aim is to use the classical theoretical decomposition of the elastodynamic displacement into two potentials referring to the pressure wave and the shear wave, and use it in a numerical context. Last year, a method has been proposed for solving the Dirichlet problem (clamped boundary), successfully analyzed and implemented. For free boundary conditions, we have proposed an original method considereing these boundary conditions as a perturbation of the Dirichlet conditions. The natural adaptation of the variational formulation used in the case of the Dirichlet problems presents nice theoretical properties and leads to satisfactory numerical results for the time harmonic problem. However, the implementation for the time dependent problem reveals severe instability phenomena that seem to be already present in the semi-discrete (in space) problem. In order to understand the cause of these instability (and possibly remedy them) we are currently performing the Kreiss analysis of the half-space problems in the case where Q_1 finite elements are used on the same uniform square grid for both P-waves and S-waves potentials.

6.1.3. Time domain analysis of Maxwell's equations in Lorentz materials

Participants: Maxence Cassier, Lucas Chesnel, Christophe Hazard, Patrick Joly, Valentin Vinoles.

This is the time-domain counterpart of the research done at Poems about frequency domain analysis of metamaterials (see also the section 6.2.7) in the framework of the ANR Project Metamath. One fundamental question is the link between the two problems via the limiting amplitude principle, in particular in the cases where the time harmonic problem fails to be well posed problem in the standard framework. This occurs at certain frequencies (see section) when one considers a transmission problem between a Lorentz material and a standard one.

We are investigating this question from both theoretical and numerical points of view. This is also the object of a collaboration with B. Gralak from the Institut Fresnel in Marseille.

6.1.4. Modeling and numerical simulation of a piano.

Participants: Juliette Chabassier, Marc Duruflé, Sébastien Imperiale, Patrick Joly.

The defense of the PhD thesis of Juliette Chabassier, in March, has marked one of the most spectacular achievements in Poems for the past years, concerning the "complete" physical and mathematical modeling of a grand piano and its computer simulation. This is the result of a quite interdisciplinary work in collaboration with Antoine Chaigne (UME, ENSTA). We refer the reader to the three previous activity reports of Poems for a more detailed description of the scientific developments that have led to the implementation of a parallel code for the simulation of the piano. Using this code, M. Duruflé and J. Chabassier have realized a bank of synthetic sounds that can be used for playing scoreboards (using MIDI files for instance). For more details, and also other additional information about the work, we refer the reader to the Web page : http://modelisation.piano.free.fr.

Although already quite satiafactory, the results obtained by the present version of the code show that there is still room for the improvement of our piano model. One of the ideas consists in improving the quality of the model for the hammers and that is why J. Chabassier and M. Duruflé have proposed an enriched model involving the virations of the hammer's shank. We expect to achieve further progress in this direction through our participation to the ITN (Initial Training Network) European project BATWOMAN (Basic Acoustics Training and Workprogram on Methodologies for Acoustics Network) that has been submitted lst November. This projects regroups 11 partners from 7 different contries and gathers academic people with industrials of the donain, including Steinway.

As a theoretical complement to the numerical developments, we have led a systematic theoretical study of the numerical method used in our code for computing string's vibrations. Our concern was to develop a new implicit time discretization, which is associated with finite element methods in space, in order to reduce numerical dispersion while allowing the use of a large time step. We proposed a new θ-scheme based on different θ-approximations for the flexural and shear terms of the equations, which allows to reduce numerical dispersion while relaxing the stability condition. In particular, we gave some insights of innovative proofs of stability by energy techniques that provide uniform estimates with respect to the CFL number. Theoretical results have been illustrated with numerical experiments corresponding to the simulation of a realistic piano string.

6.1.5. Numerical methods in electromagnetism

Participant: Patrick Ciarlet.

Collaborations with Eric Chung, Tang Fei Yu and Jun Zou (Chinese University of Hong Kong, China), Philippe Ciarlet (City University of Hong Kong, China) Haijun Wu (Nanjing University, China), Stefan Sauter and Corina Simian (Universität Zürich).
The numerical approximation of electromagnetic fields is still a very active branch of research. Below, three lines of work are briefly reported.
Edge finite elements are widely used in 2D/3D electromagnetics, however they approximate very weakly the divergence of the fields. In a recent work with H. Wu & J. Zou, we proposed a method that allows one to approximate the divergence accurately in H^{-s}-norms $(1/2 < s < 1)$.
Discontinuous Galerkin finite elements are also very popular, as they allow one to design fast (and accurate) methods to solve PDEs. Jointly with E. Chung and T. F. Yu, we designed a numerical method to solve the 2D/3D time-dependent Maxwell equations, using a high order staggered DG method in the spirit of those introduced by E. Chung and B. Engquist. The method has been analyzed on Cartesian meshes and its generalization to unstructured meshes is under way.

A few years ago, we proposed with Philippe Ciarlet a method to solve some problems in linear elasticity intrinsically. With S. Sauter, C. Simian and Philippe Ciarlet, we studied a similar approach that can be applied to 2D electrostatics. It consists in solving the problem in the electric field directly, using exact or local curl-free approximation of the field. Within this framework, we have been able to derive a general method that allows one to derive intrinsic conforming and non-conforming finite element spaces to compute the electrostatic potential. Generalization to 3D electrostatics and linear elasticity is under way.

6.2. Time-harmonic diffraction problems

6.2.1. Numerical computation of variational integral equation methods

Participants: Marc Lenoir, Nicolas Salles.

The dramatic increase of the efficiency of the variational integral equation methods for the solution of scattering problems must not hide the difficulties remaining for an accurate numerical computation of some influence coefficients, especially when the panels are close and almost parallel.

The formulas have been extended to double layer potentials and, for self influence coefficients, to affine basis functions. Their efficiency for the solution of Maxwell equations has been proved in the framework of a collaboration with CERFACS.

6.2.2. Formulation and Fast Evaluation of the Multipole Expansions of the Elastic Half-Space Fundamental Solutions

Participants: Marc Bonnet, Stéphanie Chaillat.

The use of the elastodynamic half-space Green's tensor in the FM-BEM is a very promising avenue for enhancing the computational performances of 3D BEM applied to analyses arising from e.g. soil-structure interaction or seismology. This ongoing work is concerned with a formulation and computation algorithm for the elastodynamic Green's tensor for the traction-free half-space allowing its use within a Fast Multipole Boundary Element Method (FM-BEM). Due to the implicit satisfaction of the traction-free boundary condition achieved by the Green's tensor, discretization of (parts of) the free surface is no longer required. Unlike the full-space fundamental solution, the elastodynamic half-space Green's tensor cannot be expressed in terms of usual kernels such as $e^{\mathrm{i}kr}/r$ or $1/r$. Its multipole expansion thus cannot be deduced from known expansions, and is formulated in this work using a spatial two-dimensional Fourier transform approach. The latter achieves the separation of variables which is required by the FMM. To address the critical need of an efficient quadrature for the 2D Fourier integral, whose singular and oscillatory character precludes using usual (e.g. Gaussian) rules, generalized Gaussian quadrature rules have been used instead. The latter were generated by tailoring for the present needs the methodology of Rokhlin's group. Numerical tests have been conducted to demonstrate the accuracy and numerical efficiency of the proposed FMM. In particular, a complexity significantly lower than that of the non-multipole version was shown to be achieved. A full FM-BEM based on the proposed acceleration method for the half-space Green's tensor is currently under way.

6.2.3. Domain decomposition methods for time harmonic wave propagation

Participants: Francis Collino, Patrick Joly, Mathieu Lecouvez.

This work is motivated by a collaboration with the CEA-CESTA (B. Stupfel) through the PhD thesis of M. Lecouvez that has started at the beginning of the year.

We are interested in the diffraction of time harmonic electromagnetic waves by perfectly conducting objects covered by multi-layered (possibly thin) dielectric coatings. This problem is computationally hard when the size of the object is large (typically 100 times larger) with respect to the incident wavelength. In such a situation is to use a domain decomposition method in which each layer would contitute a subdomain. More precisely, we want to use a non overlaping iterative domain decomposition method based on the use of Robin type transmission conditions, a subject to which people at Poems gave substantial contributions in the 90's through the works of Collino, Desprès, and Joly.

The novelty of our approach consists in using new transmission conditions using some specific impedance operators in order to improve the convergence properties of the method (with respect to more standard Robin conditions). Provided that such operators have appropriate functional analytic properties, the theory shows that one achieves geometric convergence (in opposition the the slow algebraic convergence obtained with standard methods). These properties prevent the use of local impedance operator, a choice that was commonly done for the quest of optimized transmission conditions (following for instance the works of Gander, Japhet, Nataf). We propose a solution that uses nonlocal integral operators using appropriate Riesz potentials. To overcome the disadvantage of dealing with completely nonlocal operators, we suggest to work with truncated kernels, i.e. with operators of the form (Γ represents one interface)

$$u(x) \quad \longrightarrow \quad \int_\Gamma K\left(\,|x-y|\,\right)\ \chi\left(\ \frac{|x-y|}{\lambda}\ \right)\ u(y)\ d\sigma(y)$$

where $K(|x|)$ is an appropriate singlar kernel (typically $K(|x|) = |x|^{-\gamma}$) and $\chi(\rho)$ an adequate smooth cut-off function. Playing with a few parameters such as the size of the support of χ, we expect to achieve an optimal compromise between the reduction of the number of iterations of the method and the cost of each iteration.

6.2.4. Time harmonic aeroacoustics

Participants: Anne-Sophie Bonnet-Ben Dhia, Jean-François Mercier.

We are still working on the numerical simulation of the acoustic radiation and scattering in presence of a mean flow. This is the object of the ANR project AEROSON, in collaboration with Florence Millot and Sébastien Pernet at CERFACS, Nolwenn Balin at EADS and Vincent Pagneux at the Laboratoire d'Acoustique de l'Université du Maine. Let us recall that our method combines, a Finite Element resolution of the augmented Galbrun equation and of the coupled vorticity transport equation, and the use of Perfectly Matched Layers (PML) to bound the computational domain. The main recent improvements concern the test of the method in presence of unstable modes.

When determining the aeroacoustics modes propagating in a flow, unstable modes exist for certain types of flows: when an inflection point exists in the velocity profile and when the shear in this point is strong enough. Such modes grow exponentially in space. Up to recently, our numerical simulations have been performed for stable flows. We have tested the behavior of PML in the presence of unstable modes, which usually convert a propagating field in a decaying field. Therefore we do not have a theoretical framework to characterize the behavior of PML in the presence of spatially growing modes but the various conducted numerical tests have shown that our numerical method is still able to select the outgoing solution, even in the presence of instabilities, if the attenuation in the PML is strong enough.

6.2.5. Multiple scattering in a duct

Participant: Jean-François Mercier.

This topis is developed in collaboration with Agnès Maurel (Langevin Institute ESPCI).
The objective of this work, part of the ANR Procomedia, is to develop analytical methods to describe the propagation of acoustic waves in 2D waveguides containing penetrable inclusions. Scatterers of arbitrary shape with a contrast in both density and sound speed are considered. A modal approach is adopted, in which the wave equation is projected onto the transverse modes of the homogeneous guide. For each mode a 1D wave equation is obtained with a source term which characterizes the scatterers and couples modes together. In weak scattering regime (small scatterers or low contrasts or low frequency), the Born approximation is used to solve analytically this family of coupled ODE. This gives an explicit prediction for the scatterered field, in particular the reflection and transmission coefficients are obtained in two cases of interest: periodically or randomly distributed scatterers. In both cases, expressions similar to those in free space (available only for low frequencies) are obtained without frequency limit, thanks to the presence of a shape factor sensitive to the geometry of the scatterers at high frequencies.

Recently the obtained analytical expressions have been exploited to develop a very simple imaging method in a heterogeneous waveguide. Measurements of low-frequency reflection and transmission allow to find the position of the object while the higher frequency measurements give access to the shape and to the physical characteristics of the scatterers. The results are good in the case of low contrast and small scatterers, for which the Born approximation is perfectly valid.

6.2.6. Localization in perturbed periodic metamaterials

Participant: Jean-François Mercier.

This topis is developed in collaboration with Agnès Maurel, Abdelwaheb Ourir (Langevin Institute ESPCI) and Vincent Pagneux (LAUM).
The aim of this work, part of the ANR Procomedia, is to study the propagation of electromagnetic waves through 1D perturbed periodic media. The attenuation length in a medium consisting of alternating materials of optical indices $n_1 > 0$ and $n_2 < 0$ (metamaterials) is determined. When such medium is randomly disturbed, the localization properties differ significantly from those obtained in a classical disturbed medium: in the homogeneous case $n_1 = n_2$, a random perturbation of the indices induces the Anderson localization with a strong field attenuation. In contrast, in the case $n_1 = -n_2$, it was recently shown that the introduction of disorder on the permittivities ϵ_1 and ϵ_2 gave rise to an "anomaly", the suppression of the Anderson localization. This anomaly results in a significant increase of the attenuation length l_N for large sample sizes N.
We have made two improvements to existing works: simple analytical expressions of the attenuation length have been determined, valid over a wide range of frequencies and of number of layers. In addition we considered realistic metamaterials by taking into account disorder in both the permittivity and the permeability μ. When only the permeability is disturbed (or only the permittivity), our analytical expression can explain the transition to the abnormal behavior when the number of layers increases. Furthermore we show that the anomaly is strongly affected when disturbances in permeability and permittivity are jointly considered: the coupling of the two effects is capable of reseting the usual localization.

6.2.7. Modeling of meta-materials in electromagnetism

Participants: Anne-Sophie Bonnet-Ben Dhia, Camille Carvalho, Patrick Ciarlet, Lucas Chesnel.

This topis is developed in collaboration with Eric Chung (Chinese Univ. of Hong Kong) and Xavier Claeys (Paris VI).
Meta-materials can be seen as particular media whose dielectric and/or magnetic constant are negative, at least for a certain range of frequencies. This type of behavior can be obtained, for instance, with particular periodic structures. Of special interest is the transmission of an electromagnetic wave between two media with opposite sign dielectric and/or magnetic constants. As a matter of fact, applied mathematicians have to address challenging issues, both from the theoretical and the discretization points of view. The year 2012 saw the completion of Lucas Chesnel PhD thesis. We present below the main results obtained these last three years. The first topic we considered a few years ago was: when is the (simplified) scalar model well-posed in the classical H^1 framework? It turned out this issue could be solved with the help of the so-called T-coercivity framework. While numerically, we proved that the (simplified) scalar model could be solved efficiently by the most “naive” discretization, still using T-coercivity. Recently, we have been able to provide sharp conditions for the T-coercivity to hold in general 2D and 3D geometries, which involve explicit estimates in simplified geometries together with localization arguments. We then analyzed the discretization of the scalar problem with a classical, H^1 conforming, finite element method, and proved the convergence under the same sharp conditions. We also showed that the problem can be solved with the help of a Discontinuous Galerkin discretization, which allows one to approximate both the field and its gradient (with E. Chung).
As a second topic, we investigated the case of a 2D corner which can be ill-posed (in the classical H^1 framework). Using the Mellin transform, we showed that a radiation condition at the corner has to be imposed to restore well-posedness (with X. Claeys). Indeed there exists a wave which takes an infinite time to reach the corner: this “black hole” phenomenon is observed in other situations (elastic wedges for example). We proposed a numerical approach to approximate the solution which consists in adding some PMLs in the neighbourhood of the corner.

Last, we studied the transmission problem in a purely 3D electromagnetic setting from a theoretical point of view. We proved that the Maxwell problem is well-posed if and only if the two associated scalar problems (with Dirichlet and Neumann boundary conditions) are well-posed. Of course, these scalar problems involves sign-changing coefficients but they can be studied using simple scalar T-coercivity approach.
C. Carvalho started her PhD thesis this fall in the continuation of these works.

6.2.8. Numerical MicroLocal Analysis

Participants: Jean-David Benamou, Francis Collino, Simon Marmorat.

Numerical microlocal analysis of harmonic wavefields is based on a family of linear filters using Bessel functions and applied to wave data collected on a circle of fixed radius r_0 around the observation point x_0 where we want to estimate the Geometric Optics/ High Frequency components. The data can easily be reconstructed from more conventional line array or grid geometry. The output is an angular function presenting picks of amplitudes in the direction angles of rays.

The original NMLA algorithm relied on a local plane wave assumption for the data. For arbitrary waves, it meant linearization errors and accuracy limitations. Also, only the directions of the (multiple) rays are recovered but the traveltime and amplitudes are not reliably computed. We recently introduced a new "impedant" observable which allows to prove a stability theorem. Numerical results confirm that the new NMLA filter is robust to random and correlated noise.

Using asymptotic expansion on NMLA filtered point sources data, we designed a correction method for the angle which also estimates the wavefront curvature. It can be used to correct the linearization errors mentioned above and provides a second order correction in the Taylor approximation of the traveltime.

The parameters of the method (size of observation circle, discretization) are automatically optimized and a posteriori quantitative error on angles and curvature are available. Numerical studies validate the stability result and confirm the superior accuracy of the curvature corrected NMLA version over image processing methods.

When some bandwith is available we can also compute the traveltime. The amplitude remains polluted by phase errors. Its determination is still open.

6.3. Absorbing boundary conditions and absorbing layers

6.3.1. Evolution problems in perturbed infinite periodic media

Participant: Sonia Fliss.

For parabolic problems set in locally perturbed periodic media, we have developed an approach to determine the time-domain DtN operator. The principle is to apply the Laplace Transform in time to the equation and use the construction of the DtN operator for stationary equations. The main difficulty is the computation of the inverse of the Laplace Transform, more precisely to understand how to deal with the unbounded interval of integration and the choice of the discretization of the laplace variable. To deal with the first difficulty for waveguide problem, we have studied the asymptotic behavior of the DtN operator in the laplace domain when the laplace variable tends to $p_0 \pm \infty$. To deal with the second difficulty, we have used the Z-Transformation and its properties. The numerical study is still in progress. This work enters in the framework of the ANR PRoject MicroWave (Sonia Fliss is an external collaborator), in collaboration with Karim Ramdani (Institut Elie Cartan de Nancy, UMR CNRS 7502), Christophe Besse and Ingrid Violet (Laboratoire Paul Painlevé, UMR CNRS 8524).

6.3.2. New transparent boundary conditions for time harmonic acoustic problem in anisotropic media

Participants: Anne-Sophie Bonnet-Ben Dhia, Sonia Fliss, Antoine Tonnoir.

This topis is developed in collaboration with Vahan Baronian (CEA). Many industrial applications require to check the quality of structures such as plates, for instance in aircraft design. A common way to inspect structures is to propagate ultrasonic waves and detect from the experimental results the presence or not of a defect or a crack. However, in aeronautics, structures are often complex media like anisotropic elastic plates for which the interpretation of this results is complicated. Therefore, efficient and accurate numerical methods of simulation are required. In our work, we want to study the diffraction of a time harmonic wave by a bounded defect in an anisotropic elastic media. In order to study the diffraction properties of the defect, we consider it in as infinite. Since the defect has an arbitrary geometry, we want to use a finite element method in a box that surround the defect. On the boundary of this artificial box, we need to find transparent conditions to simulate an infinite domain.

- We first have considered waveguides. The transparent boundary conditions are often written by using the so-called Dirichlet-to-Neumann maps which can be expressed thanks to a modal decomposition. However, classical iterative method does not converge necessarily. In this work, we introduce a new Dirichlet-to-Neumann operator which links the trace of the solution on a section of the waveguide to the normal trace on a different one. This operator can also be expressed analytically via a modal decomposition. Its main advantage is that, because of the overlapping, it becomes compact and this is exactly why we think an iterative resolution has more chance to converge. Other advantages will appear with the elasticity application. Indeed, in the formulation of the transparent boundary condition without overlapping, appears a lagrange multiplier which makes the resolution more costly. This additional unknown will be avoided with an overlap. For now, the theory is done for the scalar acoustic waveguide and the method has been implemented in the Melina code for the acoustic and the elastic case. The redaction of an article is in progress.

  item We then have studied scattering problem in locally perturbed anisotropic plate. The classical methods to derive transparent boundary conditions for acoustic isotropic media are based on the Green function (boundary integral formulation) or Fourier series (to determine DtN operator set on an artificial circle boundary). However, they cannot be extended for anisotropic elastic problems. Using a constructive method to determine transparent boundary conditions for periodic media developped in the laboratory, we were able to propose new exact boundary conditions which are adapted to anisotropic media and for which iterative method could converge rapidly. The numerical study is in progress for acoustic isotropic problem.

6.4. Waveguides, resonances, and scattering theory

6.4.1. Localized modes in periodic waveguides

Participants: Anne-Sophie Bonnet-Ben Dhia, Bérangère Delourme, Sonia Fliss, Sergei Nazarov, Elizaveta Vasilevskaia.

The general objective is the study of localized modes in locally perturbed periodic media. We investigate the existence theory of such modes as well as their numerical computations. We can distinguish two types of problems.

Numerical computation of guided modes in periodic media with line defects. We are interested in the propagation of guided modes that propagate in the direction of the line defect (which is parallel to one of the periodicity directions of the unperturbed medium) and decrease exponentially in the transverse directions. We aim at computing these modes and their dispersion relation. Last year, we developed a method based on the use of the DtN approach introduced in the PhD thesis of S. Fliss and the resolution of "operator pencil" eigenvalue problems. This year, in collaboration with Kersten Schmidt, we have made a numerical comparison of this new method with the more standard supercell method.

Existence of localized modes in closed periodic waveguides. We consider a propagation medium which is infinite and periodic in one space dimension and bounded in the transverse ones. We investigate the question of the influence of a local defect on the existence of localized modes. Once again this reduces to a selfadjoint eigenvalue problem in an unbounded domain.

The first problem that we studied is in the framework of the PostDoc of Bérangère Delourme. We have considered general locally perturbed periodic media for which we focus on determining sufficient conditions on the periodic media or the local defect so that it exists at least one eigenvalue below the essential spectrum of the underlying perfectly periodic operator. These sufficient conditions are based on Min-Max theory and an appropriate choice of test functions. We were able to validate these existence conditions thanks to the numerical method based on the use of DtN operators. For situations where the periodic "reference medium" is closed to a simple "limit medium" fo which all calculations can be made by hand, we show that these conditions could be really simple and explicit using perturbation theory and asymptotic expansions of the eigenvalues. We are investigating now the extension of this approach to sufficient conditions for existence of guided modes inside the essential spectrum.
The second case, that is investigated in the framework of the PhD thesis of E. Valisevskaia, is the case where the propagation medium is a thin structure (the thinness being characterized by the parameter ε) whose limit is a periodic graph. This is for instance the case of a symmetric ladder as illustrated by figure . If Neumann boundary conditions are considered, it is well known (see in particular the works by Exner, Kuchment) the the limit model when εtends to is the Helmholtz equation on the graph (1D Helmholtz equations on each branch competed by continuity and Kirchoff transimission conditions at each node) . For this limit problem, the underlying operator does not present any spectral gaps but can be written, due to the symmetry of the problem, as the sum of two operators, each of which having an infinity of spectral gaps. This allows us to look for eigenvalues in these spectral gaps, induced by symmetric and localized perturbations of the limit graph model. This can de done for instance by modifying (symetrically) the Kirchoff conditions on two symmetric nodes of the graph. In the limit process mentionned above, this would correspond to modifying the width of the rung that joins these two points in the original problem. First existence results have been obtained in this direction. In a further step, one can expect, by asymptotic analysis, to get corresponding existence results for the original problem, at least for ε small enough.

6.4.2. A new approach for the numerical computation of non linear modes of vibrating systems

Participants: Anne-Sophie Bonnet-Ben Dhia, Jean-François Mercier.

A collaboration with Cyril Touzé and François Blanc (Unité de Mécanique, ENSTA). The simulation of vibrations of large amplitude of thin plates or shells requires the expensive solution of a non-linear finite element model. The main objective of the proposed study is to develop a reliable numerical method which reduces drastically the number of degrees of freedom. The main idea is the use of the so-called non-linear modes to project the dynamics on invariant subspaces, in order to generate accurate reduced-order models. Cyril Touzé from the Unité de Mécanique of ENSTA has derived an asymptotic method of calculation of the non-linear modes for both conservative and damped systems. But the asymptotically computed solution remains accurate only for moderate amplitudes. This motivates the present study which consists in developing a numerical method for the computation of the non-linear modes, without any asymptotic assumption. This is the object of a collaboration with Cyril Touzé, and new results have been obtained during the post-doc of François Blanc in the Unité de Mécanique of ENSTA. The partial differential equations defining the invariant manifold of the non-linear mode are seen as a vectorial transport problem : the variables are the amplitude and the phase (a, φ) where the phase φ plays the role of the time. In the case of conservative systems, a finite difference scheme is used and an iterative algorithm is written, to take into account the 2π-periodicity in φ which is seen as a constraint. An adjoint state approach has been introduced to evaluate the gradient of the coast function. The method has been validated in a simple example with two degrees of freedom. Good agreement with an alternative method, the continuation of periodic solutions method, has been found. Currently the method is extended to the case of damped systems. The main difficulty is that, due to a change of variables, the 2π-periodicity does not hold anymore and new constraints more complicated to implement must be considered.

6.4.3. Harmonic wave propagation in locally perturbed periodic waveguide

Participants: Sonia Fliss, Patrick Joly.

We work on the expression and the asymptotic behaviour of the Green function for time harmonic wave equation in two-dimensional periodic waveguide. This enables us to define a radiation condition and show well-posedness of the Helmholtz equation set in a periodic waveguide. The redaction of an article is ongoing. This analysis is one of the main tool to solve inverse problems in locally perturbed periodic waveguide (see section 6.6.1) when the data are far field measurements of scattering problems.
One challenging perspective of this work is to extend these results to periodic problems in free space.

6.4.4. Finite element approximation of modes of elastic waveguides immersed in an infinite fluid

Participants: Anne-Sophie Bonnet-Ben Dhia, Cédric Doucet, Christophe hazard.

This work is developped in collaboration with Vahan Baronian (CEA). We are developping numerical tools to simulate ultrasonic non-destructive testing in elastic waveguides. This particular topic aims at finding an efficient way of coupling semi-analytical finite element methods and perfectly matched layers (PMLs) to compute modes of elastic waveguides embedded in an infinite fluid.
During our numerical investigations, we noticed that the semi-analytical mixed finite element formulation proposed in the PhD thesis of V. Baronian may lead to the computation of spurious modes. We overcame this problem in the following way: instead of approximating components of stress tensors by means of first-order finite elements of class $\mathcal{C}^0$, we decided to use zeroth-order discontinuous ones. This simple modification seems not only to stabilize the discretization step, but also to approximate modes more accurately in comparison with the classical semi-analytical finite element formulation. Last but not least, we observed a meaningful improvement of the approximation of the continuous spectrum of stretched operators related to PMLs. Besides, previous results (in the PhD thesis of B. Goursaud) about the best way of designing PMLs to simulate wave propagation in open acoustic waveguides have been confirmed by our numerical experiments on immersed elastic structures.
Further investigations need to be carried out to explain these phenomena. Especially, a theoretical analysis still remains to be done.

6.5. Asymptotic methods and approximate models

6.5.1. Effective boundary conditions for thin periodic coatings

Participants: Mathieu Chamaillard, Patrick Joly.

This topic is the object of a collaboration with Houssem Haddar. We are interested in the construction of "equivalent" boundary condition for the diffraction of waves by an obstacle with smooth boundary Γ covered with a thin coating of width δ whose physical characteristics vary "periodically along Γ with a period proportional to the small parameter δ. For a general boundary Γ, the notion of periodicity is ambiguous: we have chosen to define the coating as the image, or the deformation, by a smooth mapping of a flat layer of width delta (the reference configuration) that preserves the normals, which appears consistent with a manufactoring process. The electromagnetic parameters in the coating are then defined as the images through Φ_Γ of periodic functions in the reference configuration.

We have first considered the case of the scalar wave equation. Using an asymptotic analysis in δ, which combines homogenization and matched asymptotic expansions, we have been able to establish a second order boundary condition of the form

$$\partial_\nu u + \left(\delta\, B_\Gamma^1 + \delta^2\, B_\Gamma^2\right) u = 0$$

where B_Γ^1 and B_Γ^2 are second order tangential differential operators along Γ whose coefficients depend on both the geometrical characteristics of Γ (through the curvature tensor) and the material properties of the coating, through the resolution of particular cell problems in the flat reference configuration. When the coating is homogeneous, we have checked that one recovers the well known second order thin layer condition. This new condition is expected to provide $O(\delta^3)$ accuracy. Its implementation and its rigorous analysis (error estimates) are ongoing.

6.5.2. Thin Layers in Isotropic Elastodynamics

Participants: Marc Bonnet, Aliénor Burel, Patrick Joly.

This research is developed in the framework the numerical modeling of non-destructive testing experiments using ultrasonic waves. Most realistis propagation media involves thin layers of resin (typically for gluing together different homogeneous media), which are, until now, difficult to take into account numerically, the principal issue being the very small space step needed for meshing such a thin layer. An idea to get rid of this complication is to use asymptotic analysis in order to establish effective transmission conditions. We have studied the simple model problem in two dimensions, with an infinite flat layer of thickness ε. Using a formal approach based on a scaling inside the layer and an power series expansion in ε solution as a polynomial in ε, we have established first and second order conditions. Energy techniques parmit to guaranty the stability of our approximation.

6.5.3. Homogenization and metamaterials

Participants: Sonia Fliss, Patrick Joly, Valentin Vinoles.

This topic is developped in collaboration with Xavier Claeys (LJLL, Paris VI).
The mathematical modeling of electromagnetic metamaterials and the homogenization theory are intimately related because metamaterials are precisely constructed by a periodic assembly of small microstructures involving dielectric materials presenting a high contrast with respect to a reference medium. As a consequence, each microstructure behaves as a resonator which induces surprising properties to the effective or homogenized material such as negative permittivity and / or permeability at certain frequencies. The relevant theoretical approach to this question is the non standard (or high contrast) homogenization theory developed in particular in France by G. Bouchitté.

In the framework of the ANR Metamath, we wish to deepen this question by looking carefully at the treatment of boundaries and interfaces that are generally poorly taken into account by the first order homogenization. This is developed in collaboration with X. Claeys (Paris VI).

This question is already relevant for standard homogenization for which taking into account the presence of a boundary induces a loss of accuracy due to the inadequation of the standard homogenization approach to take into account the boundary layers induced by the boundary. Our objective is to construct approximate effective boundary conditions that would restore the desired accuracy.

With the PhD thesis of V. Vinoles, we aim at extending the previous approach to the treatment of metamaterials via high contrast homogenization. In particular, we intend to treat the challenging question of interfaces between metamaterials and standard materials (see also sections).

6.5.4. Asymptotic analysis and negative materials

Participants: Lucas Chesnel, Sergei Nazarov.

This topic is developped in collaboration with Xavier Claeys (LJLL, Paris VI) and S.A. Nazarov (IPME RAS, St Petersburg, Russia).

One of the applications of negative materials (metals at optical frequencies or negative metamaterials) is the construction of subwavelength cavities. In this kind of application, the idea is to use the following result: an inclusion of a negative material in a positive material changes radically the spectrum of the Maxwell's operators. We demonstrated this result for the scalar operator in a configuration where a positive material contains a small negative inclusion whose size tends to zero. As a second topic, we proved an instability result for a configuration where the interface between the positive and the negative material has a rounded corner. It appears that the solution depends critically on the value of the rounding parameter and does not converge when the rounded corner tends to the actual corner. We also studied the spectrum of the scalar operator in this configuration. This spectrum does not converge but seems (for the moment, the proof is not complete) to oscillate like $\ln \delta$ where $\delta \to 0$ is the rounding parameter.

6.5.5. Modelling of non-homogeneous lossy coaxial cable for time domain simulation.

Participants: Geoffrey Beck, Sébastien Imperiale, Patrick Joly, Martina Novelinkova.

This topic, initiated at the end of the PhD thesis of S. Imperiale, has been the subject of the internship of M. Novelinkova and is the subject of the PhD thesis of G. Bech which started in October.

We investigate the question of the electromagnetic propagation in thin electric cables from a mathematical point of view via an asymptotic analysis with respect to the (small) transverse dimension of the cable: as it has been done in the past in mechanics for the beam theory from 3D elasticity, we use such an approach for deriving simplified effective 1D models from 3D Maxwell's equations. Doing so, we have been able to derive a generalized telegraphist's equation, a 1D wave equation with additional time convolution terms that results from the conjugated effect of electromagnetic losses and heterogeneity of the cross section. This new model has been fully justified through error estimates. We are currently working on a higher order generalized telegraphist's equation that would include dispersive effects through nonlocal capacity and inductance operators.

From the pratical point of view, a code that computes the coefficients (including the convolution kernel) of the effective model and solves the generalized telegraphist's equation has been implemented. It has been exploited to measure the presence of localized defects on the propagation of electromagnetic waves. This application has been motivated by the ANR project SODDA, in collaboration with CEA-LETI, about the non destructive trsting of networks of electric cables (a subject that we are investigating in collaboration with M. Sorine from Inria Rocquencourt).

6.5.6. Elastic wave propagation in strongly heterogeneous media

Participants: Patrick Joly, Simon Marmorat.

This subject enters our long term collaboration with CEA-LIST on the development on numerical methods for time-domain non destructive testing experiments using ultra-sounds. This is also the subject of the PhD thesis of Simon Marmorat. Our objective is to develop an efficient numerical approach for the propagation of elastic waves in a medium which is made of many small inclusions / heterogeneities embedded in a smooth (or piecewise smooth) background medium, without any particular assumption (such as periodicity) on the spatial distribution of these heterogeneities. Our idea is to exploit the smallness of the inclusions (with respect to the wavelength in the background medium) to derive a simplified approximate model in which each inclusion would be described by very few parameters (functions of time) coupled to the displacement field in background medium for which we could use a computational mesh that ignores the presence of the heterogeneities. For deriving such a model. we intend to use and adapt the asymptotic methods previously developed at Poems (such as matched asymptotic expansions).

6.5.7. Multiple scattering by small scatterers

Participants: Maxence Cassier, Christophe Hazard.

We consider the scattering of an acoustic time-harmonic wave by an arbitrary number of sound-soft obstacles located in a homogeneous medium. When the size of the obstacles is small compared with the wavelength, the numerical simulation of such a problem by classical methods (e.g., integral equation techniques or methods based on a Dirichlet-to Neumann map) can become highly time-consuming, particularly when the number of scatterers is large. In this case, the use of an asymptotic model may reduce considerably the numerical cost. Such a model was introduced by Foldy and Lax in the middle of the last century to study multiple isotropic scattering in a medium which contains randomly distributed small scatterers. Their asymptotic model is based on the fact that the scattered wave can be approximated by a wave emitted by point sources placed at the centers of the scatterers; the amplitudes of the sources are calculated by solving a linear system which represents the interactions between the scatterers. Nowadays, the FoldyñLax model is still used in numerous physical and numerical applications to approximate the scattered wave in a deterministic media. But to the best of our knowledge, there was no mathematical justification of this asymptotic model. We have proposed such a justification which provides local error estimates for the two-dimensional problem in the case of circular obstacles. An article on this subject has been accepted and will be published in Wave Motion in January 2013.

6.6. Imaging and inverse problems

6.6.1. Sampling methods in waveguides

Participants: Laurent Bourgeois, Anne-Claire Egloffe, Sonia Fliss, Mathieu Guenel, Eric Lunéville.

First, we have adapted the modal formulation of sampling methods (Linear Sampling Method and Factorization Method) to the case of a periodic waveguide in the acoustic case. This study is based on the analysis of the far field of scattering solutions in cylindrical waveguides, in particular for the fundamental solution, which enables us to obtain a far field formulation of sampling methods, and then a modal formulation of such methods. The aim of the inverse problem is to retrieve a defect from the scattered fields which correspond to the incident fields formed by the Floquet modes. The corresponding numerical implementation was the subject of the Master internship of Mathieu Guenel who obtained some first promising results.
Secondly, going back to the homogeneous waveguide in the acoustic case, we have started a study of the sampling methods in the time domain. This will be the subject of Anne-Claire Egloffe's post-doc. The aim is to use the modal formulation of the sampling methods at all frequencies and recompose the best possible image of the defect.

6.6.2. The exterior approach to retrieve obstacles

Participant: Laurent Bourgeois.

This theme is a collaboration with Jérémi Dardé from IMT (Toulouse).
We have adapted the exterior approach developped for the Laplace equation to the Stokes system. The aim is to find a fixed Dirichlet obstacle in a fluid which is governed by the Stokes system with the help of boundary measurements. The exterior approach consists in defining a decreasing sequence of domains that converge in some sense to the obstacle. More precisely, such iterative approach is based on a combination of a quasi-reversibility method to update the solution of the ill-posed Cauchy problem outside the obstacle obtained at previous iteration and of a level set method to update the obstacle with the help of the solution obtained at previous iteration. In particular, we have introduced two different mixed formulations of quasi-reversibility for the ill-posed Stokes systems in order to use standard Lagrange finite elements.

6.6.3. Inverse scattering with generalized impedance boundary conditions

Participants: Laurent Bourgeois, Mathieu Chamaillard, Nicolas Chaulet.

This work is a collaboration between POEMS and DEFI projects (more precisely Houssem Haddar) and constitutes the subject of the PhD thesis of N. Chaulet, which was defended on the 27/11/2012. We are concerned with the identification of some obstacle and some Generalized Impedance Boundary Conditions (GIBC) on the boundary of such obstacle from far field measurements generated by the scattering of harmonic incident waves. The GIBCs are approximate models for thin coatings, corrugated surfaces, rough surfaces or imperfectly conducting media.

During this last year, we complemented our previous work in two directions. First, we justified the use of the Factorization method to solve the inverse obstacle problem in the presence of GIBCs. This method gives a uniqueness proof as well as a fast algorithm to reconstruct the obstacle from the knowledge of the far field produced by incident plane waves for all the directions of incidence at a given frequency. We also provided some numerical reconstructions of obstacles for several impedance operators.

Meanwhile, we studied the application of non linear optimization techniques to solve the inverse problem for the $3D$ Maxwell's equations. The main advantage of this type of method is that they can be applied with much less data than the Factorization method. Nevertheless, we had to compute the partial derivatives of the electromagnetic field with respect to the parameters we want to reconstruct. In our case, these parameters are the coefficients that define the impedance operator and the shape of the obstacle. We characterized these derivatives in the case where the GIBC is defined by a second order surface operator. The applicability of such methods has been illustrated by some numerical experiments in dimension 3 in which we reconstructed the shape of the scatterer as well as the coefficients that characterize the impedance operator. As demonstrated in the two dimensional case, we think that the GIBCs could be efficiently used to identify the shape of coated objects as well as the parameters of the coating in the $3D$ Maxwell case.

6.6.4. Linear sampling methods in the time domain

Participant: Simon Marmorat.

This work is developed in collaboration with H. Haddar (DEFI, Inria Saclay) and A. Lechleiter (Bremen University). We are concerned with the inverse problem of reconstructing obstacles from the knowledge of scattered acoustic waves in the time domain. We tackle this problem using a linear sampling method that directly acts on time domain data: this imaging technique yields a picture of the scatterer by solving a linear operator equation involving the measured data for many right-hand sides given by singular solutions to the wave equation. We have illustrated the method on numerical examples and have shown a good behaviour with respect to aperture (the quality of reconstruction is better than in the frequency case in the case of limited aperture) and the ability of simultaneously reconstructing obstacles with different boundary conditions among the Dirichlet, Neumann and Robin-Fourier ones.

6.6.5. Space-time focusing on unknown scatterers

Participants: Maxence Cassier, Patrick Joly, Christophe Hazard.

This topic concerns the studies started two years ago about time-reversal in the context of Maxence Cassier's thesis. The main question is to generate a time-dependent wave that focuses on one given scatterer not only in space, but also in time. Our recent works concern two items. On one hand, we have proposed a way to construct such a focusing wave which does not require an a priori knowledge of the location of the obstacle. This wave is represented by a suitable superposition of the eigenvectors of the so-called time-reversal operator in the frequency domain. Numerical results show the focusing properties of such a wave. On the other hand, we try to understand how to translate the physical idea of ìfocusingî into mathematical terms. We proposed and and implemented energy criterion which can be used in numerical experiments in order to evaluate the quality of the focus.

6.6.6. Asymptotic analysis of the interior transmission eigenvalues related to coated obstacles

Participant: Nicolas Chaulet.

This work is a collaboration with Fioralba Cakoni from the University of Delaware (USA) and Houssem Haddar from the DEFI project. The interior transmission eigenvalues play an important role in the area of inverse scattering problems. These eigenvalues can actually be determined by multi-static far field data. Thus, they could be used for non destructive testing. We focused on the case where the obstacle is a perfectly conducting body coated by some thin dielectric material. We derived and justified the asymptotic expansion of the first interior transmission eigenvalue with respect to the thickness of the coating for the TM electromagnetic polarization. This expansion provided interesting qualitative information about the behavior of these eigenvalues and also gave an explicit formula to compute the thickness of the coating.

6.6.7. Interior transmission problem

Participants: Anne-Sophie Bonnet-Ben Dhia, Lucas Chesnel, Jérémi Firozaly.

This work is a collaboration with F. Cakoni from the University of Delaware (U.S.) and H. Haddar from the DEFI project at Inria Saclay. The interior transmission problem plays an important role in the inverse scattering theory for inhomogeneous media. In particular, it arises when one is interested in the reconstruction of an inclusion embedded in a background medium from multi-static measurements of diffracted fields at a given frequency. Physically, it is important to prove that, for a given frequency, there are no waves which do not scatter. Mathematically, this last property boils down to state that the frequency is not a transmission eigenvalue, that is, an eigenvalue of the interior transmission problem. An important issue is to prove that transmission eigenvalues form at most a discrete set with infinity as the only accumulation point. This is not straightforward because the operator associated with this problem exhibits a sign changing in its principal part and its study is not standard. Using the T-coercivity approach, we proved the discreteness under relatively weak assumptions both for the scalar and Maxwell cases. In particular, the simple technique we proposed allows to treat cases, which were not covered by existing methods, where the difference between the inclusion index and the background index changes sign. Now, we are trying to understand the fundamental links which exist between this problem and the transmission problem between a positive and a negative material. In some configurations, the study of the interior transmission problems leads to consider the operator $\Delta(\sigma\Delta\cdot) : H_0^2(\Omega) \to H^{-2}(\Omega)$ where Ω is the domain and σ is a coefficient which changes sign on Ω. During the internship of Jérémy Firozaly, we proved that this operator exhibits properties very different from the operator $\mathrm{div}(\sigma\nabla\cdot) : H_0^1(\Omega) \to H^{-1}(\Omega)$.

6.6.8. Flaw identification using elastodynamic topological derivative

Participants: Marc Bonnet, Rémi Cornaggia.

In collaboration with Cédric Bellis (Columbia Univ. USA), Bojan Guzina (Univ. of Minnesota, USA).
The concept of topological derivative (TD) quantifies the perturbation induced to a given cost functional by the nucleation of an infinitesimal flaw in a reference defect-free body, and may serve as a flaw indicator function. In this work, the TD is derived for three-dimensional crack identification exploiting over-determined transient elastodynamic boundary data. This entails in particular the derivation of the relevant polarization tensor, here given for infinitesimal trial cracks in homogeneous or bi-material elastic bodies. Simple and efficient adjoint-state based formulations are used for computational efficiency, allowing to compute the TD field for arbitrarily shaped elastic solids. The latter is then used as an indicator function for the spatial location of the sought crack(s). Current investigations focus on justifying the heuristic underpinning TD-based identification, which consists in deeming regions where the TD is most negative as the likeliest locations of actual flaws and on formulating higher-order topological expansions in the elastodynamic case.

6.6.9. Topological derivative in anisotropic elasticity

Participant: Marc Bonnet.

In collaboration with Gabriel Delgado (CMAP, Ecole Polytechnique).
Following up on previous work on the topological derivative (TD) of displacement-based cost functionals in anisotropic elasticity, a TD formula has been derived for general cost functionals that involve strains (or displacement gradients) rather than displacements. The small-inclusion asymptotics of such cost functionals are quite different than in the previous case, due to the fact that the strain perturbation inside an elastic inclusion remains finite no matter how small the inclusion size. Cost functionals of practical interest having this format include von Mises equivalent stress (often used in plasticity or failure criteria) and energy-norm error functionals for coefficient-identification inverse problems.

6.6.10. Energy functionals for elastic medium reconstruction using transient data

Participant: Marc Bonnet.

In collaboration with Wilkins Aquino (Cornell Univ., USA).

Energy-based misfit cost functionals, known in mechanics as error in constitutive relation (ECR) functionals, are known since a long time to be well suited to (electrostatic, elastic,...) medium reconstruction. In this ongoing work, a transient elastodynamic version of this methodology is developed, with emphasis on its applicability to large time-domain finite element modeling of the forward problem. The formulation involves coupled transient forward and adjoint solutions, a fact which greatly hinders large-scale computations. A computational approach combining an iterative treatment of the coupled problem and the adjoint to the discrete Newmark time-stepping scheme is found to perform well on large FE models, making the time-domain ECR functional a worthwhile tool for medium identification.

6.7. Other topics

6.7.1. Fast non-overlapping Schwarz domain decomposition methods for the neutron diffusion equation

Participant: Patrick Ciarlet.

A collaboration with Erell Jamelot (CEA Saclay/DEN).
Investigating numerically the steady state of a nuclear core reactor can be very expensive, in terms of memory storage and computational time. In order to address both requirements, one can use a domain decomposition method, which is then implemented on a parallel computer.
We model the problem using a mixed approach, which involves a scalar flux and a vector current. The equivalent variational formulation is then discretized with the help of Raviart-Thomas-Nédélec finite elements. The domain decomposition method is based on the Schwarz iterative algorithm with Robin interface conditions to handle communications. This method is analyzed from the continuous to the discrete point of views: well-posedness, convergence of the finite element method, optimality of the parameter appearing in the Robin interface condition and algorithms. Numerical experiments carried out on realistic 3D configurations using the APOLLO3©code (of CEA/DEN) show the parallel efficiency of the algorithm.

POPIX Exploratory Action

6. New Results

6.1. Mixture of mixed effects models

Participants: Cyprien Mbogning, Marc Lavielle.

We have proposed a new methodology for maximum likelihood estimation in mixtures of non linear mixed effects models (NLMEM). The article *Inference in mixtures of non-linear mixed effects models* was submitted in 2012. Such mixtures of models include mixtures of distributions, mixtures of structural models and mixtures of residual error models. Since the individual parameters inside the NLMEM are not observed, we propose to combine the EM algorithm usually used for mixtures models when the mixture structure concerns an observed variable, with the Stochastic Approximation EM (SAEM) algorithm, which is known to be suitable for maximum likelihood estimation in NLMEM and also has nice theoretical properties. The main advantage of this hybrid procedure is to avoid a simulation step of unknown group labels required by a "full" version of SAEM. The resulting MSAEM (Mixture SAEM) algorithm is now implemented in the MONOLIX software. Several criteria for classification of subjects and estimation of individual parameters were also proposed. Numerical experiments on simulated data have shown that MSAEM performs well in a general framework of mixtures of NLMEM. Indeed, MSAEM provides an estimator close to the maximum likelihood estimator in very few iterations and is robust with regard to initialization. An application to pharmacokinetic (PK) data demonstrates the potential of the method for practical applications.

6.2. Between-subject and within-subject model mixtures for classifying HIV treatment response

Participants: Cyprien Mbogning, Kevin Bleakley, Marc Lavielle.

We have proposed a method for classifying individuals into clinically-relevant population subgroups [5]. This is achieved by treating "subgroup" as a categorical covariate whose value is unknown for each individual, and predicting its value using mixtures of models that represent "typical" longitudinal data from each subgroup. Under a nonlinear mixed effects model framework, two types of model mixtures were developed:

- *Between-Subject Model Mixtures* (BSMM) assume that each individual's longitudinal data follows one of M "base" models, but we do not necessarily know *a priori* which one. Individual i thus has a label $z_i = m \in \{1, ..., M\}$ referring to the model that is supposed to have generated it. We have shown how to extract *a posteriori* estimates of the probability that each individual was generated by each of the base models; this can be used to predict which type of patient we have: non-responder, responder or rebounder.
- *Within-Subject Model Mixtures* (WSMM) make the hypothesis that the model mixture occurs *within each individual*. In the HIV example, this means that we consider that each patient is partially a non-responder, partially a responder and partially a rebounder. This is perhaps more biologically plausible than BSMMs in the sense that each individual's response may be due to their own particular combination of virus strains, cell populations, etc. Within the NLMEM framework, this means including individual "model proportion" parameters into the model and having to estimate them along with the other parameters of the NLMEM. It turns out that this does not require any mathematical extensions to a typical NLMEM. But we can use the estimated proportions to help categorize patients, especially those who do not naturally fall into one of the three "typical" categories.

An application to longitudinal viral load data for HIV-positive patients were used to predict whether they are responding – completely, partially or not at all – to a new drug treatment.

6.3. Joint modeling of longitudinal and repeated time-to-event data

Participants: Cyprien Mbogning, Kevin Bleakley, Marc Lavielle.

We have proposed a nonlinear mixed-effects framework to jointly model longitudinal and repeated time-to-event data. The article *Joint modeling of longitudinal and repeated time-to-event data with maximum likelihood estimation via the SAEM algorithm* was submitted in 2012. A parametric nonlinear mixed-effects model is used for the longitudinal observations and a parametric mixed-effects hazard model for repeated event times. We have shown the importance for parameter estimation of properly calculating the conditional density of the observations (given the individual parameters) in the presence of interval and/or right censoring. Parameters are estimated by maximizing the exact joint likelihood with the Stochastic Approximation Expectation-Maximization algorithm.

We have illustrated the use of these modeling methods in two real data examples: patient survival in primary biliary cirrhosis, and repeated epileptic seizure count data from a clinical trial.

This workflow for joint models is now implemented in the MONOLIX software

6.4. A new Bayesian Information Criteria for mixed-effects models

Participants: Maud Delattre, Marie-Anne Poursat, Marc Lavielle.

The Bayesian Information Criterion (BIC) is widely used for variable selection in mixed effects models. However, its expression is unclear in typical situations of mixed effects models, where simple definition of the sample size is not meaningful. Yet, in the mixed effects model literature, the BIC penalty usually involves the total number of observations $\log n_{\text{tot}}$. From a practical point of view, the $\log n_{\text{tot}}$ penalty is implemented in the R package nlme and in the SPSS procedure MIXED while the $\log N$ penalty, where N is the number of subjects, is used in MONOLIX, saemix or in the SAS proc NLMIXED.

We have derived an appropriate BIC expression that is consistent with the random effect structure of the mixed effects model [7]. We have illustrated the behavior of the proposed criterion through a simulation study. The use of this new version of BIC is recommended as an alternative to various existing BIC versions that are implemented in available software.

6.5. Inference in mixed hidden Markov models

Participants: Maud Delattre, Marc Lavielle.

Mixed hidden Markov models have been recently defined in the literature as an extension of hidden Markov models for dealing with population studies. The notion of mixed hidden Markov models is particularly relevant for modeling longitudinal data collected during clinical trials, especially when distinct disease stages can be considered. However, parameter estimation in such models is complex, especially due to their highly nonlinear structure and the presence of unobserved states. Moreover, existing inference algorithms are extremely time consuming when the model includes several random effects.

We have proposed new inference procedures for estimating population parameters, individual parameters and sequences of hidden states in mixed hidden Markov models [1]. The main contribution consists of a specific version of the stochastic approximation EM algorithm coupled with the Baum-Welch algorithm for estimating population parameters. The properties of this algorithm were investigated via a Monte-Carlo simulation study.

An application of mixed hidden Markov models to the description of daily seizure counts in epileptic patients was then considered. We proposed to describe exposure-response relationship of gabapentin in epileptic patients using MHMM approach. Longitudinal seizure frequency data from six clinical studies were available for the analysis. The model describes daily seizure frequencies to be governed by an unobserved, yet present, underlying disease dynamics, defined by states of high or low epileptic activity. Individual day-to-day states are dependent exhibiting their own dynamics with patients transitioning between disease states, according to a set of transition probabilities. MHMM estimates both unobserved disease dynamics and daily seizure frequencies in all disease states. Novel drug action modes are achievable: drug may influence both seizure frequencies

and transition probabilities. The model showed that gabapentin significantly reduced seizure frequencies in both disease states, without altering disease dynamics. Novel methodology offers additional insights into understanding epilepsy time course, gabapentin mode of action and provides a tool for realistic clinical trial simulations.

6.6. Inference in mixed-effects diffusion models

Participants: Maud Delattre, Marc Lavielle.

The structure of mixed effects models allows a suitable consideration of the whole variability characterizing such data, which is usually split into some intra-individual variability - i.e., the variability occurring within the dynamics of each individual - and some between-subjects variability. In a mixed-effects model, the same structural model is used for describing each individual sequence of observations, but the parameters of this model vary randomly among the individuals, which allows a correct account of the differences between subjects. In a mixed-effects diffusion model, the description of each individual series of observations is based on stochastic differential equations (SDEs). Diffusion is known to be a relevant tool for describing random variability in dynamical systems, and is widely used in applications in many domains.

Although many methods are available for the inference in classical fixed-effects diffusion models, there is still a need for a general, fast and easy to implement method for the inference in mixed-effects diffusion models. Indeed, except in very specific classes of mixed-effects diffusion models, the likelihood of the observations does not have any closed-form expression, making maximum likelihood estimation of the model parameters an intricate issue. The difficulty is twofold for computing the observed likelihood since it involves the transition densities of the underlying individual diffusion processes and integrals over the unobserved individual parameters that can rarely be computed in a closed form. Specific versions of the SAEM algorithm have already been proposed for estimating the population parameters in mixed-effects diffusion models (using for instance an Euler-Maruyama approximation of the individual processes or some particle Markov Chain Monte-Carlo methods). In these two versions of SAEM however, simulation of both the random individual parameters and the individual latent processes is required at simulation step, which is computationally cumbersome.

We have proposed a new inference methodology for mixed-effects diffusion models which consists in coupling the SAEM algorithm with the extended Kalman filter for estimating the population parameters. The relevant article has been submitted in 2012. In this new version of the SAEM algorithm, we only need to simulate the individual parameters at each iteration. We also provide tools for estimating the individual parameters and the individual diffusion trajectories.

6.7. Random threshold for linear model selection

Participant: Marc Lavielle.

We have in a previous work introduced a random thresholding method to select the significant, or non-null, mean terms from a collection of independent random variables, and applied it to the problem of recovering the significant coefficients in nonordered model selection.

We have improved this method by introducing a simple modification which removes the dependency of the proposed estimator on a window parameter while maintaining its asymptotic properties [4]. A simulation study suggests that both procedures compare favorably to standard thresholding approaches, such as multiple testing or model-based clustering, in terms of the binary classification risk. An application to the problem of activation detection on functional magnetic resonance imaging (fMRI) data was used to illustrate the performance of the proposed method.

REGULARITY Project-Team

6. New Results

6.1. A multifractional Hull and White model

Participants: Joachim Lebovits, Jacques Lévy Véhel.

In collaboration with Sylvain Corlay (Paris 6 University).

We have considered the following model, which is an extension of the fractional Hull and White model proposed in [55]: under the risk-neutral measure, the forward price of a risky asset is the solution of the S.D.E.

$$\begin{aligned} dF_t &= F_t \sigma_t dW_t, \\ d\ln(\sigma_t) &= \theta(\mu - \ln(\sigma_t))\,dt + \gamma_h d^\diamond B_t^h + \gamma_\sigma dW_t^\sigma, \quad \sigma_0 > 0, \theta > 0, \end{aligned}$$

where B_t^h is a multifractional Brownian motion with regularity function h, and W_t, W_t^σ are standard Brownian motions. This SDE is interpgreted in the Wick-Itô sense.

Using functional quantization techniques, it is possible to compute numerically implied forward start volatilities for this model. Using an adequate h function estimated from SP500 data, we have shown that this model is able to reproduce to some extent the volatility surface observed on the market [34].

6.2. Markov characterization of the set-indexed Lévy process

Participant: Erick Herbin.

In collaboration with Prof. Ely Merzbach (Bar Ilan university, Israel).

In [21], the class of set-indexed Lévy processes is considered using the stationarity property defined for the set-indexed fractional Brownian motion in [20]. The general framework of Ivanoff-Merzbach allows to consider standard properties of stochastic processes (e.g. martingale and Markov properties) in the set-indexed context. Processes are indexed by a collection $\mathcal{A}$ of compact subsets of a metric space $\mathcal{T}$ equipped with a Radon measure m, which satisfies several stability conditions. Each process $\{X_U;\ U \in \mathcal{A}\}$ is assumed to admit an increment process $\{\Delta X_C;\ C \in \mathcal{C}\}$ defined as an additive extension of X to the collections $\mathcal{C}_0 = \{U \setminus V;\ U, V \in \mathcal{A}\}$ and

$$\mathcal{C} = \left\{ U \setminus \bigcup_{1 \le i \le n} V_i;\ n \in \mathbf{N}; U, V_1, \cdots, V_n \in \mathcal{A} \right\}.$$

A set-indexed process $X = \{X_U;\ U \in \mathcal{A}\}$ is called a *set-indexed Lévy process* if the following conditions hold

1. $X_{\varnothing'} = 0$ almost surely, where $\varnothing' = \bigcap_{U \in \mathcal{A}} U$.
2. the increments of X are independent: for all pairwise disjoint $C_1, \cdots, C_n$ in $\mathcal{C}$, the random variables $\Delta X_{C_1}, \cdots, \Delta X_{C_n}$ are independent.
3. X has m-stationary $\mathcal{C}_0$-increments, i.e. for all integer n, all $V \in \mathcal{A}$ and for all increasing sequences $(U_i)_i$ and $(A_i)_i$ in $\mathcal{A}$, we have

$$[\forall i,\ m(U_i \setminus V) = m(A_i)] \Rightarrow (\Delta X_{U_1 \setminus V}, \cdots, \Delta X_{U_n \setminus V}) \overset{(d)}{=} (\Delta X_{A_1}, \cdots, \Delta X_{A_n})$$

4. X is continuous in probability: if $(U_n)_{n \in \mathbf{N}}$ is a sequence in $\mathcal{A}$ such that

$$\overline{\bigcup_n \bigcap_{k\geq n} U_k} = \bigcap_n \overline{\bigcup_{k\geq n} U_k} = A \in \mathcal{A}$$

then

$$\lim_{n\to\infty} P\left\{|X_{U_n} - X_A| > \epsilon\right\} = 0$$

On the contrary to previous works of Adler and Feigin (1984) on one hand, and Bass and Pyke (1984) one the other hand, the increment stationarity property allows to obtain explicit expressions for the finite-dimensional distributions of a set-indexed Lévy process. From these, we obtained a complete characterization in terms of Markov properties.

Among the various definitions for Markov property of a SI process, we considered the $\mathcal{Q}$-Markov property. A collection $\mathcal{Q}$ of functions

$$\begin{aligned} \mathbf{R}\times\mathcal{B}(\mathbf{R}) &\to \mathbf{R}_+ \\ (x,B) &\mapsto Q_{U,V}(x,B) \end{aligned}$$

where $U, V \in \mathcal{A}(u)$ are s.t. $U \subseteq V$, is called a *transition system* if the following conditions are satisfied:

1. $Q_{U,V}(\bullet, B)$ is a random variable for all $B \in \mathcal{B}(\mathbf{R})$.
2. $Q_{U,V}(x, \bullet)$ is a probability measure for all $x \in \mathbf{R}$.
3. For all $U \in \mathcal{A}(u)$, $x \in \mathbf{R}$ and $B \in \mathcal{B}(\mathbf{R})$, $Q_{U,U}(x,B) = \delta_x(B)$.
4. For all $U \subseteq V \subseteq W \in \mathcal{A}(u)$,

$$\int_{\mathbf{R}} Q_{U,V}(x,dy) Q_{V,W}(y,B) = Q_{U,W}(x,B).$$

A transition system $\mathcal{Q}$ is said

- *spatially homogeneous* if for all $U \subset V$,

$$\forall x \in \mathbf{R}, \forall B \in \mathcal{B}(\mathbf{R}), \quad Q_{U,V}(x,B) = Q_{U,V}(0, B-x);$$

- *m-homogeneous* if $Q_{U,V}$ only depends on $m(V \setminus U)$,
  i.e. $\forall U, V, U', V' \in \mathcal{A}(u)$ such that $U \subset V$ and $U' \subset V'$,

$$m(V \setminus U) = m(V' \setminus U') \Rightarrow Q_{U,V} = Q_{U',V'}.$$

A set-indexed process $X := \{X_U;\ U \in \mathcal{A}\}$ is called $\mathcal{Q}$-*Markov* if $\forall U, V \in \mathcal{A}(u)$, $U \subseteq V$

$$\forall B \in \mathcal{B}(\mathbf{R}), \quad P[\Delta X_V \in \Gamma \mid \mathcal{F}_U] = Q_{U,V}(\Delta X_U; \Gamma),$$

where $(\mathcal{F}_U)_{U\in\mathcal{A}(u)}$ is the minimal filtration of the process X.

Balan-Ivanoff (2002) proved that any SI process with independent increments is a $\mathcal{Q}$-Markov process with a spatially homogeneous transition system. The following result proved in [21] shows that the converse is true.

Theorem *Let $X = \{X_U;\ U \in \mathcal{A}\}$ be a set-indexed process with definite increments. The two following assertions are equivalent:*

1. *X is a $\mathcal{Q}$-Markov process with a spatially homogeneous transition system $\mathcal{Q}$;*
2. *X has independent increments.*

This result is strengthened in the following characterization of set-indexed Lévy processes as Markov processes with homogeneous transition systems.
Theorem *Let $X = \{X_U;\ U \in \mathcal{A}\}$ be a set-indexed process with definite increments and satisfying the stochastic continuity property.*

The two following assertions are equivalent:

1. *X is a set-indexed Lévy process ;*
2. *X is a $\mathcal{Q}$-Markov process such that $X_\varnothing = 0$ and the transition system $\mathcal{Q}$ is spatially homogeneous and m-homogeneous.*

Consequently, if $\mathcal{Q}$ is a transition system which is both spatially homogeneous and m-homogeneous, then there exists a set-indexed process X which is a $\mathcal{Q}$-Markov process.

6.3. Local Hölder regularity of Set-Indexed processes

Participants: Erick Herbin, Alexandre Richard.

In the set-indexed framework of Ivanoff and Merzbach ([62]), stochastic processes can be indexed not only by $\mathbf{R}$ but by a collection $\mathcal{A}$ of subsets of a measure and metric space $(\mathcal{T}, d, m)$, with some assumptions on $\mathcal{A}$. In we introduce and study some assumptions (A_1) and (A_2) on the metric indexing collection $(\mathcal{A}, d_\mathcal{A})$ in order to obtain a Kolmogorov criterion for continuous modifications of SI stochastic processes. Under this assumption, the collection is totally bounded and a set-indexed process with good incremental moments will have a modification whose sample paths are almost surely Hölder continuous, for the distance $d_\mathcal{A}$. Once this condition is established, we investigate the definition of Hölder coefficients for SI processes. We shall denote $\widetilde{\alpha}_X(t)$ and $\alpha_X(t)$ for the local and pointwise Hölder exponents of X at t, and $\widetilde{\alpha}_X(t)$ and $\alpha_X(t)$ for their deterministic counterpart in case X is Gaussian.

In [18], a set-indexed extension for fractional Brownian motion has been defined and studied. A mean-zero Gaussian process $\mathbf{B}^H = \left\{\mathbf{B}^H_U, U \in \mathcal{A}\right\}$ is called a *set-indexed fractional Brownian motion (SIfBm for short)* on $(\mathcal{T}, \mathcal{A}, m)$ if

$$\forall U, V \in \mathcal{A}, \quad \mathbf{E}\left[\mathbf{B}^H_U \mathbf{B}^H_V\right] = \frac{1}{2}\left[m(U)^{2H} + m(V)^{2H} - m(U \bigtriangleup V)^{2H}\right], \tag{5}$$

where $H \in (0, 1/2]$ is the index of self-similarity of the process.

In [12], $\widetilde{\alpha}_X$ and $\widetilde{\alpha}_X$ have been determined for the particular case of an SIfBm indexed by the collection $\{[0,t];\ t \in \mathbf{R}^N_+\} \cup \{\varnothing\}$, called the *multiparameter fractional Brownian motion*. If X denotes the $\mathbf{R}^N_+$-indexed process defined by $X_t = \mathbf{B}^H_{[0,t]}$ for all $t \in \mathbf{R}^N_+$, it is proved that for all $t_0 \in \mathbf{R}^N_+$, $\widetilde{\alpha}_X(t_0) = H$ and with probability one, for all $t_0 \in \mathbf{R}^N_+$, $\widetilde{\alpha}_X(t_0) = H$. A theorem of allows one to extend these results to SIfBm indexed by a more general class than the sole collection of rectangles of $\mathbf{R}^N_+$.

Theorem 0.1 Let $\mathbf{B}^H$ be a set-indexed fractional Brownian motion on $(\mathcal{T}, \mathcal{A}, m)$, $H \in (0, 1/2]$. Assume that the subclasses $(\mathcal{A}_n)_{n\in\mathbf{N}}$ satisfy Assumption (A_1).
Then, the local and pointwise Hölder exponents of $\mathbf{B}^H$ at any $U_0 \in \mathcal{A}$, defined with respect to the distance d_m or any equivalent distance, satisfy

$$\mathbf{P}\left(\forall U_0 \in \mathcal{A},\ \widetilde{\alpha}_{\mathbf{B}^H}(U_0) = H\right) = 1$$

and if Assumption (A_2) holds,

$$\mathbf{P}\left(\forall U_0 \in \mathcal{A},\ \alpha_{\mathbf{B}^H}(U_0) = H\right) = 1.$$

Consequently, since the collection $\mathcal{A}$ of rectangles of $\mathbf{R}_+^N$ with m the Lebesgue measure satisfies (A_1) and (A_2), we obtained a new result on a classical multiparameter process: the multiparameter fractional Brownian motion $\mathbf{B}^H$ satisfy, for $T \in \mathbf{R}_+^N$:

$$\mathbf{P}\left(\forall t \in [0,T],\ \alpha_{\mathbf{B}^H}([0,t]) = \widetilde{\alpha}_{\mathbf{B}^H}([0,t]) = H\right) = 1.$$

6.4. Separability of Set-Indexed Processes

Participant: Alexandre Richard.

A classical result states that any (multiparameter) stochastic process has a separable modification, thus ensuring the measurability property of the sample paths. We extend this result to set-indexed processes.
Let $(T, \mathcal{O})$ be a topological space. We assume that this space is *second-countable*, ie there exists a countable subset $\widetilde{\mathcal{O}} \subseteq \mathcal{O}$ such that any open set of $\mathcal{O}$ can be expressed as a union of elements of $\widetilde{\mathcal{O}}$.

A process $\{X_t, t \in T\}$ is *separable* if there exists an at most countable set $S \subset T$ and a null set Λ such that for all closed sets $F \subset \mathbf{R}$ and all open set $O \in \mathcal{O}$,

$$\{\omega : X_s(\omega) \in F \text{forall} s \in O \cap S\} \setminus \{\omega : X_s(\omega) \in F \text{forall} s \in O\} \subset \Lambda.$$

This definition is different of the one found in [57], where the space is "linear", in that this author considers the previous equation only when O is an interval. It happens that this notion needs not be defined in a general topological space. However when restricted to a vector space, our definition implies the previous one.

Theorem 0.2 (Doob's separability theorem) Any T-indexed stochastic process $X = \{X_t;\ t \in T\}$ has a separable modification.

If T is an indexing collection in the sense of [62], the topology induced by the distance d_T has to be second-countable. This happens for instance when (T, d_T) is totally bounded, which is the case in

6.5. An increment type set-indexed Markov property

Participant: Paul Balança.

[1] investigates a new approach for the definition of a set-indexed Markov property, named $\mathcal{C}$-*Markov*. The study is based on Merzbach and Ivanoff's set-indexed formalism, i.e. $\mathcal{A}$ denotes a set-indexed collection and $\mathcal{C}$ the family of increments $C = A \setminus B$, where $A \in \mathcal{A}$ and $B \in \mathcal{A}(u)$ (finite unions of sets from $\mathcal{A}$). Moreover, for any $C = A \setminus B$, $B = \cup_{i=1}^k A_i$, $\mathcal{A}_{\mathbf{C}}$ is defined as the following subset of $\mathcal{A}$:

$$\mathcal{A}_{\mathbf{C}} = \{U \in \mathcal{A}_\ell;\ U \not\subseteq B^\circ\} := \{U_C^1, \cdots, U_C^p\}, \qquad \text{where } p = |\mathcal{A}_{\mathbf{C}}|$$

and $\mathcal{A}_\ell$ corresponds to the semilattice $\{A_1 \cap \cdots \cap A_k, \cdots, A_1 \cap A_2, A_1 \cdots, A_k\} \subset \mathcal{A}$. The notation $\mathbf{X_C}$ refers to a random vector $\mathbf{X_C} = \left(X_{U_C^1}, \cdots, X_{U_C^p}\right)$. Similarly, $\mathbf{x_C}$ is used to denote a vector of variables $(x_{U_C^1}, \cdots, x_{U_C^p})$.

Then, an E-valued set-indexed process $(X_A)_{A\in\mathcal{A}}$ is said to be $\mathcal{C}$-*Markov* with respect to a filtration $(\mathcal{F}_A)_{A\in\mathcal{A}}$ if it is adapted to $(\mathcal{F}_A)_{A\in\mathcal{A}}$ and if it satisfies

$$\mathbb{E}[f(X_A)\,|\,\mathcal{G}_C^*] = \mathbb{E}[f(X_A)\,|\,\mathbf{X_C}] \quad \mathbb{P}\text{-a.s.} \tag{6}$$

for all $C = A \setminus B \in \mathcal{C}$ and any bounded measurable function $f : E \to \mathbf{R}$. The sigma-algebra $\mathcal{G}_C^*$ is usually called the strong history of $(\mathcal{F}_A)_{A\in\mathcal{A}}$ and is defined as $\mathcal{G}_C^* = \bigvee_{A\in\mathcal{A}, A\cap C=\varnothing} \mathcal{F}_A$.

The $\mathcal{C}$-Markov approach has several advantages compared to existing set-indexed Markov literature (mainly $\mathcal{Q}$-Markov described in [48]). It appears to be a natural extension of the classic one-parameter Markov property. In particular, the concept of transition system can easily extended to our formalism: for any $\mathcal{C}$-Markov process X, one can defined $\mathcal{P} = \{P_C(\mathbf{x_C}; \mathrm{d}x_A); C \in \mathcal{C}\}$ as

$$\forall \mathbf{x_C} \in E^{|\mathcal{A}_\mathbf{C}|}, \Gamma \in \mathcal{E}; \quad P_C(\mathbf{x_C}; \Gamma) := \mathbb{P}(X_A \in \Gamma \,|\, \mathbf{X_C} = \mathbf{x_C}).$$

A $\mathcal{C}$-transition system $\mathcal{P}$ happens to satisfy a set-indexed Chapman-Kolmogorov equation,

$$\forall C \in \mathcal{C},\ A' \in \mathcal{A}; \qquad P_C f = P_{C'} P_{C''} f \quad \text{where} \quad C' = C \cap A',\ C'' = C \setminus A' \tag{7}$$

and f is a bounded measurable function.

Similarly to the classic Markovian theory, is is proved in [1] that the initial distribution μ and $\mathcal{P}$ characterize entirely the law of a $\mathcal{C}$-Markov process, and that conversely, for any initial law and any $\mathcal{C}$-transition system, a corresponding canonical set-indexed $\mathcal{C}$-Markov process can be constructed. $\mathcal{C}$-Markov processes enjoy several other properties such as

1. Projections on elementary flows are Markovian;
2. Conditional independence of natural filtrations;
3. Strong Markov property.

The class of set-indexed Lévy processes defined and studied in [21] offers examples of $\mathcal{C}$-Markov processes whose transition probabilities correspond to

$$\forall C = A \setminus B \in \mathcal{C}, \quad \forall \Gamma \in \mathcal{E}; \quad P_C(\mathbf{x_C}; \Gamma) = \mu^{m(C)}(\Gamma - \Delta x_B), \tag{8}$$

where m is a measure on $\mathcal{T}$ and μ the infinitely divisible probability measure that characterizes the Lévy process. We note that the transition system related the $\mathcal{Q}$-Markov property has a different form, even if it is related.

Another non-trivial example of $\mathcal{C}$-Markov process is the set-indexed Ornstein-Uhlenbeck process that has been introduced and studied in [32]. It is a Gaussian Markovian process whose transition densities are given by

$$p_C(\mathbf{x_C}; y) = \frac{1}{\sigma_C\sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma_C^2}\left(y - e^{-\lambda m(A)}\left[\sum_{i=1}^{n} (-1)^{\varepsilon_i}\, x_{U_C^i}\, e^{\lambda m(U_C^i)}\right]\right)^2\right], \tag{9}$$

where λ and σ are positive parameters, m is a measure on $\mathcal{T}$ and

$$\sigma_C^2 = \frac{\sigma^2}{2\lambda}\left(1 - e^{-2\lambda m(A)}\left[\sum_{i=1}^{n} (-1)^{\varepsilon_i} e^{2\lambda m(U_C^i)}\right]\right).$$

In the particular case of multiparameter processes, corresponding to the indexing collection $\mathcal{A} = \{[0,t]; t \in \mathbf{R}_+^N\}$, the $\mathcal{C}$-Markov formalism is related to several existing works. It generalizes the two-parameter $*$-Markov property introduced in [53] and also embraces the multiparameter Markov property investigated recently in [68]. Finally, under some Feller assumption on the transition system, a multiparameter $\mathcal{C}$-Markov process is proved to admit a modification with right-continuous sample paths.

6.6. Fine regularity of Lévy processes

Participant: Paul Balança.

This ongoing work focuses on the fine regularity of one-parameter Lévy processes. The main idea of this study is to use the framework of stochastic 2-microlocal analysis (introduced and developed in [16],[33]) to refine sample paths results obtained in [65].

The latter describes entirely the multifractal spectrum of Lévy processes, i.e. the Hausdorff geometry of level sets $(E_h)_{h\in\mathbf{R}_+}$ of the pointwise exponent. These are usually called the *iso-Hölder sets* of X and are given by

$$E_h = \{t \in \mathbf{R} : \alpha_{X,t} = h\} \quad \text{for every } h \in \mathbf{R}_+ \cup \{+\infty\}.$$

The multifractal spectrum is itself defined as the localized Hausdorff dimension of the previous sets, i.e.

$$d_X(h,V) = \dim_{\mathcal{H}}(E_h \cap V) \quad \text{for every } h \in \mathbf{R}_+ \cup \{+\infty\} \text{ and } V \in \mathcal{O} \text{ (open sets in } \mathbf{R}). \tag{10}$$

[65] states that under a mild assumption on the Lévy measure π, a Lévy process X with no Brownian component almost surely satisfies

$$\forall V \in \mathcal{O}; \quad d_X(h,V) = \begin{cases} \beta h & \text{if } h \in [0,1/\beta]; \\ -\infty & \text{if } h \in (1/\beta, +\infty], \end{cases} \tag{11}$$

where the Blumenthal-Getoor exponent β is given by

$$\beta = \inf\left\{\delta \geq 0 : \int_{\mathbf{R}^d} \left(1 \wedge \|x\|^\delta\right) \pi(\mathrm{d}x) < \infty\right\}. \tag{12}$$

Since classic multifractal analysis focuses on the pointwise exponent, it is natural from our point of view to integrate the 2-microlocal frontier into this description. More precisely, we focus on the dichotomy usual/unusual regularity, corresponding to the sets $(\widetilde{E}_h)_{h\in\mathbf{R}_+}$ and $(\widehat{E}_h)_{h\in\mathbf{R}_+}$:

$$\widetilde{E}_h = \{t \in E_h : \forall s' \in \mathbf{R};\ \sigma_{X,t}(s') = (h+s') \wedge 0\} \quad \text{and} \quad \widehat{E}_h = E_h \setminus \widetilde{E}_h.$$

The collection $(\widetilde{E}_h)_{h\in\mathbf{R}_+}$ represents times at which the 2-microlocal behaviour is rather common (i.e. the slope is equal to one), whereas at points which belong $(\widehat{E}_h)_{h\in\mathbf{R}_+}$, the 2-microlocal frontier has an unusual form.

Then, our main result states that sample paths of a Lévy process X with no Brownian component almost surely satisfy

$$\forall V \in \mathcal{O}; \quad \dim_{\mathcal{H}}(\widetilde{E}_h \cap V) = \begin{cases} \beta h & \text{if } h \in [0,1/\beta]; \\ -\infty & \text{if } h \in (1/\beta, +\infty]. \end{cases} \tag{13}$$

Furthermore, the collection of sets $(\widehat{E}_h)_{h\in\mathbf{R}_+}$ enjoys almost surely

$$\forall V\in\mathcal{O};\quad \dim_{\mathcal{H}}(\widehat{E}_h\cap V)\leq\begin{cases}2\beta h-1 & \text{if } h\in(1/2\beta,1/\beta)\,;\\ -\infty & \text{if } h\in[0,1/2\beta]\cup[1/\beta,+\infty].\end{cases}\tag{14}$$

These results clearly extend those obtained in [65] since we know that the pointwise exponent is completely characterize by the 2-microlocal frontier. Moreover, it also proves that from a Hausdorff dimension point of view, the common regularity is a 2-microlocal frontier with a slope equal to one.

Nevertheless, equation (15) also exhibits some unusual behaviours, corresponding to times $(\widehat{E}_h)_{h\in\mathbf{R}_+}$, that are not captured by the classic multifractal spectrum. The existence of such particular times highly depends on the structure of the Lévy measure, and not only the value of the Blumenthal-Getoor exponent which is therefore not sufficient to characterize entirely the fine regularity. This last aspect of the study illustrates the fact that 2-microlocal analysis is an interesting tool for the study of stochastic processes' regularity since some sample paths' properties can not be captured by common tools such as Hölder exponents.

6.7. A class of self-similar processes with stationary increments in higher order Wiener chaoses.

Participant: Benjamin Arras.

Self similar processes with stationary increments (SSSI processes) have been studied for a long time due to their importance both in theory and in practice. Such processes appear as limits in various renormalisation procedures [69]. In applications, they occur in various fields such as hydrology, biomedicine and image processing. The simplest SSSI processes are simply Brownian motion and, more generally, Lévy stable motions. Apart from these cases, the best known such process is probably fractional Brownian motion (fBm). A construction of SSSI processes that generalizes fBm to higher order Wiener chaoses was proposed in [73]. These processes read

$$\forall t\in\mathbb{R}_+\quad X_t=\int_{\mathbb{R}^d}h_t^H(x_1,...,x_d)dB_{x_1}...dB_{x_d}$$

where h_t^H verifies:

1. $h_t^H\in\widehat{L}^2(\mathbb{R}^d)$,
2. $\forall c>0,\quad h_{ct}^H(cx_1,...,cx_d)=c^{H-\frac{d}{2}}h_t^H(x_1,...,x_d)$,
3. $\forall\rho\geq 0,\quad h_{t+\rho}^H(x_1,...,x_d)-h_t^H(x_1,...,x_d)=h_\rho^H(x_1-t,...,x_d-t)$.

In [41], we define a class of such processes by the following multiple Wiener-Itô integral representation:

$$X_t^\alpha=\int_{\mathbb{R}^d}\left[||\mathbf{t}^*-\mathbf{x}||_2^{H-\frac{d}{2}}-||\mathbf{x}||_2^{H-\frac{d}{2}}\right]dB_{x_1}...dB_{x_d}\tag{15}$$

where $t\in[0,1]$, $\mathbf{t}^*=(t,...,t)$ and $\alpha=H-1+\frac{d}{2}$. When $d=1$, this is just fBm. In order to study the local regularity of this class of processes as well as the asymptotic behaviour at infinity, we use wavelet's methods. More precisely, following ideas from [46], we obtain the following wavelet-like expansion:
Almost surely,

$$\forall t\in[0,1]\quad X_t^\alpha=\sum_{j\in\mathbb{Z}}\sum_{\mathbf{k}\in\mathbb{Z}^d}\sum_{\epsilon\in E}2^{-jH}\left[I^{\alpha+1}(\psi^{(\epsilon)})(2^j\mathbf{t}^*-\mathbf{k})-I^{\alpha+1}(\psi^{(\epsilon)})(-\mathbf{k})\right]I_d(\psi_{j,\mathbf{k}}^{(\epsilon)}).$$

From this representation, we get several results about this class of processes. Namely:

- There exists a strictly positive random variable A_d of finite moments of any order and a constant, $b_d > 1$, such that:

$$\forall \omega \in \Omega^* \quad \sup_{(s,t)\in[0,1]} \frac{|X_t^\alpha(\omega) - X_s^\alpha(\omega)|}{|t-s|^H (\log(b_d + |t-s|^{-1}))^{\frac{d}{2}}} \leq A_d(\omega).$$

- There exists a strictly positive random variable B_d of finite moments of any order and a constant $c_d > 3$, such that:

$$\forall \omega \in \Omega^* \quad \sup_{t\in\mathbb{R}_+} \frac{|X_t^\alpha(\omega)|}{(1+|t|)^H (\log\log(c_d + |t|))^{\frac{d}{2}}} \leq B_d(\omega).$$

Using an estimate from [54], we compute the uniform almost sure pointwise Hölder exponent of X^α defined by:

$$\gamma_{X^\alpha}(t) = \sup\{\gamma > 0 : \quad \limsup_{\rho\to 0} \frac{|X_{t+\rho}^\alpha - X_t^\alpha|}{|\rho|^\gamma} < +\infty\}.$$

We get the following result:
Almost surely,

$$\forall t \in (0,1), \ \gamma_{X^\alpha}(t) = H.$$

In the last part of [41], we give general bounds on the Hausdorff dimension of the range and graphs of multidimensional anisotropic SSSI processes defined by multiple Wiener integrals. Let $Y_t^H = \gamma(H,d) I_d(h_t^H)$ where $\gamma(H,d)$ is a normalizing positive constant such that $\mathbb{E}[|Y_1^H|^2] = 1$. Let $\frac{1}{2} < H_1 \leq ... \leq H_N < 1$. Let $\{\mathbb{Y}_t^H\}$ be the multidimensional process defined by:

$$\{\mathbb{Y}_t^H\} = \{(Y_t^{H_1}, ..., Y_t^{H_N}) : t \in \mathbb{R}_+\}$$

where the coordinates are independent copies of the process Y_t^H. Following classical ideas from [78] and using again the estimate from [54], we obtain:
Almost surely,

$$dim_{\mathcal{H}} R_E(\mathbb{Y}^H) \geq \min\left(N; \frac{dim_{\mathcal{H}} E + \frac{\sum_{j=1}^k (H_k - H_j)}{d}}{H_k}, k = 1, ..., N\right),$$

$$dim_{\mathcal{H}} Gr_E(\mathbb{Y}^H) \geq \min\left(\frac{dim_{\mathcal{H}} E + \frac{\sum_{j=1}^k (H_k - H_j)}{d}}{H_k}, k = 1, ..., N, dim_{\mathcal{H}} E + \sum_{i=1}^N \frac{(1-H_i)}{d}\right).$$

And,

$$dim_{\mathcal{H}} R_E(\mathbb{Y}^H) \leq \min\left(N; \frac{dim_{\mathcal{H}} E + \sum_{j=1}^{k}(H_k - H_j)}{H_k}, k = 1, ..., N\right),$$

$$dim_{\mathcal{H}} Gr_E(\mathbb{Y}^H) \leq \min\left(\frac{dim_{\mathcal{H}} E + \sum_{j=1}^{k}(H_k - H_j)}{H_k}, k = 1, ..., N; dim_{\mathcal{H}} E + \sum_{i=1}^{N}(1 - H_i)\right).$$

where $E \subset \mathbb{R}_+$.

6.8. Economic growth models

Participants: Jacques Lévy Véhel, Lining Liu.

In collaboration with D. La Torre, University of Milan.

We study certain economic growth models where we add a source of randomness to make the evolution equations more realistic. We have studied two particular models:

- An augmented Uzawa-Lucas growth model where technological progress is modelled as the solution of a stochastic differential equation driven by a Lévy or an additive process. This allows for a more faithful description of reality by taking into account discontinuities in the evolution of the level of technology. In details, we consider a closed economy in which there is single good which is produced by combining physical capital $K(t)$ and human capital $H(t)$. The laws of motions of $K(t)$ and $H(t)$ are:

$$\dot{K}(t) = A(t)^{\gamma}[u(t)H(t)]^{\xi}K(t)^{1-\xi-\gamma} - \beta_K K(t) - C(t), \tag{16}$$

$$K(0) = K_0;$$

$$\dot{H}(t) = (\eta(1 - u(t)) - \beta_H)H(t), \tag{17}$$

$$H(0) = H_0,$$

where $A(t)$ is the level of technology, $H(t)$ is the total stock of human capital, $u(t)$ is the proportion to the production of good, $\gamma \in (0, 1)$, $\xi \in (0, 1)$ and $1 - \xi - \gamma \in (0, 1)$ are the shares of income accruing to $A(t)$, $u(t)H(t)$ and $K(t)$, respectively, $\beta_K \in [0, 1]$ is the constant rate of depreciation of physical capital, $\beta_H \in [0, 1]$ is the rate of depreciation of human capital and $\eta \geq 0$ is the productivity of human capital.

We assume that the level of technology evolves according to the following stochastic differential equation:

$$dA(t) = \mu A(t)dt + \sigma A(t)dW(t) + \delta \int A(t^-)z(\widetilde{N}(dt, dz) - \nu(dt, dz)), \tag{18}$$

where $\mu \in \mathbb{R}$ is the drift rate, $\sigma > 0$ is the volatility, $0 \leq \delta \leq 1$, W is a standard Brownian motion and $\widetilde{N}$ is Poisson random measure with intensity measure ν which satisfies

$$\lim_{s\to t^+}\frac{1}{s-t}\int_t^s\int_{-1}^1 z^2\nu(dz,dx)+\lim_{s\to t^+}\frac{1}{s-t}\int_t^s\int_1^\infty z\nu(dz,dx)<\infty,$$

and

$$\int_0^t\int_{-1}^1 z^2\nu(dz,dx)+\int_0^t\int_1^\infty z\nu(dz,dx)<\infty,$$

for $t>0$.

With a CIES utility function, the optional inter-temporal decision problem can be formulated as

$$\max_{[C,u]}\mathbb{E}\left[\int_0^\infty\frac{C(t)^{1-\phi}-1}{1-\phi}e^{-\rho t}dt\right], \tag{19}$$

where $\rho>0$ is the rate of time preference and $\phi>0$. We denote $V(H,K,A)$ the maximum value function associated with the stochastic optimisation problem. For given t, the maximum expect utility up to time t obtained when applying the stochastic control $[C(t),u(t)]$ is defined by

$$V(H(t),K(t),A(t))=\max_{[C,u]}\mathbb{E}\left[\int_0^t\frac{C(x)^{1-\phi}-1}{1-\phi}e^{-\rho x}dx\right]. \tag{20}$$

We have been able to solve this program under some simplifying assumptions. Numerical simulations allow one to assess precisely the effect of (tempered) multistable noise on the model.

- A stochastic demographic jump shocks in a multi-sector growth model with physical and human capital accumulation. This models allows one to take into account sudden changes in population size, due for instance to wars or natural catastrophes. The laws of motions of physical capital $K(t)$ and human capital $H(t)$ are:

$$\dot{K}(t)=AM(t)^{1-\xi-\beta}[u(t)H(t)]^\beta K(t)^\xi-\eta_K K(t)-c(t)M(t), \tag{21}$$

$$\dot{H}(t)=B(1-u(t))H(t)-\eta_H H(t), \tag{22}$$

with initial conditions $K(0)=K_0$ and $H(0)=H_0$, where $M(t)$ is the population size, $H(t)$ is the human capital, $u(t)$ is the share of human capital employed in production, $\beta\in(0,1)$, $\xi\in(0,1)$ and $1-\xi-\beta\in(0,1)$ are the shares accruing to $M(t)$, $u(t)H(t)$ and $K(t)$, respectively, $\eta_K\in[0,1]$ is the constant rate of depreciation of physical capital, $\eta_H\in[0,1]$ is the rate of depreciation of human capital and $A\geq 0$, $B\geq 0$ are the productivities of physical capital and human capital.

We assume that the population size evolves according to the following stochastic differential equation:

$$dM(t)=\mu M(t)dt+\sigma M(t)dW(t)+\delta\int M(t^-)z(\widetilde{N}(dt,dz)-\nu(dt,dz))$$

with initial condition $M(0)=M_0$, where $\mu\in\mathbb{R}$ is the drift rate, $\sigma>0$ is the volatility, $0\leq\delta\leq 1$, W is a standard Brownian motion and $\widetilde{N}$ is Poisson random measure with intensity measure $\nu(dt,dz)$.

Here again, we are able to solve an optimisation program under some simplifying assumptions. This sheds light on the effect of demographic shocks on macroeconomic growth.

SECSI Project-Team

6. New Results

6.1. Dishonest keys (Objective 2)

Participants: Hubert Comon-Lundh, Guillaume Scerri.

One of the main issues in the formal verification of the security protocols is the validity (and scope) of the formal model. Otherwise, it may happen that a protocol is proved and later someone finds an attack. This paradoxical situation may happen when the formal model used in the proof is too abstract.

A main stream of research therefore consists in proving full abstraction results (also called *soundness*): if the protocol is secure in the (symbolic) model, then an attack can only occur with negligible probability in a computational model. Such results have two main drawbacks: first they are very complicated, and have to be completed again and again for each combination of security primitives. Second, they require strong hypotheses on the primitives, some of which are not realistic. For instance, it is assumed that the attacker cannot forge his own keys (or that all keys come with their certificates, even for symmetric encryption keys).

Hubert Comon-Lundh, Véronique Cortier and Guillaume Scerri [31] propose an extension of the symbolic model, and prove it computationally sound, without this restriction on the dishonest keys.

6.2. Unconditional Soundness (Objective 2)

Participant: Hubert Comon-Lundh.

Hubert Comon-Lundh, Véronique Cortier and Guillaume Scerri [31] show how one can drop one of the assumptions of computational soundness results. However, the proofs remain very complicated and there are still assumptions such as the absence of key cycles, or no dynamic corruption... that are still necessary for all these results.

Gergei Bana and Hubert Comon-Lundh investigated a completely different approach to formal security proofs [25], which does not make any such assumptions. The idea can be stated in a nutshell: whereas all existing formal models state the attacker's abilities, they propose to formally state what the attacker *cannot* do.

This makes a big difference, since the soundness need only to be proved formula by formula and only the very necessary assumptions are used for such formulas (for instance, no absence of key cycles is needed). This does not need to be proved again when a primitive is added.

The counterpart of this nice approach is the difficulty of the automation: a tool is required for checking the consistency of a set of axioms, together with the conditions accumulated along a trace. This problem is the subject of research for the next year(s).

6.3. QRB-Domains (Objective 4)

Participant: Jean Goubault-Larrecq [correspondant].

One of the outstanding problems that remains in the denotational semantics of higher-order programming languages with probabilistic choice is the existence of a suitable, convenient category of domains for defining the denotations of types. Technically, a category of so-called continuous domains is sought after, which would be Cartesian-closed and stable by the action of the probabilistic powerdomain functor. This is not known to exist, and is part of the Jung-Tix conjecture. Jean Goubault-Larrecq found out that relaxing continuity to quasi-continuity helped gaining stability by the action of the probabilistic powerdomain functor [20]. This is an extended version of previous work published at the LICS'10 conference.

6.4. Complete WSTS

Participant: Jean Goubault-Larrecq [correspondant].

Well-structured transition systems form a large class of infinite-state transition systems on which one can decide coverability (a slightly relaxed form of reachability). These include Petri nets, lossy channel systems, and various process algebras.

With Alain Finkel, Jean Goubault-Larrecq developed a theory of *complete* well-structured transition systems, allowing one to generalize Karp and Miller's coverability tree construction for Petri nets to all well-structured transition systems. This work culminated in [19], following two conference papers (STACS'09, ICALP'09). The general theory was the topic of the invited talk [34].

6.5. Static Analysis of Programs with Imprecise Probabilities

Participant: Jean Goubault-Larrecq [correspondant].

Static analyses allows one to obtain guarantees about the behavior of programs, without running them. Programs that handle numerical data such as feedback control loops pose a challenge in this area. This gets even harder when one considers programs that read numerical data from sensors, and write to actuators, as these data are imprecise, and are governed by probability distributions that may themselves be unknown, and only know to fall into some interval of distributions. As part of the ANR projet blanc CPP, an efficient static analysis framework that deals with this kind of programs was proposed [16], based on P-boxes and Dempster-Shafer structures to handle imprecise probabilities. This is based on work first presented at the SCAN'11 conference.

6.6. New Attacks on RSA PKCS#1v1.5 (Objective 2)

Participants: Graham Steel [correspondant], Romain Bardou.

RSA PKCS#1v1.5 is the most commonly used standard for public key encryption, used for example in TLS/SSL. It has been known to be vulnerable to a so-called padding-oracle attack since 1998 when Bleichenbacher described the vulnerability at CRYPTO. The attack, known was the "million message attack" was not thought to present a practical threat, due in part to the large number of oracle messages required. In a paper published at CRYPTO 2012 [26] we gave original modifications showing how the attack can be completed in a median of just 15 000 messages. The results led to widespread interest, indicated by over 1400 downloads of the long version of the paper from the HAL webpage and articles in the New York Times, Boston Globe and Süddeutscher Zeitung.

6.7. Deciding trace equivalence (Objectives 1, 3)

Participants: Vincent Cheval, Hubert Comon-Lundh, Stéphanie Delaune, Rémy Chrétien.

Most existing results focus on trace properties like secrecy or authentication. There are however several security properties, which cannot be defined (or cannot be naturally defined) as trace properties and require the notion of indistinguishably. Typical examples are anonymity, privacy related properties or statements closer to security properties used in cryptography.

In the framework of the applied pi-calculus [44], as in similar languages based on equational logics, indistinguishability corresponds to a relation called trace equivalence. Roughly, two processes are trace equivalent when an observer cannot see any difference between the two processes. Static equivalence applies only to observations on finite sets of messages, and does not take into account the dynamic behavior of a process, whereas trace equivalence is more general and takes into account this aspect.

6.7.1. Static equivalence.

As explained above, static equivalence is a cornerstone to provide decision procedures for observational equivalence.

Stéphanie Delaune, in collaboration with Mathieur Baudet and Véronique Cortier, has designed a generic procedure for deducibility and static equivalence that takes as input any convergent rewrite system [15]. They have shown that their algorithm covers most of the existing decision procedures for convergent theories. They also provide an efficient implementation, and compare it briefly with the tools ProVerif and KiSs. This paper is a journal version of the work presented in [47].

In [17], Ştefan Ciobâcă, Stéphanie Delaune and Steve Kremer propose a representation of deducible terms to overcome the limitation of the procedure mentionned above. This new procedure terminates on a wide range of equational theories. In particular, they obtain a new decidability result for the theory of trapdoor bit commitment encountered when studying electronic voting protocols. The algorithm has been implemented in the KiSs tool. This paper is a journal version of the work presented in [49].

In [18], Stéphanie Delaune, in collaboration with Véronique Cortier (LORIA, France), shows that existing decidability results can be easily combined for any disjoint equational theories: if the deducibility and indistinguishability relations are decidable for two disjoint theories, they are also decidable for their union. They also propose a general setting for solving deducibility and indistinguishability for an important class (called *monoidal*) of equational theories involving AC operators. This paper is a journal version of the works presented in [45], [50].

6.7.2. Trace equivalence.

When processes under study do not contain replication, trace equivalence can be reduced to the problem of deciding symbolic equivalence, an equivalence relation introduced by M. Baudet [46].

Stéphanie Delaune, Steve Kremer, and Daniel Pasaila study this symbolic equivalence problem when cryptographic primitives are modeled using a group equational theory, a special case of monoidal equational theories. The results strongly rely on the correspondance between group theories and rings. This allows them to reduce the problem under study to the problem of solving systems of equations over rings. This result was published at IJCAR'12 [33],

When processes under study contain replication, the approach relying on symbolic equivalence does not work anymore. Moreover, since it is well-known that deciding reachability properties is undecidable under various restrictions, there is actually no hope to do better for equivalence-based properties. Rémy Chrétien, Véronique Cortier, and Stéphanie Delaune provide the first results of (un)decidability for certain classes of protocols for the equivalence problem. They consider a class of protocols shown to be decidable for reachability properties, and establish a first undecidability result. Then, they restrained the class of protocols a step further by making the protocols deterministic in some sense and preventing it from disclosing secret keys. This tighter class of protocols was then shown to be decidable after reduction to an equivalence between deterministic pushdown automata (see [42])

To deal with replication, another approach was studied by Vincent Cheval in collaboration with Bruno Blanchet. They propose an extension of the automatic protocol verifier ProVerif. ProVerif can prove observational equivalence between processes that have the same structure but differ by the messages they contain. In order to extend the class of equivalences that ProVerif handles, they extend the language of terms by defining more functions (destructors) by rewrite rules. These extensions have been implemented in ProVerif and allow one to automatically prove anonymity in the private authentication protocol by Abadi and Fournet. This work is currently under submission [40].

6.8. Mobile ad-hoc networks (Objectives 1, 3)

Participants: Rémy Chrétien, Stéphanie Delaune, Graham Steel.

Mobile ad hoc networks consist of mobile wireless devices which autonomously organize their communication infrastructure: each node provides the function of a router and relays packets on paths to other nodes. Finding these paths in an a priori unknown and constantly changing network topology is a crucial functionality of any ad hoc network. Specific protocols, called *routing protocols*, are designed to ensure this functionality known as *route discovery*. Secured versions of routing protocols have been proposed to provide more guarantees on the resulting routes, and some of them have been designed to protect the privacy of the users.

However, existing results and tools do not apply to routing protocols. This is due in particular to the fact that all possible topologies (infinitely many) have to be considered. Véronique Cortier, Jan Degrieck, and Stéphanie Delaune propose a simple reduction result: when looking for attacks on properties such as the validity of the route, it is sufficient to consider topologies with only four nodes, resulting in a number of just five distinct topologies to consider. As an application, several routing protocols, such as the SRP applied to DSR and the SDMSR protocols, have been analysed using the ProVerif tool. This work was published at POST'12 [32].

Rémy Chrétien and Stéphanie Delaune propose a framework for analysing privacy-type properties for routing protocols. They use the notion of equivalence between traces to formalise three security properties related to privacy, namely indistinguishability, unlinkability, and anonymity. They study the relationship between these definitions and we illustrate them using two versions of the ANODR routing protocol. This work is currently under submission [43].

In the context of vehicular ad-hoc networks, to improve road safety, a vehicle-to-vehicle communication platform is currently being developed by consortia of car manufacturers and legislators. In [51], Morten Dahl, Stéphanie Delaune and Graham Steel propose a framework for formal analysis of privacy in location based services such as anonymous electronic toll collection. They give a formal definition of privacy, and apply it to the VPriv scheme for vehicular services. They analyse the resulting model using the ProVerif tool, concluding that the privacy property holds only if certain conditions are met by the implementation. Their analysis includes some novel features such as the formal modelling of privacy for a protocol that relies on interactive zero-knowledge proofs of knowledge and list permutations.

6.9. Composition results (Objective 1)

Participants: Vincent Cheval, Stéphanie Delaune.

Formal methods have proved their usefulness for analysing the security of protocols. However, protocols are often analysed in isolation, and this is well-known to be not sufficient as soon as the protocols share some keys. Nowdays, several composition results exist for trace-based properties, but there is a lack of composition results for equivalence-based properties.

Myrto Arapinis, Vincent Cheval, and Stéphanie Delaune study the notion of trace equivalence and we show how to establish such an equivalence relation in a modular way. They show that composition works even when the processes share secrets provided that they satisfy some reasonable conditions. Their composition result allows one to prove various equivalence-based properties in a modular way, and works in a quite general setting. In particular, they consider arbitrary cryptographic primitives and processes that use non-trivial else branches. As an example, they consider the ICAO e-passport standard, and they show how the privacy guarantees of the whole application can be derived from the privacy guarantees of its sub-protocols. This work was published at CSF'12 [22].

SELECT Project-Team

6. New Results

6.1. Model selection in Regression and Classification

Participants: Gilles Celeux, Mohammed El Anbari, Clément Levrard, Erwan Le Pennec, Lucie Montuelle, Pascal Massart, Caroline Meynet, Jean-Michel Poggi, Adrien Saumard.

Erwan Le Pennec is still working with Serge Cohen (IPANEMA Soleil) on hyperspectral image segmentation based on a spatialized Gaussian Mixture Model. Their scheme is supported by some theoretical investigation [6] and have been applied in pratice with an efficient minimization algorithm combining EM algorithm, dynamic programming and model selection implemented with MIXMOD. Lucie Montuelle is studying extensions of this model that comprise parametric logistic weights and regression mixtures.

In collaboration with Marie-Laure Martin-Magniette (URGV et UMR AgroParisTech/INRA MIA 518) and Cathy Maugis (INSA Toulouse) Gilles Celeux has extended their variable selection procedure for model-based clustering and supervised classification to deal with high dimensional data sets with a backward selection procedure which is more efficient that the previous forward selection procedure in this context. Moreover they have analysed the differences between the model-based approach and geometrical approach to select variable for clustering. Through numerical experiments, they showed the advantage of the model-based appraoch when many variables are highly correlated. These variable selection procedures are in particular used for genomics applications which is the result of a collaboration with researchers of of URGV (Evry Genopole).

Caroline Meynet provided an ℓ_1-oracle inequality satisfied by the Lasso estimator with the Kullback-Leibler loss in the framework of a finite mixture of Gaussian regressions model for high-dimensional heterogeneous data where the number of covariates may be much larger than the sample size. In particular, she has given a condition on the regularization parameter of the Lasso to obtain such an oracle inequality. This oracle inequality extends the ℓ_1-oracle inequality established by Massart and Meynet in the homogeneous Gaussian linear regression case. It is deduced from a finite mixture Gaussian regression model selection theorem for ℓ_1-penalized maximum likelihood conditional density estimation, which is inspired from Vapnik's method of structural risk minimization and from the theory on model selection for maximum likelihood estimators developed by Massart.

From an practical point of view, Caroline Meynet has introduced a procedure to select variables in model-based clustering in a high-dimensional context. In order to tackle with the problem of high-dimension, she has proposed to first use the Lasso in order to select different sets of variables and then estimate the density by a standard EM algorithm by reducing the inference to the linear space of the selected variables by the Lasso. Numerical experiments show that this method can outperform direct estimation by the Lasso.

In collaboration with Jean-Patrick Baudry (Paris 6) and Margarida Cardoso, Ana Ferreira and Maria-José Amorim (Lisbon University], Gilles Celeux has proposed an approach to select, in the model-based clustering context, a model and a number of clusters in order to get a partition which both provides a good fit with the data and is related to the external categorical variables. This approach makes use of the integrated joint likelihood of the data, the partition derived from the mixture model and the known partitions. It is worth noticing that the external categorical variables are only used to select a relevant mixture model. Each mixture model is fitted by the maximum likelihood methodology from the observed data. Numerical experiments illustrate the promising behaviour of the derived criterion [29].

Since September 2008, Pascal Massart is the cosupervisor with Frédéric Chazal (GEOMETRICA) of the thesis of Claire Caillerie (GEOMETRICA). The project intends to explore and to develop new researches at the crossing of information geometry, computational geometry and statistics.

Tim van Erven is studying Model Selection for the Long Term. When a model selection procedure forms an integrated part of a company's day-to-day activities, its performance should be measured not on a single day, but on average over a longer period, like for example a year. Taking this long-term perspective, it is possible to aggregate model predictions optimally even when the data probability distribution is so irregular that no statistical guarantees can be given for any individual day seperately. He studies the relation between model selection for individual days and for the long term, and how the geometry of the models affects both. This work has potential applications in model aggregation for the forecasting of electrical load consumption at EDF.

Adrien saumard has worked on the theoretical validation of the slope heuristics, a practical method of penalties calibration derived in a Gaussian setting by Birgé and Massart in 2006 and extended to bounded M-estimation by Arlot and Massart in 2010. He was able to prove the validity of this heuristics in bounded heteroscedastic regression with random design when the considered models where linear spans made of piecewise polynomials. A preliminary work on a fixed model was necessary and published in [9], while the validation of the slope heuristics itself - as well as the validation of a cross-validation approach - can be found in a preprint.

6.2. Statistical learning methodology and theory

Participants: Gilles Celeux, Christine Keribin, Erwan Le Pennec, Pascal Massart, Lucie Montuelle, Jean-Michel Poggi, Adrien Saumard, Solenne Thivin.

Unsupervised segmentation is an issue similar to unsupervised classification with an added spatial aspect. Functional data is acquired on points in a spatial domain and the goal is to segment the domain in homogeneous domain. The range of applications includes hyperspectral images in conservation sciences, fMRi data and all spatialized functional data. Erwan Le Pennec and Lucie Montuelle are focusing on the questions of the way to handle the spatial component from both the theoretical and the practical point of views. They study in particular the choice of the number of clusters. Furthermore, as functional data require heavy computation, they are required to propose numerically efficient algorithms. They have also extend the model to regression mixture.

Gilles Celeux, Christine Keribin and the Ph D. student Vincent Brault continue their work on the Latent Block Model (LBM). They compared several model selection criteria for binary tables [19]. However, the SEM-VEM Gibbs algorithm used to estimate LBM is subject to spurious solutions (empty clusters). To tackle this drawback, they have proposed to use Bayesian inference through Gibbs Sampling and studied the influence of the calibration of non informative prior distributions. They showed on numerical experiment the advantages of coupling Gibbs sampling with a Variational Bayes algorithm to get pointwise estimators [17]. Furthermore, they extended the previous studies from binary to categorical data [32].

Christine Keribin has proposed to compare, on genomics applications, the use of LBM with other methodologies (variable selection procedure of Maugis and Martin Magniette, component analysis). She supervised an internship (Master 1) on the use of principal component analysis for gene expression data (Inria funding). This has been done on data of the SONATA project (leaded by URGV - Evry Genopole), in collaboration with Marie-Laure Martin-Magniette.

Erwan Le Pennec is supervising Solenne Thivin in her CIFRE with Michel Prenat and Thales Optronique. The aim is target detection on complex background such as clouds or sea. Their approach is a local test approach based on the test decision theory. A key issue is to learn good discrimant features and their probabilistic properties. So far, they have worked on cloud images given by Thales. They focus on a Markovian modeling of the clouds.

Considering the case of maximum likelihood density estimation on histograms, Adrien saumard has investigated both theory and methodology. On the one hand, he has shown that AIC is twice the minimal penalty in the sense of Birgé and Massart, which by consequence implies the asymptotic optimality of the slope heuristics based on a linear shape. On the other hand, he investigated the methodology of the small to moderate sample size setting in this case. The robustness of the slope heurisitics compared to AIC is shown on simulated examples and a new overpenalization of Akaike's criterion is proposed, which outperforms the criterion AICc of

Hurvitch and Tsai and shows comparable results to the procedure proposed by Birgé and Rozenholc in 2006. The benefits of the derived procedure here is its theoretical background and interpretation. This work is still in process and some of the results can be found in a preprint.

6.3. Reliability and Computer Experiments

Participants: Yves Auffray, Gilles Celeux, Rémy Fouchereau, Shuai Fu.

In the computer experiments field, the goal is to approximate an expensive black box function from a limited number of evaluations. The choice of these evaluations i.e. the choice of a design of (computer) experiments is a major issue.

Following the previous work of the past three years, Shuai Fu has concluded her Ph.D thesis under the direction of Gilles Celeux [1]. This year, the work was focused on controlling four main error quantities, in order to validate the methodology in the industrial framework. More precisely, the DAC criterion (Data Agreement Criterion), which has been proposed for assessing the relevance of the design of experiments (DOE) and the prior choice with the observed data was applied to a complex hydrological model, coding and testing the relevant algorithms [30]. For the purpose of controlling the emulator error in an adaptive kriging algorithm, two Bayesian criteria have been proposed for searching and adding new points into the current DOE. The computation time remains important, which makes the method meaningful only in the case where we have a really time-consuming code.

In the framework of a CIFRE convention with Snecma-SAFRAN Rémy Fouchereau has started a thesis on the modeling of fatigue damage for Inco718 supervised by Gilles Celeux. Inco718 is a Zinc-based alloy. To determine its minimum lifetime, a lot of stress tests are made. The alloy lifetimes are reported as function of the stress. The aim is to propose a stochastic models for fatigue lifetime prediction based on a fracture mechanics-based approach. A mixture model with a lognormal component and a sum of two lognormals components is considered. Since the sum of two or more lognormal distribution is not closed form, inference on this model needs Monte Carlo integration within the EM algorithm. Thus, we have provided engineers with a probabilistic tool for reliability design of mechanical parts, but also with a diagnostic tool for material elaboration.

6.4. Statistical analysis of genomic data

Participant: Gilles Celeux.

In collaboration with Florence Jaffrezic and Andrea Rau (INRA, département de génétique animale) Gilles Celeux initiated modelling genomics networks from RNA-seq data. It was the subject of the internship of Mélina Gallpin who is starting a thesis on this subject. To day the performance of overdispersed Poisson models has been investigated. The results are somewhat poor especially for large numbers of genes.

6.5. Curves classification, denoising and forecasting

Participants: Émilie Devijver, Pascal Massart, Jean-Michel Poggi.

In collaboration with Farouk Mhamdi and Meriem Jaidane (ENIT, Tunis, Tunisia), Jean-Michel Poggi proposeda method for trend extraction from seasonal time series through the Empirical Mode Decomposition (EMD). Experimental comparison of trend extraction based on EMD, X11, X12 and Hodrick Prescott filter are conducted. First results show the eligibility of the blind EMD trend extraction method. Tunisian real peak load is also used to illustrate the extraction of the intrinsic trend.

In collaboration with Mina Aminghafari (Amirkabir University, Teheran), Jean-Michel Poggi made uses of wavelets in a statistical forecasting purpose for time series. Recent approaches involve wavelet decompositions in order to handle non stationary time series. They study and extended an approach proposed by Renaud et al., to estimate the prediction equation by direct regression of the process on the Haar non-decimated wavelet coefficients depending on its past values. The new variants are used first for stationary data and after for stationary data contaminated by a deterministic trend.

Jean-Michel Poggi was the supervisor (with A. Antoniadis) of the PhD Thesis of Jairo Cugliari-Duhalde which takes place in a CIFRE convention with EDF. It is strongly related to the use of wavelets together with curves clustering in order to perform accurate load comsumption forecasting. The thesis develops methodological and applied aspects linked to the electrical context as well as theoretical ones by introducing exogeneous variables in the context of nonparametric forecasting time series.

Jean-Michel Poggi, co-supervising with Anestis Antoniadis (Université Joseph Fourier Grenoble) the PhD thesis of Vincent Thouvenot, funded by a CIFRE with EDF. The industrial motivation of this work is the recent development of new technologies for measuring power consumption by EDF to acquire consumption data for different mesh network. The thesis will focus on the development of new statistical methods for predicting power consumption by exploiting the different levels of aggregation of network data collection. From the mathematical point of view, the work is to develop generalized additive models for this type of kind of aggregated data for the modeling of functional data, associating closely nonparametric estimation and variable selection using various penalization methods.

Jean-Michel Poggi and Pascal Massart are the co-advisors of the PhD thesis of Emilie Devijver, strongly motivated by the same kind of industrial forecasting problems in electricity, is dedicated to curves clustering for the prediction. A natural framework to explore this question is mixture of regression models for functional data. The theoretical subject of the thesis is to extend to functional data the recent work by Bühlmann et al. dealing with the simultaneous estimation of mixture regression models in the scalar case using Lasso type methods. Of course, it will be based on the technical tools of the work of Caroline Meynet (which completes his thesis Orsay under the direction of P. Massart), which deals with the clustering of functional data using Lasso methods choosing simultaneously number of clusters and selecting significant wavelet coefficients.

6.6. Neuroimaging, Statistical analysis of fMRI data

Participants: Gilles Celeux, Christine Keribin.

This research takes place as part of a collaboration with Neurospin on brain functional Magnetic Resonance Imaging (fMRI) data. (http://www.math.u-psud.fr/select/reunions/neurospin/Welcome.html). and concerns essentially regularisation in a supervised clustering methodology that includes spatial information in the prediction framework, and yields clustered weighted maps.

TAO Project-Team

6. New Results

6.1. Realistic step sizes for optimization algorithms

Many theoretical results about objective improvement in the process of continuous optimization rely on the assumption that the steps of the algorithm are infinitesimally small, the only situation in which theoretical guarantees of improvement can be given. Y. Akimoto and Y. Ollivier have waived the necessity for such an assumption in a whole class of continuous optimization algorithms, thanks to the use of information geometry [20]. This takes theory closer to the practice of actual optimization algorithms.

6.2. Noisy Optimization Bounds with Constant Noise Variance

Many bounds in noisy evolutionary optimization are based on low variance assumptions (in particular, variance of noise converging to 0 close to the optima). Other bounds in the optimization literature consider difficult objective functions. We prove some new bounds, in the following setting[55]:

- without assuming that the variance is going to zero at the optimum;
- following some debates on the COCO mailing list (see 5.4), assuming that sampling far from the optimum (we had earlier results without this assumption; new results emphasize the contrast).

6.3. Extensions of Upper Confidence Trees

We developed extensions of Upper Confidence Trees to continuous or large domains (states and/or actions) and to domains with high expertise or strong structure[37], [31], [38] (incidentally realizing performances on MineSweeper); we recently submitted a proof of a variant of UCT with consistency proof in the continuous domains (both actions and random variables are allowed to be continuous). Another extension is to the difficult setting with no possibility to "undo" a decision or duplicate a state; see [63]. Yet another extension aims at multi-objective optimization [56].

6.4. Mixing myopic fast algorithms and asymptotically optimal algorithms

We made several works based on combining in sequential decision making:

- a fast algorithm providing quickly good heuristic results;
- an asymptotically optimal, too slow for real size problems.

Results are published in [31] and [38], outperforming the state of the art for MineSweeper in reasonable time; an application to energy has been done, and a new one is under work (see Section 4.1). We believe that this diea of combining fast approximate solutions and slow asymptotically optimal algorithms is a key for improving the state of the art in high dimensional combinatorial planning and that our results on MineSweeper and moderate size energy problem are a solid first step in this direction.

6.5. Adaptive Metropolis with Online Relabeling

In [23] we proposed a novel adaptive MCMC algorithm named AMOR (Adaptive Metropolis with Online Relabeling) for efficiently simulating from permutation-invariant targets occurring in, for example, Bayesian analysis of mixture models. An important feature of the algorithm is to tie the adaptation of the proposal distribution to the choice of a particular restriction of the target to a domain where label switching cannot occur. The algorithm relies on a stochastic approximation procedure for which we design a Lyapunov function that formally defines the criterion used for selecting the relabeling rule. This criterion reveals an interesting connection with the problem of optimal quantifier design in vector quantization which was only implicit in previous works on the label switching problem. In benchmark examples, the algorithm turns out to be fast-converging and efficient at selecting meaningful non-trivial relabeling rules to allow accurate parameter inference. In [24] the algorithm was applied to a synthetic mixture model inspired by the muonic water Cherenkov signal of the surface detectors in the Pierre Auger Experiment.

6.6. Reinforcement learning for frugal cascade learning

In [32] we propose an algorithm that builds sparse decision DAGs (directed acyclic graphs) from a list of base classifiers provided by an external learning method such as AdaBoost. The basic idea is to cast the DAG design task as a Markov decision process. Each instance can decide to use or to skip each base classifier, based on the current state of the classifier being built. The result is a sparse decision DAG where the base classifiers are selected in a data-dependent way. The method has a single hyperparameter with a clear semantics of controlling the accuracy/speed trade-off. The algorithm is competitive with state-of-the-art cascade detectors on three object-detection benchmarks, and it clearly outperforms them when there is a small number of base classifiers. Unlike cascades, it is also readily applicable for multi-class classification. Using the multi-class setup, we show on a benchmark web page ranking data set that we can significantly improve the decision speed without harming the performance of the ranker. Beside outperforming classical cascade designs on benchmark data sets, the algorithm also produces interesting deep structures where similar input data follows the same path in the DAG, and subpaths of increasing length represent features of increasing complexity.

TOCCATA Team

6. New Results

6.1. Proofs of (Imperative) Programs

- A. Charguéraud has extended his ICFP'11 paper [70] into a journal paper, which is currently under review. This paper describes in more details the theory of characteristic formulae and the tool *CFML*, which supports the verification of *OCaml* programs through interactive *Coq* proofs.
- J.-C. Filliâtre has verified a two lines C program (solving the N-queens puzzle) using *Why3*. This case study has been presented at VSTTE 2012 [27].
- With M .Pereira and S. Melo de Sousa (Universidade da Beira Interior, Covilhã, Portugal), J.-C. Filliâtre developed an environment for proving ARM assembly code. It uses *Why3* as an intermediate VC generator. It was presented at the Inforum conference [34] (best student paper).
- F. Bobot and J.-C. Filliâtre have presented the notion of separation predicates introduced in the PhD of F. Bobot (defended December 2011) at ICFEM 2012 [21].
- S. Conchon and A. Mesbout, in collaboration with F. Zaïdi (Fortesse team, LRI) and A. Goel and S. Krstić (Strategic Cad Labs, INTEL), have presented a tool paper about the Cubicle model checker at CAV 2012 [24]. A more detailed description of the main algorithms implemented in Cubicle will be presented during the JFLA 2013 [73].
- A significant effort was dedicated to the development of *Why3*, with 3 public releases [39], [40], [41]. Associated with this activity, we actively participe to the new trend (that emerged in 2010-2011) of construction of international program verification benchmarks and organization of program verification competitions. We participated to the joint paper that reports on the first FoVeOOS competition [23] (http://proval.lri.fr/gallery/cost11comp.en.html). J.-C. Filliâtre and A. Paskevich wrote a detailed report [33] on the 2nd competition VSTTE competition (https://sites.google.com/site/vstte2012/compet) that they organized, published in the proceedings of the COMPARE workshop. This paper describes the competition, presents the five problems that were proposed to the participants, and gives an overview of the solutions sent by the 29 teams that entered the competition.

  Our own gallery of verified programs (http://toccata.lri.fr/gallery/index.en.html) was augmented significantly, with now approximately 100 examples, classified by topics, tools, etc.

6.2. Floating-Point and Numerical Programs

- The PhD thesis of T. Nguyen was defended in June [12]. It includes an improved version of the former approach [102] that we proposed for proving floating-point programs while taking into account architecture- and compiler-dependent features, such as the use of the x87 stack in Intel micro-processors. The underlying tool analyzes the assembly code generated by the compiler. It also includes a preliminary and independent approach for proving floating-point programs involving bit-level operations.
- C. Lelay, under the supervision of S. Boldo and G. Melquiond, has worked on easing proofs of differentiability and integrability in *Coq*. The use case was the existence of a solution to the wave equation thanks to D'Alembert's formula; the goal was to automate the process as much as possible [30]. While a major improvement with respect to *Coq* standard library, this first approach was not user-friendly enough for parametric intervals. So a different approach based on the pervasive use of total functions has been experimented with [22].
- S. Boldo, F. Clément, J.-C. Filliâtre, M. Mayero, G. Melquiond and P. Weis finished the formal proof of a numerical analysis program: the second order centered finite difference scheme for the one-dimensional acoustic wave [14].

- S. Boldo has developed a formal proof of an algorithm for computing the area of a triangle, an improvement of its error bound and new investigations in case of underflow [60].
- S. Boldo, J.-H. Jourdan, X. Leroy, and G. Melquiond have extended CompCert to get the first formally verified compiler that provably preserves the semantics of floating-point programs [63].
- G. Melquiond has kept improving the floating-point and interval theories used to perform proofs by computations in *Coq* [16].

6.3. Automated Deduction

- In collaboration with Assia Mahboubi (from Typical Inria project-team), and Guillaume Melquiond, the group involved in the development of *Alt-Ergo*, implemented and proved the correctness of a novel decision procedure for quantifier-free linear integer arithmetic [20]. This algorithm tries to bridge the gap between projection and branching/cutting methods: it interleaves an exhaustive search for a model with bounds inference. These bounds are computed provided an oracle capable of finding constant positive linear combinations of affine forms. An efficient oracle based on the Simplex procedure has been designed. Our algorithm is proved sound, complete, and terminating and is implemented in the *Alt-Ergo* theorem prover.
- In their LMCS journal paper [15], S. Conchon, É. Contejean and M. Iguernelala present a modular extension of ground AC-completion for deciding formulas in the combination of the theory of equality with user-defined AC symbols, uninterpreted symbols and an arbitrary signature disjoint Shostak theory X. This paper extends the results presented in [72] by showing that a simple preprocessing step allows to get rid of a full AC-compatible reduction ordering, and to simply use a partial multiset extension of a *non necessarily AC-compatible* ordering.
- In [31], S. Conchon, G. Melquiond and C. Roux described a dedicated procedure for a theory of floating-point numbers which allows reasoning on approximation errors. This procedure is based on the approach of the Gappa tool: it performs saturation of consequences of the axioms, in order to refine bounds on expressions. In addition to the original approach, bounds are further refined by a constraint solver for linear arithmetic. This procedure has been implemented in *Alt-Ergo*.
- In [42], [32], C. Dross and J. Kanig from AdaCore, in collaboration with S. Conchon and A. Paskevich propose a generic framework for adding a decision procedure for a theory or a combination of theories to an SMT prover. This mechanism is based on the notion of instantiation patterns, or *triggers*, which restrict instantiation of universal premises and can effectively prevent a combinatorial explosion. A user provides an axiomatization with triggers, along with a proof of completeness and termination in our framework, and obtains in return a sound, complete and terminating solver for his theory. A prototype implementation was realized in the Alt-Ergo prover. As a case study, a feature-rich axiomatization of doubly-linked lists was proved comlpete and terminating.
- In [38], A. Paskevich in collaboration with J. Blanchette from TU München, introduced a new format in the TPTP family (http://tptp.org), called TFF1, which extends the earlier TFF0 format (many-sorted first-order logic) with rank-1 type polymorphism. The technical report presents the syntax, typing rules, and semantics, as well as a sound and complete translation from TFF1 to TFF0. The format is designed to be easy to process by existing reasoning tools that support ML-style polymorphism. It opens the door to useful middleware, such as monomorphizers and other translation tools that encode polymorphism in FOF or TFF0. Ultimately, the hope is that TFF1 will be implemented in popular automatic theorem provers.
- A. Paskevich and J.-C. Filliâtre implemented a new *Coq* tactic that is able call an automated prover from *Coq* environment. It uses *Why3* as an intermediate tool. This new tactic brings a very significant improvement of proof automation within *Coq*. For example, the development of a certified VC generator in *Why3* made an intensive use of this tactic. The combination of automatic and interactive theorem proving was the subject of invited talks given by J.-C. Filliâtre at the workshop "Automation in Proof Assistants" [17] (satellite workshop of ETAPS 2012) and at the international workshop on Intermediate Verification Languages [18] (BOOGIE 2012, Berkeley, California, USA, July 2012).

- Together with O. Hermant (ISEP, Paris), D. Cousineau studied the cut elimination property for deduction modulo theories. They were able to show a strong relationship the syntactic cut-elimination property and the semantic construction of pre-models: they made a full semantic proof that the existence of a pre-model entails the cut elimination property for the considered theory in deduction modulo. This is published at the RTA Conference [26].
- *TLA+* is a specification language based on standard set theory and temporal logic, developed by the TLA groupe of Microsoft Research (http://research.microsoft.com/en-us/um/people/lamport/tla/tla.html). During the first part of his post-doc, D. Cousineau finalized a work on describing how to write *TLA+* proofs and check them with *TLAPS*, the *TLA+* Proof System. It was published as a tool description at FM Conference [25].
- S. Conchon defended his *habilitation à diriger des recherches* in December 2012. The memoir [11] provides a very good and useful survey of the scientific work of the past 10 years, around the SMT solving techniques, that leaded to the tools *Alt-Ergo* and Cubicle as they are nowadays.

6.4. Certification

- P. Herms, together with C. Marché and B. Monate (CEA List), developed a certified VC generator, using Coq. The program for VC calculus and its specifications are both written in Coq, but the code is crafted so that it can be extracted automatically into a stand-alone executable. It is also designed in a way that allows the use of arbitrary first-order theorem provers to discharge the generated obligations [28].
- On top of the previous generic VC generator, P. Herms developed a certified VC generator for C source code annotated using ACSL. This work is the main result of his PhD thesis which will be defended in January 2013.
- A. Tafat and C. Marché started experiments of development of a certified VC generator using Whyt instead of Coq. The challenge was to formalize the operational semantics of an imperative language, and a corresponding weakest precondition calculus, without the possibility to use Coq advanced features such as dependent types nor higher-order functions. The classical issues with local bindings, names and substitutions were solved by identifying appropriate lemmas. It was shown that *Why3* can offer a very significantly higher amount of proof automation compared to Coq [43]. This will be presented at the JFLA conference in February 2013 [95]
- The work that we started in 2011, about the use of the *Why3* environment and its back-end provers as an alternative to the built-in prover of "Atelier B", was published at the ABZ conference [29]. This work continues in the context of the new ANR project BWare.
- With J. Almeida, M. Barbosa, J. Pinto and B. Vieira (University do Minho, Braga, Portugal), J.-C. Filliâtre developed a method for certifying programs involving cryptographic methods. It uses *Why* as an intermediate language. A journal article will appear on *Science of Computer Programming* [13].
- Watermarking techniques are used to help identify copies of publicly released information. They consist in applying a slight and secret modification to the data before its release, in a way that should remain recognizable even in (reasonably) modified copies of the data. Using the *Coq*ALEA library, which formalizes probability theory and probabilistic programs, D. Baelde together with P. Courtieu, D. Gross-Amblard from Rennes and C. Paulin have established new results about the robustness of watermarking schemes against arbitrary attackers. The technique for proving robustness is adapted from methods commonly used for cryptographic protocols and our work illustrates the strengths and particularities of the induced style of reasoning about probabilistic programs. This work has been presented at the conference ITP 2012 [19].
- Supervised by J. Falcou and C. Paulin during his M2 internship, N. Lupinski developed a formalisation of a skeleton language for automated generation of parallel programs. A kernel of the language has been identified, its semantics has been formalised in *Coq* where a construction is interpreted by a

relation between lists of entries and lists of outputs. A transformation scheme from the skeleton language towards JOCaml programs has been proposed and proven correct with respect to the relational semantics. This work is described in [44].

- A. Charguéraud is currently working on the JsCert project (http://jscert.org), which aims at the formalization of the semantics of the JavaScript programming language (as described in *ECMAScript Language Specification, version 5.1*) and the development of a verified JavaScript interpreter. This project is joint work with Philippa Gardner, Sergio Maffeis, Gareth Smith, Daniele Filaretti and Daiva Naudziuniene from Imperial College, and Alan Schmitt and Martin Bodin from Inria Rennes - Bretagne Atlantique. As of today, the formalization already covers a substantial amount of the JavaScript language, and the verified interpreter is able to execute a number of benchmarks taken from standard JavaScript test suites.

  The formalization of the semantics of JavaScript makes use of a novel technique, called *pretty-big-step semantics*, for representing reduction rules in big-step style without suffering from a duplication of several premises accross different rules. This duplication is indeed typical in big-step semantics describing the behavior of exceptions and of divergence. The pretty-big-step semantics is described by A. Charguéraud in a paper to appear at ESOP 2013 [71].

TYPICAL Project-Team

5. New Results

5.1. Feit-Thompson

The Feit-Thompson is an important theorem stating that every finite group of odd order is solvable. It is an important step in the classification of finite groups. Its proof is remarkable through its difficulty and its length (more than 1000 pages of dense mathematical text).

This proof was entirely formalized in Coq. This effort was started six years ago, as a joint project of the project teams Typical, Marelle (Sophia-Antipolis) and the Inria-MSR joint center, under the supervision of Georges Gonthier. The proof was finished in september 2012 and is considered a remarkable achievement. It also gave birth to several side products, such as enhancements of the SSReflect proof language. For Typical, Assia Mahboubi, Enrico Tassi and Cyril Cohen were instrumental in this effort.

5.2. Formal Semantics of Type Theory

Bruno Barras finished an extensive formalization of Coq's type theory in Coq, as well as a large formalization of set theory. This work includes several new results and insights in the study of Type Theory and is the body of Barras' habilitation thesis to be defended early in 2013.

5.3. Study of Type Theories

Bruno Barras finished an extensive formalization of Coq's type theory in Coq, as well as a large formalization of set theory. This work includes several new results and insights in the study of Type Theory and is the body of Barras' habilitation thesis to be defended early in 2013.

Chantal Keller, with Marc Lasson, has presented a notion of parametricity in impredicative type theories, which yields some possible application in proof search [18].

5.4. Formal and computable algebra

Cyril Cohen and Assia Mahboubi have worked on representing various algebraic objects in Coq, in a way that allows computation. In particular, Cohen proposed and developed a representation of algebraic numbers in Coq, as presented in [16]. Assia Mahboubi has collaborated with Frédéric Chyzak (Inria Paris-Roquencourt, Algo team) on the certification of algorithms for D-finite objects.

5.5. Certifiable real optimization

Under the joint supervision of Stéphane Gaubert and Benjamin Werner, with Xavier Allamigeon, Victor Magron is investigating ways to check difficult real inequalities, over bounded domains, in ways which can be re-checked by proof systems like Coq. One such algorithm, combining convex optimization and Max-plus techniques is submitted for publication at ECC 2013.

5.6. Binder representation in Coq

Benjamin Werner has developed a generic tree datatype in Coq, which can encode any language with fixed-arity operators with binders. The application towards smoother formal treatment of such languages is still in progress.

5.7. SMT and Coq

Chantal Keller has enhanced the performances of her SMT-Coq interface based automatic tactic. More precisely, the code has been made more modular which allowed:

- A first interfacing with the renowned Z3 SMT prover from Microsoft Research,
- Extending SMT-Coq to the theory of Coq's native 31 bits integers.

5.8. Automated decision procedures

Assia Mahboubi has woked with members of the Proval team on a new decision procedure for integer arithmetics now intergrated in the Alt-Ergo SMT solver. Assia Mahboubi has worked with Stéphane Lengrand and Mahfuza Farooque on the design of a sequent calculus with focussing and on the conception of a proof search strategy in this calculus which simulates the Davis-Putman-Logemann-Loveland algorithme modulo theory (DPPL(T)) which is implemented by modern SMT-solvers. An implementation developped by Stéphane Lengrand illustrate this approach on standard SMT benchmarks.