RESEARCH CENTER
Saclay - Île-de-France

FIELD

Activity Report 2014

Section New Results

Edition: 2015-03-24

COMETE Project-Team

6. New Results

6.1. Highlights of the Year

- Prix de thèse de l'Ecole Polytechnique 2014 for the thesis "The Epistemic View of Concurrency Theory" by Sophia Knight (Defended 20 September, 2013).
- Catuscia Palamidessi has been invited keynote speaker at the joint conferences CONCUR 2014 and TGC 2014. Rome, September 2014.

6.2. Foundations of information hiding

Information hiding refers to the problem of protecting private information while performing certain tasks or interactions, and trying to avoid that an adversary can infer such information. This is one of the main areas of research in Comète; we are exploring several topics, described below.

6.2.1. Additive and multiplicative notions of leakage, and their capacities

Protecting sensitive information from improper disclosure is a fundamental security goal. It is complicated, and difficult to achieve, often because of unavoidable or even unpredictable operating conditions that can lead to breaches in planned security defences. An attractive approach is to frame the goal as a quantitative problem, and then to design methods that measure system vulnerabilities in terms of the amount of information they leak. A consequence is that the precise operating conditions, and assumptions about prior knowledge, can play a crucial role in assessing the severity of any measured vunerability.

In [20] we developed this theme by concentrating on vulnerability measures that are *robust* in the sense of allowing general leakage bounds to be placed on a program, bounds that apply whatever its operating conditions and whatever the prior knowledge might be. In particular we proposed a theory of channel capacity, generalising the Shannon capacity of information theory, that can apply both to additive and to multiplicative forms of a recently-proposed measure known as g-leakage. Further, we explored the computational aspects of calculating these (new) capacities: one of these scenarios can be solved efficiently by expressing it as a Kantorovich distance, but another turns out to be NP-complete.

We also found capacity bounds for arbitrary correlations with data not directly accessed by the channel, as in the scenario of Dalenius's Desideratum.

6.2.2. Compositionality Results for Quantitative Information Flow

In the min-entropy approach to quantitative information flow, the leakage is defined in terms of a minimization problem, which, in case of large systems, can be computationally rather heavy. The same happens for the recently proposed generalization called g-vulnerability. In [28] we studied the case in which the channel associated to the system can be decomposed into simpler channels, which typically happens when the observables consist of several components. Our main contribution was the derivation of bounds on the g-leakage of the whole system in terms of the g-leakages of its components.

6.2.3. LeakWatch: Estimating Information Leakage from Java Programs

Programs that process secret data may inadvertently reveal information about those secrets in their publicly-observable output. In [23] we presented LeakWatch, a quantitative information leakage analysis tool for the Java programming language; it is based on a flexible "point-to-point" information leakage model, where secret and publicly-observable data may occur at any time during a program's execution. LeakWatch repeatedly executes a Java program containing both secret and publicly-observable data and uses robust statistical techniques to provide estimates, with confidence intervals, for min-entropy leakage (using a new theoretical result presented in this paper) and mutual information. We demonstrated how LeakWatch can be used to estimate the size of information leaks in a range of real-world Java programs.

6.2.4. On the information leakage of differentially-private mechanisms

Differential privacy aims at protecting the privacy of participants in statistical databases. Roughly, a mechanism satisfies differential privacy if the presence or value of a single individual in the database does not significantly change the likelihood of obtaining a certain answer to any statistical query posed by a data analyst. Differentially-private mechanisms are often oblivious: first the query is processed on the database to produce a true answer, and then this answer is adequately randomized before being reported to the data analyst. Ideally, a mechanism should minimize leakage—i.e., obfuscate as much as possible the link between reported answers and individuals' data—while maximizing utility—i.e., report answers as similar as possible to the true ones. These two goals are, however, conflicting, and a trade-off between privacy and utility is imposed.

In [13] we used quantitative information flow principles to analyze leakage and utility in oblivious differentially-private mechanisms. We introduced a technique that exploits graph-symmetries of the adjacency relation on databases to derive bounds on the min-entropy leakage of the mechanism. We evaluated utility using identity gain functions, which are closely related to min-entropy leakage, and we derived bounds for it. Finally, given some graph-symmetries, we provided a mechanism that maximizes utility while preserving the required level of differential privacy.

6.2.5. Metric-based approaches for privacy in concurrent systems

In a series of two papers we investigated metric-based techniques for varifying differential privacy in the context of concurrent systems.

The first work [30] was motivated from the one of Tschantz et al., who proposed a verification method based on proving the existence of a stratified family of bijections between states, that can track the privacy leakage, ensuring that it does not exceed a given leakage budget. We improved this technique by investigating state properties which are more permissive and still imply differential privacy. We introduced a new pseudometric, still based on the existence of a family of bijections, but relaxing the relation between them by integrating the notion of amortization, and showed that this results to a more parsimonious use of the privacy budget. We also showed that for the new pseudometric the level of differential privacy is continuous on the distance between the starting states, which makes it suitable for verification.

Continuing this line of work, we studied the pseudometric based on the Kantorovich lifting, which is one of the most popular notions of distance between probabilistic processes proposed in the literature. However, its application in verification is limited to linear properties. In [19], we proposed a generalization which allows to deal with a wider class of properties, such as those used in security and privacy. More precisely, we proposed a family of pseudometrics, parametrized on a notion of distance which depends on the property we want to verify. Furthermore, we showed that the members of this family still characterize bisimilarity in terms of their kernel, and provided a bound on the corresponding distance between trace distributions. Finally, we studied the instance corresponding to differential privacy, and we showed that it has a dual form, easier to compute. We also proved that the typical process-algebra constructs are non-expansive, thus paving the way to a modular approach to verification.

6.2.6. Optimal Geo-Indistinguishable Mechanisms for Location Privacy

With location-based services becoming increasingly more popular, serious concerns are being raised about the potential privacy breaches that the disclosure of location information may induce. In [21] we considered two approaches that have been proposed to limit and control the privacy loss: one is the *geo-indistinguishability* notion developed within Comète, which is inspired by differential privacy, and like the latter it is independent from the side knowledge of the adversary and robust with respect to composition of attacks. The other one is the mechanism of Shokri et al., which offers an optimal trade-off between the loss of quality of service and the privacy protection with respect to a given Bayesian adversary.

We showed that it is possible to combine the advantages of the two approaches: given a minimum threshold for the degree of geo-indistinguishability, we construct a mechanism that offers the maximal utility, as the solution of a linear program. Thanks to the fact that geo-indistinguishability is insensitive to the remapping of a Bayesian adversary, the mechanism so constructed is optimal also in the sense of Shokri et al. Furthermore

we proposed a method to reduce the number of constraints of the linear program from cubic to quadratic (with respect to the number of locations), maintaining the privacy guarantees without affecting significantly the utility of the generated mechanism. This lowers considerably the time required to solve the linear program, thus enlarging significantly the size of location sets for which the optimal trade-off mechanisms can still be computed.

6.2.7. A Predictive Differentially-Private Mechanism for Mobility Traces

With the increasing popularity of GPS-enabled handheld devices, location based applications and services have access to accurate and real-time location information, raising serious privacy concerns for their millions of users. Trying to address these issues, the notion of *geo-indistinguishability* was recently introduced, adapting the well-known concept of Differential Privacy to the area of location-based systems. A Laplace-based obfuscation mechanism satisfying this privacy notion works well in the case of a *sporadic* use; Under repeated use, however, *independently* applying noise leads to a quick loss of privacy due to the correlation between the location in the trace.

In [22] we showed that correlations in the trace can be in fact exploited in terms of a *prediction function* that tries to guess the new location based on the previously reported locations. The proposed mechanism tests the quality of the predicted location using a private test; in case of success the prediction is reported otherwise the location is sanitized with new noise. If there is considerable correlation in the input trace, the extra cost of the test is small compared to the savings in budget, leading to a more efficient mechanism.

We evaluated the mechanism in the case of a user accessing a location-based service while moving around in a city. Using a simple prediction function and two budget spending strategies, optimizing either the utility or the budget consumption rate, we showed that the predictive mechanism can offer substantial improvements over the independently applied noise.

6.2.8. A differentially private mechanism of optimal utility for a region of priors

Differential privacy is a notion of privacy that was initially designed for statistical databases, and has been recently extended to a more general class of domains. Both differential privacy and its generalized version can be achieved by adding random noise to the reported data. Thus, privacy is obtained at the cost of reducing the data's accuracy, and therefore their *utility*.

In [31] we considered the problem of identifying *optimal* mechanisms for generalized differential privacy, i.e. mechanisms that maximize the utility for a given level of privacy. The utility usually depends on a prior distribution of the data, and naturally it would be desirable to design mechanisms that are *universally optimal*, i.e., optimal for all priors. However it is already known that such mechanisms do not exist in general. We then characterized maximal *classes of priors* for which a mechanism which is optimal for all the priors of the class *does exist*. We showed that such classes can be defined as convex polytopes in the priors space.

As an application, we considered the problem of privacy that arises when using, for instance, location-based services, and we showed how to define mechanisms that maximize the quality of service while preserving the desired level of geo-indistinguishability.

6.2.9. Compositional analysis of information hiding

Systems concerned with information hiding often use randomization to obfuscate the link between the observables and the information to be protected. The degree of protection provided by a system can be expressed in terms of the probability of error associated to the inference of the secret information. In [14] we considered a probabilistic process calculus to specify such systems, and we studied how the operators affect the probability of error. In particular, we characterized constructs that have the property of not decreasing the degree of protection, and that can therefore be considered safe in the modular construction of these systems. As a case study, we applied these techniques to the Dining Cryptographers, and we derived a generalization of Chaum's strong anonymity result.

6.3. Foundations of Concurrency

Distributed systems have changed substantially in the recent past with the advent of phenomena like social networks and cloud computing. In the previous incarnation of distributed computing the emphasis was on consistency, fault tolerance, resource management and related topics; these were all characterized by *interaction between processes*. Research proceeded along two lines: the algorithmic side which dominated the Principles Of Distributed Computing conferences and the more process algebraic approach epitomized by CONCUR where the emphasis was on developing compositional reasoning principles. What marks the new era of distributed systems is an emphasis on managing access to information to a much greater degree than before.

6.3.1. A Concurrent Pattern Calculus

In [16] we detailed how Concurrent pattern calculus (CPC) drives interaction between processes by comparing data structures, just as sequential pattern calculus drives computation. By generalising from pattern matching to pattern unification, interaction becomes symmetrical, with information flowing in both directions. CPC provides a natural language to express trade where information exchange is pivotal to interaction. The unification allows some patterns to be more discriminating than others; hence, the behavioural theory must take this aspect into account, so that bisimulation becomes subject to compatibility of patterns. Many popular process calculi can be encoded in CPC; this allows for a gain in expressiveness, formalised through encodings.

6.3.2. An Intensional Concurrent Faithful Encoding of Turing Machines

The benchmark for computation is typically given as Turing computability; the ability for a computation to be performed by a Turing Machine. Many languages exploit (indirect) encodings of Turing Machines to demonstrate their ability to support arbitrary computation. However, these encodings are usually by simulating the entire Turing Machine within the language, or by encoding a language that does an encoding or simulation itself. This second category is typical for process calculi that show an encoding of lambda-calculus (often with restrictions) that in turn simulates a Turing Machine. Such approaches lead to indirect encodings of Turing Machines that are complex, unclear, and only weakly equivalent after computation. In [25] we developed an approach to encoding Turing Machines into intensional process calculi that is faithful, reduction preserving, and structurally equivalent. The encoding is demonstrated in a simple asymmetric concurrent pattern calculus before generalised to simplify infinite terms, and to show encodings into Concurrent Pattern Calculus and Psi Calculi.

6.3.3. Expressiveness via Intensionality and Concurrency

Computation can be considered by taking into account two dimensions: extensional versus intensional, and sequential versus concurrent. Traditionally sequential extensional computation can be captured by the lambda-calculus. However, recent work shows that there are more expressive intensional calculi such as SF-calculus. Traditionally process calculi capture computation by encoding the lambda-calculus, such as in the pi-calculus. Following this increased expressiveness via intensionality, other recent work has shown that concurrent pattern calculus is more expressive than pi-calculus. In [26] we formalised the relative expressiveness of all four of these calculi by placing them on a square whose edges are irreversible encodings. This square is representative of a more general result: that expressiveness increases with both intensionality and concurrency.

6.3.4. On the Expressiveness of Intensional Communication

The expressiveness of communication primitives has been explored in a common framework based on the pi-calculus by considering four features: synchronism (asynchronous vs synchronous), arity (monadic vs polyadic data), communication medium (shared dataspaces vs channel-based), and pattern-matching (binding to a name vs testing name equality). In [27] pattern-matching is generalised to account for terms with internal structure such as in recent calculi like Spi calculi, Concurrent Pattern Calculus and Psi calculi. This exploreD intensionality upon terms, in particular communication primitives that can match upon both names and structures. By means of possibility/impossibility of encodings, we showed that intensionality alone can encode synchronism, arity, communication-medium, and pattern-matching, yet no combination of these without intensionality can encode any intensional language.

6.3.5. Weak CCP Bisimilarity with Strong Procedures

Concurrent constraint programming (CCP) is a well-established model for concurrency that singles out the fundamental aspects of asynchronous systems whose agents (or processes) evolve by posting and querying (partial) information in a global medium. Bisimilarity is a standard behavioral equivalence in concurrency theory. However, only recently a well-behaved notion of bisimilarity for CCP, and a CCP partition refinement algorithm for deciding the strong version of this equivalence have been proposed. Weak bisimilarity is a central behavioral equivalence in process calculi and it is obtained from the strong case by taking into account only the actions that are observable in the system. Typically, the standard partition refinement can also be used for deciding weak bisimilarity simply by using Milner's reduction from weak to strong bisimilarity; a technique referred to as saturation. In [17] we demonstrated that, because of its involved labeled transitions, the above-mentioned saturation technique does not work for CCP. We gave an alternative reduction from weak CCP bisimilarity to the strong one that allows us to use the CCP partition refinement algorithm for deciding this equivalence.

6.3.6. Efficient Algorithms for Program Equivalence for Confluent Concurrent Constraint Programming

While the foundations and principles of CCP e.g., semantics, proof systems, axiomatizations, have been thoroughly studied for over the last two decades. In contrast, the development of algorithms and automatic verification procedures for CCP have hitherto been far too little considered. To the best of our knowledge there is only one existing verification algorithm for the standard notion of CCP program (observational) equivalence. In [18] we first showed that this verification algorithm has an exponential-time complexity even for programs from a representative sub-language of CCP; the summation-free fragment (CCP+). We then significantly improved on the complexity of this algorithm by providing two alternative polynomial-time decision procedures for CCP+ program equivalence. Each of these two procedures has an advantage over the other. One has a better time complexity. The other can be easily adapted for the full language of CCP to produce significant state space reductions. The relevance of both procedures derives from the importance of CCP+. This fragment, which has been the subject of many theoretical studies, has strong ties to first-order logic and an elegant denotational semantics, and it can be used to model real-world situations. Its most distinctive feature is that of confluence, a property we exploited to obtain our polynomial procedures.

6.3.7. A Behavioral Congruence for Concurrent Constraint Programming with Nondeterministic Choice

Weak bisimilarity is one of the most representative notions of behavioral equivalence for models of concurrency. As we mentioned earlier, a notion of weak bisimilarity, called weak saturated barbed bisimilarity (wsbb), was recently proposed for CCP. This equivalence improves on previous bisimilarity notions for CCP that were too discriminating and it is a congruence for the choice-free fragment of CCP. In [29], however, we showed that wsbb is not a congruence for CCP with nondeterministic choice. We then introduced a new notion of bisimilarity, called weak full bisimilarity (wfb), and showed that it is a congruence for the full language of CCP. We also showed the adequacy of wfb by establishing that it coincides with the congruence induced by closing wsbb under all contexts. The advantage of the new definition is that, unlike the congruence induced by wsbb, it does not require quantifying over infinitely many contexts.

6.3.8. Abstract Interpretation of Temporal Concurrent Constraint Programs

Timed Concurrent Constraint Programming (tcc) is a declarative model for concurrency offering a logic for specifying reactive systems, i.e. systems that continuously interact with the environment. The universal tcc formalism (utcc) is an extension of tcc with the ability to express mobility. Here mobility is understood as communication of private names as typically done for mobile systems and security protocols. In [15] we considered the denotational semantics for tcc, and we extended it to a "collecting" semantics for utcc based on closure operators over sequences of constraints. Relying on this semantics, we formalized a general framework for data flow analyses of tcc and utcc programs by abstract interpretation techniques. The concrete and abstract semantics we proposed are compositional, thus allowing us to reduce the complexity of data flow analyses. We

showed that our method is sound and parametric with respect to the abstract domain. Thus, different analyses can be performed by instantiating the framework. We illustrated how it is possible to reuse abstract domains previously defined for logic programming to perform, for instance, a groundness analysis for tcc programs. We showed the applicability of this analysis in the context of reactive systems. Furthermore, we made use of the abstract semantics to exhibit a secrecy flaw in a security protocol. We also showed how it is possible to make an analysis which may show that tcc programs are suspension free. This can be useful for several purposes, such as for optimizing compilation or for debugging.

6.3.9. Bisimulation for Markov Decision Processes through Families of Functional Expressions

In [24], we transfered a notion of quantitative bisimilarity for labelled Markov processes to Markov decision processes with continuous state spaces. This notion takes the form of a pseudometric on the system states, cast in terms of the equivalence of a family of functional expressions evaluated on those states and interpreted as a real-valued modal logic. Our proof amounted to a slight modification of previous techniques used to prove equivalence with a fixed-point pseudometric on the state-space of a labelled Markov process and making heavy use of the Kantorovich probability metric. Indeed, we again demonstrated equivalence with a fixed-point pseudometric defined on Markov decision processes; what is novel is that we recasted this proof in terms of integral probability metrics defined through the family of functional expressions, shifting emphasis back to properties of such families. The hope is that a judicious choice of family might lead to something more computationally tractable than bisimilarity whilst maintaining its pleasing theoretical guarantees. Moreover, we used a trick from descriptive set theory to extend our results to MDPs with bounded measurable reward functions, dropping a previous continuity constraint on rewards and Markov kernels.

GEOMETRICA Project-Team

6. New Results

6.1. Highlights of the Year

[10] was elected among the notable articles of 2013 by ACM and Computing Reviews (see http://computingreviews.com/recommend/bestof/notableitems_2013.cfm).

6.2. Mesh Generation and Geometry Processing

6.2.1. A Surface Reconstruction Method for In-Detail Underwater 3D Optical Mapping

Participant: Mariette Yvinec.

In collaboration with Pierre Alliez (EPI Titane), Ricard Campos (University of Girona), Raphael Garcia (University of Girona)

Underwater range scanning techniques are starting to gain interest in underwater exploration, providing new tools to represent the seafloor. These scans (often) acquired by underwater robots usually result in an unstructured point cloud, but given the common downward-looking or forward-looking configuration of these sensors with respect to the scene, the problem of recovering a piecewise linear approximation representing the scene is normally solved by approximating these 3D points using a heightmap (2.5D). Nevertheless, this representation is not able to correctly represent complex structures, especially those presenting arbitrary concavities normally exhibited in underwater objects. We present a method devoted to full 3D surface reconstruction that does not assume any specific sensor configuration. The method presented is robust to common defects in raw scanned data such as outliers and noise often present in extreme environments such as underwater, both for sonar and optical surveys. Moreover, the proposed method does not need a manual preprocessing step. It is also generic as it does not need any information other than the points themselves to work. This property leads to its wide application to any kind of range scanning technologies and we demonstrate its versatility by using it on synthetic data, controlled laser-scans, and multibeam sonar surveys. Finally, and given the unbeatable level of detail that optical methods can provide, we analyze the application of this method on optical datasets related to biology, geology and archeology. [23]

6.2.2. A Transfer Principle and Applications to Eigenvalue Estimates for Graphs

Participant: David Cohen-Steiner.

In collaboration with Omid Amini (ENS),

In this paper, we prove a variant of the Burger-Brooks transfer principle which, combined with recent eigenvalue bounds for surfaces, allows to obtain upper bounds on the eigenvalues of graphs as a function of their genus. More precisely, we show the existence of a universal constants C such that the k-th eigenvalue λ_k of the normalized Laplacian of a graph G of (geometric) genus g on n vertices satisfies $\lambda_k \leq C d_{max}(g+k)/n$ where d_max denotes the maximum valence of vertices of the graph. This result is tight up to a change in the value of the constant C. We also use our transfer theorem to relate eigenvalues of the Laplacian on a metric graph to the eigenvalues of its simple graph models, and discuss an application to the mesh partitioning problem. [44]

6.3. Topological and Geometric Inference

6.3.1. Only distances are required to reconstruct submanifolds

Participants: Jean-Daniel Boissonnat, Steve Oudot.

In collaboration with Ramsay Dyer (Johann Bernouilli Institute, University of Groningen, Pays Bas) and Arijit Ghosh (Max-Planck-Institut für Informatik, Saarbrücken, Germany).

In [45], we give the first algorithm that outputs a faithful reconstruction of a submanifold of Euclidean space without maintaining or even constructing complicated data structures such as Voronoi diagrams or Delaunay complexes. Our algorithm uses the witness complex and relies on the stability of *power protection*, a notion introduced in this paper. The complexity of the algorithm depends exponentially on the intrinsic dimension of the manifold, rather than the dimension of ambient space, and linearly on the dimension of the ambient space. Another interesting feature of this work is that no explicit coordinates of the points in the point sample is needed. The algorithm only needs the *distance matrix* as input, i.e., only distance between points in the point sample as input.

6.3.2. Computing Persistent Homology with Various Coefficient Fields in a Single Pass

Participants: Jean-Daniel Boissonnat, Clément Maria.

In [32], we introduce an algorithm to compute the persistent homology of a filtered complex with various coefficient fields in a single matrix reduction. The algorithm is output-sensitive in the total number of distinct persistent homological features in the diagrams for the different coefficient fields. This computation allows us to infer the prime divisors of the torsion coefficients of the integral homology groups of the topological space at any scale, hence furnishing a more informative description of topology than persistence in a single coefficient field. We provide theoretical complexity analysis as well as detailed experimental results.

6.3.3. Recognizing shrinkable complexes is NP-complete

Participants: Olivier Devillers, Marc Glisse.

In collaboration with Dominique Attali (Gipsa-lab, Grenoble), Sylvain Lazard (Inria Nancy - Grand Est)

We say that a simplicial complex is shrinkable if there exists a sequence of admissible edge contractions that reduces the complex to a single vertex. We prove [31] that it is NP-complete to decide whether a (three-dimensional) simplicial complex is shrinkable. Along the way, we describe examples of contractible complexes which are not shrinkable.

6.3.4. Zigzag Zoology: Rips Zigzags for Homology Inference

Participant: Steve Oudot.

In collaboration with Donald Sheehy (University of Connecticut)

For points sampled near a compact set X, the persistence barcode of the Rips filtration built from the sample contains information about the homology of X as long as X satisfies some geometric assumptions. The Rips filtration is prohibitively large, however zigzag persistence can be used to keep the size linear. We present [28] several species of Rips-like zigzags and compare them with respect to the signal-to-noise ratio, a measure of how well the underlying homology is represented in the persistence barcode relative to the noise in the barcode at the relevant scales. Some of these Rips-like zigzags have been available as part of the Dionysus library for several years while others are new. Interestingly, we show that some species of Rips zigzags will exhibit less noise than the (non-zigzag) Rips filtration itself. Thus, the Rips zigzag can offer improvements in both size complexity and signal-to-noise ratio. Along the way, we develop new techniques for manipulating and comparing persistence barcodes from zigzag modules. We give methods for reversing arrows and removing spaces from a zigzag. We also discuss factoring zigzags and a kind of interleaving of two zigzags that allows their barcodes to be compared. These techniques were developed to provide our theoretical analysis of the signal-to-noise ratio of Rips-like zigzags, but they are of independent interest as they apply to zigzag modules generally.

6.3.5. Zigzag Persistence via Reflections and Transpositions

Participants: Clément Maria, Steve Oudot.

We introduce [40] a simple algorithm for computing zigzag persistence, designed in the same spirit as the standard persistence algorithm. Our algorithm reduces a single matrix, maintains an explicit set of chains encoding the persistent homology of the current zigzag, and updates it under simplex insertions and removals. The total worst-case running time matches the usual cubic bound. A noticeable difference with the standard persistence algorithm is that we do not insert or remove new simplices "at the end" of the zigzag, but rather "in the middle". To do so, we use arrow reflections and transpositions, in the same spirit as reflection functors in quiver theory. Our analysis introduces a new kind of reflection called the "weak-diamond", for which we are able to predict the changes in the interval decomposition and associated compatible bases. Arrow transpositions have been studied previously in the context of standard persistent homology, and we extend the study to the context of zigzag persistence. For both types of transformations, we provide simple procedures to update the interval decomposition and associated compatible homology basis.

6.3.6. Topological analysis of scalar fields with outliers

Participants: Mickaël Buchet, Frédéric Chazal, Steve Oudot.

In collaboration with Tamal K. Dey (University of Ohio) Fengtao Fan (University of Ohio) Yusu Wang (University of Ohio)

We extend [57] the notion of the distance to a measure from Euclidean space to probability measures on general metric spaces as a way to do topological data analysis in a way that is robust to noise and outliers. We then give an efficient way to approximate the sub-level sets of this function by a union of metric balls and extend previous results on sparse Rips filtrations to this setting. This robust and efficient approach to topological data analysis is illustrated with several examples from an implementation.

6.3.7. Efficient and Robust Persistent Homology for Measures.

Participants: Mickaël Buchet, Frédéric Chazal, Steve Oudot.

In collaboration with Donald Sheehy (University of Connecticut)

In [34], we extend the notion of the distance to a measure from Euclidean space to probability measures on general metric spaces as a way to do topological data analysis in a way that is robust to noise and outliers. We then give an efficient way to approximate the sub-level sets of this function by a union of metric balls and extend previous results on sparse Rips filtrations to this setting. This robust and efficient approach to topological data analysis is illustrated with several examples from an implementation.

6.3.8. Persistence-based Structural Recognition

Participants: Frédéric Chazal, Maksims Ovsjanikovs.

In collaboration with Chunyuan Li (former intern in Saclay in 2013)

In [39] we present a framework for object recognition using topological persistence. In particular, we show that the so-called persistence diagrams built from functions defined on the objects can serve as compact and informative descriptors for images and shapes. Complementary to the bag-of-features representation, which captures the distribution of values of a given function, persistence diagrams can be used to characterize its structural properties, reflecting spatial information in an invariant way. In practice, the choice of function is simple: each dimension of the feature vector can be viewed as a function. The proposed method is general: it can work on various multimedia data, including 2D shapes, textures and triangle meshes. Extensive experiments on 3D shape retrieval, hand gesture recognition and texture classification demonstrate the performance of the proposed method in comparison with state-of-the-art methods. Additionally, our approach yields higher recognition accuracy when used in conjunction with the bag-offeatures.

6.3.9. Convergence rates for persistence diagram estimation in Topological Data Analysis

Participants: Frédéric Chazal, Marc Glisse, Bertrand Michel.

In collaboration with Catherine Labruère (University of Burgundy)

Computational topology has recently known an important development toward data analysis, giving birth to the field of topological data analysis. Topological persistence, or persistent homology, appears as a fundamental tool in this field. In [36], we study topological persistence in general metric spaces, with a statistical approach. We show that the use of persistent homology can be naturally considered in general statistical frameworks and persistence diagrams can be used as statistics with interesting convergence properties. Some numerical experiments are performed in various contexts to illustrate our results.

6.3.10. Stochastic Convergence of Persistence Landscapes and Silhouettes

Participant: Frédéric Chazal.

In collaboration with Brittany Fasy (Tulane University) Fabrizio Lecci (Carnegie Mellon University) Alessandro Rinaldo (Carnegie Mellon University) Larry Wasserman (Carnegie Mellon University)

Persistent homology is a widely used tool in Topological Data Analysis that encodes multiscale topological information as a multi-set of points in the plane called a persistence diagram. It is difficult to apply statistical theory directly to a random sample of diagrams. Instead, we can summarize the persistent homology with the persistence landscape, introduced by Bubenik, which converts a diagram into a well-behaved real-valued function. In [35], we investigate the statistical properties of landscapes, such as weak convergence of the average landscapes and convergence of the bootstrap. In addition, we introduce an alternate functional summary of persistent homology, which we call the silhouette, and derive an analogous statistical theory.

6.3.11. Subsampling Methods for Persistent Homology

Participants: Frédéric Chazal, Bertrand Michel.

In collaboration with Brittany Fasy (Tulane University) Fabrizio Lecci (Carnegie Mellon University) Alessandro Rinaldo (Carnegie Mellon University) Larry Wasserman (Carnegie Mellon University)

Persistent homology is a multiscale method for analyzing the shape of sets and functions from point cloud data arising from an unknown distribution supported on those sets. When the size of the sample is large, direct computation of the persistent homology is prohibitive due to the combinatorial nature of the existing algorithms. We propose to compute the persistent homology of several subsamples of the data and then combine the resulting estimates. We study the risk of two estimators and we prove that the subsampling approach carries stable topological information while achieving a great reduction in computational complexity.

6.3.12. The observable structure of persistence modules

Participant: Frédéric Chazal.

In collaboration with Vin de Silva (Pomona College) William Crawley-Boevey (University of Leeds)

In persistent topology, q-tame modules appear as a natural and large class of persistence modules indexed over the real line for which a persistence diagram is de- finable. However, unlike persistence modules indexed over a totally ordered finite set or the natural numbers, such diagrams do not provide a complete invariant of q-tame modules. The purpose of [59] is to show that the category of persistence modules can be adjusted to overcome this issue. We introduce the observable category of persis- tence modules: a localization of the usual category, in which the classical properties of q-tame modules still hold but where the persistence diagram is a complete isomorphism invariant and all q-tame modules admit an interval decomposition.

6.4. Data Structures and Robust Geometric Computation

6.4.1. Efficiently Navigating a Random Delaunay Triangulation

Participants: Olivier Devillers, Ross Hemsley.

In collaboration with Nicolas Broutin (EPI RAP*)*

Planar graph navigation is an important problem with significant implications to both point location in geometric data structures and routing in networks. Whilst many algorithms have been proposed, very little theoretical analysis is available for the properties of the paths generated or the computational resources required to generate them. In this work, we propose and analyse a new planar navigation algorithm for the Delaunay triangulation. We then demonstrate a number of strong theoretical guarantees for the algorithm when it is applied to a random set of points in a convex region [33]. In a side result, we give a new polylogarithmic bound on the maximum degree of a random Delaunay triangulation in a smooth convex, that holds with probability one as the number of points goes to infinity. In particular, our new bound holds even for points arbitrarily close to the boundary of the domain. [56]

6.4.2. A chaotic random convex hull

Participants: Olivier Devillers, Marc Glisse, Rémy Thomasse.

The asymptotic behavior of the expected size of the convex hull of uniformly random points in a convex body in $\mathcal{R}^d$ is polynomial for a smooth body and polylogarithmic for a polytope. We construct a body whose expected size of the convex hull oscillates between these two behaviors when the number of points increases [62]

6.4.3. A generator of random convex polygons in a disc

Participants: Olivier Devillers, Rémy Thomasse.

In collaboration with Philippe Duchon (LABRI)

Let $\mathcal{D}$ a disc in $\mathbb{R}^2$ with radius 1 centered at $\mathfrak{o}$, and $(x_1, \cdots, x_n)$ a sample of n points uniformly and independently distributed in $\mathcal{D}$. Let's define the polygon P_n as the convex hull of $(x_1, \cdots, x_n)$, and $f_0(P_n)$ its number of vertices. This kind of polygon has been well studied, and it is known, see [65], that

$$\mathbb{E}f_0(P_n) = c\ n^{\frac{1}{3}} + o(n^{\frac{1}{3}})$$

where $c > 0$ is constant. To generate such a polygon, one can explicitly generate n points uniformly in $\mathcal{D}$ and compute the convex hull. For a very large quantity of points, it could be interesting to generate less points to get the same polygon, for example to have some estimations on asymptotic properties, such as the distribution of the size of the edges.We propose an algorithm that generate far less points at random in order to get P_n, so that the time and the memory needed is reduced for n large. Namely [61], we generate a number of points of the same order of magnitude than the final hull, up to a polylogarithmic factor

6.4.4. On the complexity of the representation of simplicial complexes by trees

Participants: Jean-Daniel Boissonnat, Dorian Mazauric.

In [46], we investigate the problem of the representation of simplicial complexes by trees. We introduce and analyze local and global tree representations. We prove that the global tree representation is more efficient in terms of time complexity for searching a given simplex and we show that the local tree representation is more ecient in terms of size of the structure. The simplicial complexes are modeled by hypergraphs. We then prove that the associated combinatorial optimization problems are very dicult to solve and to approximate even if the set of maximal simplices induces a cubic graph, a planar graph, or a bounded degree hypergraph. However, we prove polynomial time algorithms that compute constant factor approximations and optimal solutions for some classes of instances.

6.4.5. Building Efficient and Compact Data Structures for Simplicial Complexes

Participant: Jean-Daniel Boissonnat.

In collaboration with Karthik C.S (Weizmann Institute of Science, Israël) and Sébastien Tavenas (Max-Planck-Institut für Informatik, Saarbrücken, Germany).

The Simplex Tree is a recently introduced data structure that can represent abstract simplicial complexes of any dimension and allows to efficiently implement a large range of basic operations on simplicial complexes. In this paper, we show how to optimally compress the simplex tree while retaining its functionalities. In addition, we propose two new data structures called Maximal Simplex Tree and Compact Simplex Tree. We analyze the Compressed Simplex Tree, the Maximal Simplex Tree and the Compact Simplex Tree under various settings.

6.4.6. Delaunay triangulations over finite universes

Participant: Jean-Daniel Boissonnat.

In collaboration with Ramsay Dyer (Johann Bernouilli Institute, University of Groningen, Pays Bas) and Arijit Ghosh (Max-Planck-Institut für Informatik, Saarbrücken, Germany).

The witness complex was introduced by Carlsson and de Silva as a weak form of the Delaunay complex that is suitable for finite metric spaces and is computed using only distance comparisons. The witness complex $\mathrm{Wit}(L, W)$ is defined from two sets L and W in some metric space X: a finite set of points L on which the complex is built, and a set W of witnesses that serves as an approximation of X. A fundamental result of de Silva states that $\mathrm{Wit}(L, W) = \mathrm{Del}(L)$ if $W = X = \mathbb{R}^d$. In this paper we give conditions on L that ensure that the witness complex and the Delaunay triangulation coincide when $W \subset \mathbb{R}^d$ is a finite set, and we introduce a new perturbation scheme to compute a perturbed set L' close to L such that $\mathrm{Del}(L') = \mathrm{Wit}(L', W)$. The algorithm constructs $\mathrm{Wit}(L', W)$ in time sublinear in $|W|$.

The only numerical operations used by our algorithms are (squared) distance comparisons (i.e., predicates of degree 2). In particular, we do not use orientation or in-sphere predicates, whose degree depends on the dimension d, and are difficult to implement robustly in higher dimensions. Although the algorithm does not compute any measure of simplex quality, a lower bound on the thickness of the output simplices can be guaranteed. Another novelty in the analysis is the use of the Moser-Tardos constructive proof of the general Lovász local lemma.

GRACE Project-Team

6. New Results

6.1. Highlights of the Year

- F. Morain and A. Guillevic (with their co-authors R. Barbulescu and P. Gaudry) broke the discrete logarithm world record for finite fields of the form $GF(p^2)$ with a prime p of 80 decimal digits. The new techniques form the preprint [31].
- D. Augot and M. Finiasz received the best paper award at FSE 2014 [17]. FSE is the most important conference devoted to symmetric cryptography. Grace contribution is to propose a mathematical construction which enables direct construction of so-called diffusion layers in block ciphers.
- A. Zeh, former Grace PhD student, received the special Prize of the Université Franco-Allemande (UFA) Jury 2014 at the French Ambassy in Berlin, on November 21st.

BEST PAPER AWARD :
[17] **21st International Workshop on Fast Software Encryption, FSE 2014**. D. AUGOT, M. FINIASZ.

6.2. Diffusion layers for block ciphers

MDS matrices allow the construction of optimal linear diffusion layers in block ciphers. However, MDS matrices usually have a large description (for example, they can never be sparse), and this results in costly software/hardware implementations. We can solve this problem using *recursive MDS matrices*, which can be computed as a power of a simple companion matrix—and thus have a compact description suitable for constrained environments. Until now, finding recursive MDS matrices required an exhaustive search on families of companion matrices; this clearly limited the size of MDS matrices that one could look for. We have found a new direct construction, based on shortened BCH codes, which allows us to efficiently construct these matrices for arbitrary parameter sizes [17]. D. Augot and M. Finiasz received the best paper award at FSE 2014, and were invited to submit an extended journal version to *Journal of Cryptology*.

P. Karpman started to study sub-optimal diffusion layers, which can be built using algebraic geometry codes with a large automorphism group. Preliminary work has been done, leading to promising results [18]. To properly assert the cryptanalytic properties of these codes, V. Ducet is starting to implement a method for computing efficiently the weight distribution of AG codes.

6.3. Rank metric codes over infinite fields

Rank metric and Gabidulin codes over the rationals promise interesting applications to space-time coding. We have constructed optimal codes, similar to Gabidulin codes, in the case of infinite fields. We use algebraic extensions, and we have determined the condition on the considered extension to enable this construction. For example: we can design codes with complex coefficients, using number fields and Galois automorphisms. Then, in the rank metric setting, codewords can be seen as matrices. In this setting, a channel introduces errors (a matrix of small rank r added to the codeword) and erasures (s_r rows and s_c columns of the matrix are erased). We have developed an algorithm (adapted from the Welch–Berlekamp algorithm) to recover the right codeword in the presence of an error of rank weight up to $r + s_c + s_r \leqslant d - 1$, where d is the minimal distance of the code. As opposed to the finite field case, we are confronted by coefficient size growth. We solve this problem by computing modulo prime ideals. Using these codes we can completely bypass intermediate constructions using finite fields, which were the stumbling-block in classic constructions.

We also have used this framework to build rank-metric codes over the field of rational functions, using algebraic function fields with cyclic Galois group (Kummer and Artin extensions). These codes can be seen as a generator of infinitely many convolutional codes [25].

6.4. Tensor rank of multiplication over finite fields

Determining the tensor rank of multiplication over finite fields is a problem of great interest in algebraic complexity theory, but it also has practical importance: it allows us to obtain multiplication algorithms with a low bilinear complexity, which are of crucial significance in cryptography. In collaboration with S. Ballet and J. Chaumine [35], J. Pieltant obtained new asymptotic bounds for the symmetric tensor rank of multiplication in finite extensions of finite fields $\mathbb{F}_q$. In the more general (not-necessarily-symmetric) case, J. Pieltant and H. Randriam obtained new uniform upper bounds for multiplication in extensions of $\mathbb{F}_q$. They also gave purely asymptotic bounds substantially improving those coming from uniform bounds, by using a family of Shimura curves defined over $\mathbb{F}_q$. This work will appear in Mathematics of Computation [15].

6.5. Filtration Attacks against McEliece Cryptosystem

The McEliece encryption scheme based on binary Goppa codes was one of the first public-key encryption schemes [39]. Its security rests on the difficulty of decoding an arbitrary code. The original proposal uses classical Goppa codes, and while it still remains unbroken, it requires a huge size of key. On the other hand, many derivative systems based on other families of algebraic codes have been subject to key recovery attacks. Up to now, key recovery attacks were based either on a variant of Sidelnikov and Shestakov's attack [40], where the first step involves the computation of minimum-weight codewords, or on the resolution of a system of polynomial equations using Gröbner bases.

In [10], A. Couvreur, P. Gaborit, V. Gauthier, A. Otmani and J.-P. Tillich introduced a new paradigm of attack called *filtration attacks*. The general principle decomposes in two steps:

1. **Distinguishing** the public code from a random one using the square code operation.
2. **Computing a filtration** of the public code using the distinguisher, and deriving from this filtration an efficient decoding algorithm for the public code.

This new style of attack allowed A. Couvreur, A. Otmani and J.-P. Tillich to break (in polynomial time) McEliece based on wild Goppa codes over quadratic extensions [23]; and A. Couvreur, I. Márquez-Corbella, and R. Pellikaan to break McEliece based on algebraic geometry codes from curves of arbitrary genus [22], [26].

6.6. A new bound on the number of rational points of arbitrary projective varieties

In [38], the authors asked for a general upper bound on the number of rational points of a (possibly reducible) equidimensional variety $X \subseteq \mathbf{P}^n$ of dimension d and degree δ. They conjectured that

$$|X(\mathbf{F}_q)| \leq \delta(\pi_d - \pi_{2d-n}) + \pi_{2d_n}, \tag{1}$$

where for all positive integer ℓ, π_ℓ is defined as the number of rational points of the projective space of dimension ℓ over $\mathbf{F}_q$. That is to say, $\pi_\ell = \frac{q^{\ell+1}-1}{q-1}$.

By combining algebraic geometric methods with a combinatorial method of double counting, A. Couvreur proved this conjecture [32] and got a more general upper bound on the number of rational points of arbitrary varieties (possibly non-equidimensional). In addition, he proved that (1) is sharp by providing examples of varieties reaching this bound.

6.7. New families of fast elliptic curves

B. Smith has pioneered the use of mod-p reductions of Q-curves to produce elliptic curves with efficient scalar multiplication algorithms—which translates into faster encryption, decryption, signing, and signature verification operations on these curves. A theoretical article was presented at ASIACRYPT 2013 [7], and a longer version was submitted (upon invitation) to the Journal of Cryptology. The theory was put into practice in collaboration with Craig Costello (Microsoft Research) and Huseyin Hisil (Yasar University). Their resulting publicly available implementation, which represents the state of the art in constant-time (side-channel conscious) elliptic curve scalar multiplication on 64-bit Intel platforms at the 128-bit security level, can carry out a constant-time scalar multiplication in 145k cycles on Ivy Bridge architectures. This work appeared in EUROCRYPT 2014 [21].

6.8. New results for solving the discrete logarithm problem

Recent results of R. Barbulescu, P. Gaudry, A. Joux, and E. Thomé seem to indicate that solving the discrete logarithm problem over finite fields of small characteristic is easier than was precedently thought. F. Morain and A. Guillevic, joined by R. Barbulescu and P. Gaudry, embarked on an attempt to assess the security of the discrete logarithm problem in a closely related context: that of finite fields with large characteristic and small degree. Improving on the methods of A. Joux, R. Lercier and others, they found new algorithms to select polynomials for the Number Field Sieve – the algorithm of choice in this setting. Moreover, a clever study of the algebraic properties of the fields used (e.g., algebraic units), enabled them to break the world record for the case of $GF(p^2)$, soon to be followed by new cases. This work is described in [31], and part of it is currently submitted.

6.9. Quantum Integer Factorization

Together with two researchers in quantum physics (F. Grosshans and T. Lawson), F. Morain and B. Smith have been working on the number theoretical postprocessing in Shor's algorithm. A preprint is being written.

MEXICO Project-Team

6. New Results

6.1. Highlights of the Year

6.1.1. Active Diagnosis for Probabilistic Systems

Diagnosis fits well with probabilistic systems since it is natural to model the uncertainty about the behaviour of a partially observed system by distributions. We had previously revisited the active diagnosis (which aims at controlling the system to make it diagnosable) in discrete event systems designing optimal decision and synthesis procedures [7]. This year, we have considered active diagnosis for probabilistic discrete event systems, obtaining again optimal procedures [26]. Furthermore we have refined the notion of active diagnosis by introducing the *safe active diagnosis* which ensures that after the control is applied, there is a positive probability that a fault never occurs. Interestingly this problem is undecidable but for finite memory controller we have shown that the problem becomes again decidable and we have designed optimal decision and synthesis procedures. Our approach has raised an issue that has not be observed by previous researchers: while in discrete event system, most variants of diagnosis are in fact equivalent, this is no more the case for probabilistic systems. So in [26], we have undertaken the task of classifying the different versions obtaining a complete landscape of the notions both in terms of relations and complexity. Furthermore we have proposed a new notion of diagnosis, the *prediagnosis* that combines the advantages of diagnosis and prediction.

6.1.2. Weighted automata and weighted logics

Weighted automata are a conservative quantitative extension of finite automata that enjoys applications, e.g., in language processing and speech recognition. Their expressive power, however, appears to be limited, especially when they are applied to more general structures than words, such as graphs. To address this drawback, we have introduced weighted pebble walking automata, which allow to navigate freely in the graph and may use pebbles to mark some positions.

In [20], we have shown with examples from natural language modeling and quantitative model-checking that weighted expressions and automata with pebbles are more expressive and allow much more natural and intuitive specifications than classical ones. We have extended Kleene-Schu ¨tzenberger theorem showing that weighted expressions and automata with pebbles have the same expressive power. We focussed on an efficient translation from expressions to automata. We also proved that the evaluation problem for weighted automata can be done very efficiently if the number of reusable pebbles is low.

In [18], we have studied the expressive power of these automata on words. We have proved that two-way pebble weighted automata, one-way pebble weighted automata, and our weighted logic with transitive closure are expressively equivalent. We also gave new logical characterizations of standard recognizable series.

In [30], we addressed the more general case of graphs such as nested words, trees, pictures, Mazurkiewicz traces, ... We established that weighted pebble walking automata have the same expressive power as weighted first order logic with transitive closure logic, lifting a similar result by Engelfriet and Hoogeboom from the Boolean case to a quantitative setting.

6.1.3. Verification of concurrent recursive programs

Distributed systems form a crucially important but particularly challenging domain. Designing correct distributed systems is demanding, and verifying its correctness is even more so. The main cause of difficulty here is concurrency and interaction (or communication) between various distributed components. Hence it is important to provide a framework that makes easy the design of systems as well as their analysis. There are two schools of thought on reasoning about distributed systems: one following the interleaving based semantics, and one following the visual partial-order/graph based semantics. In [23], we compare these two approaches and argue in favour of the latter. An introductory treatment of the split-width technique is also provided.

In [34], we develop a general technique based on split-width for the verification of networks of multi-threaded recursive programs communicating via reliable FIFO channels. We extend the approach of [6] to this setting. Split-width offers an intuitive visual technique to decompose our behaviour graphs such as MSCs and nested words. The decomposition is mainly a divide-and-conquer technique which naturally results in a tree decomposition. Every behaviour can now be interpreted over its decomposition tree. Properties over the behaviour naturally transfer into properties over the decomposition tree. This allows us to use tree-automata techniques to obtain decision procedures for a range of problems such as reachability, model checking against logical formalisms etc. In this way, we obtain simple, uniform and optimal decision procedures for various verification problems parametrised by split-width. Furthermore, the simple visual mechanism of split-width is as powerful as yardstick graph measures such as tree-width or clique-width. Hence it captures any class of distributed behaviours with a decidable MSO theory.

Multi-threaded recursive programs communicating via channels are turing powerful, hence their verification has focussed on under-approximation techniques. Any error detected in the under-approximation implies an error in the system. However the successful verification of the under-approximation is not as useful if the system exhibits unverified behaviours. In [24], we study controllers that observe/restrict the system so that it stays within the verified under-approximation. We identify some important properties that a good controller should satisfy. We consider an extensive under-approximation class, construct a distributed controller with the desired properties and also establish the decidability of verification problems for this class.

6.1.4. Regulation in Systems Biology

6.1.4.1. Rare events in Signalling Cascades

The visit in 2013 of Professor Monika Heiner from Cottbus University has led to a fruitful collaboration related to statistical model checking of rare events in signalling cascades (a regulatory biological system) [25]. This work has received one of the five top paper awards of the conference. In addition, we have improved the statistical methods used in our tool Cosmos.

6.1.4.2. Characterization of Reachable Attractors Using Petri Net Unfoldings

Attractors of network dynamics represent the long-term behaviours of the modelled system. Their characterization is therefore crucial for understanding the response and differentiation capabilities of a dynamical biological system. In the scope of qualitative models of interaction networks, the computation of attractors reachable from a given state of the network faces combinatorial issues due to the state space explosion.

In [33], we have presented a new algorithm that exploits the concurrency between transitions of parallel acting components in order to reduce the search space. The algorithm relies on Petri net unfoldings that can be used to compute a compact representation of the dynamics. We have illustrated the applicability of the algorithm with Petri net models of cell signalling and regulation networks, boolean and multi-valued. The proposed approach aims at being complementary to existing methods for deriving the attractors of Boolean models, while being generic since it applies to any safe Petri net.

6.2. Diagnosis

6.2.1. Diagnosability under Weak Fairness

In partially observed Petri nets, diagnosis is the task of detecting whether or not the given sequence of observed labels indicates that some unobservable fault has occurred. Diagnosability is an associated property of the Petri net, stating that in any possible execution an occurrence of a fault can eventually be diagnosed. In [35] we consider diagnosability under the weak fairness (WF) assumption, which intuitively states that no transition from a given set can stay enabled forever; it must eventually either fire or be disabled. Following our previous work [71] on how to perform *weak diagnosis* by exploiting the fact that weak fairness reveals faults in parallel with the current observation, sometimes even before their actual accurrence, we turn to the associated *diagnosability* problem in [35]. First, we show that a previous approach to WF-diagnosability in the literature has a major flaw, and present a corrected notion. Moreover, we present an efficient method for verifying WF-diagnosability based on a reduction to LTL-X model checking. An important advantage of this

method is that the LTL-X formula is fixed ? in particular, the WF assumption does not have to be expressed as a part of it (which would make the formula length proportional to the size of the specification), but rather one exploits the ability of existing model checkers to handle weak fairness directly.

6.3. Asynchronous Testing

In the final year of the TECSTES project, we have extended and completed the co-ioco - based conformance and testing theory that we had developed thus far and published in [21], in several directions:

- The testing framework now provides a test generation algorithm [21] for concurrent systems specified with true concurrency models, such as Petri nets or networks of automata. The semantic model of computation of such formalisms are labeled event structures, which allow to represent concurrency explicitly.
- Our test generation algorithm based on Petri net unfolding is able to build a complete test suite w.r.t our co-ioco conformance relation [22]. In addition we propose several coverage criteria that allow to select finite prefixes of an unfolding in order to build manageable test suites.
- We propose an extension of the *ioco* conformance relation, a standard for labeled event structures, named co-ioco, allowing to deal with strong and weak concurrency. We extend the notions of test cases and test execution to labeled event structures, and give a test generation algorithm building a complete test suite for co-ioco. Further, we have introduced and exploited [21] the notions of *strong* and *weak* concurrency: strongly concurrent events must be concurrent in the implementation, while weakly concurrent ones may eventually be ordered, leading to refine *co-ioco* into the *wsc-ioco* relation accounting for weak and strong concurrency.
- The *co-ioco* relation assumes a global control and observation of the system under test, which is not usually realistic in the case of physically distributed systems. Such systems can be partially observed at each of their points of control and observation by the sequences of inputs and outputs exchanged with their environment. Unfortunately, in general, global observation cannot be reconstructed from local ones, so global conformance cannot be decided with local tests. We showed in [39] how appending time stamps to the observable actions of the system under test in order to regain global conformance, via vector clock information, from local testing.
- The MOLE - based testing tool TOURS [42] has been developed with the help of intern Konstantinos Athanasiou, jointly supervised by Hernán Ponce de León and Stefan Schwoon of the MExICo team at LSV), and successful experiments have been conducted with a scalable benchmark example (elevator control). The results show clearly how the true-concurrency approach leads to the test case required being not only smaller individually, but also that *fewer* such test cases are necessary. In addition to the conceptual and analytical enrichment, the results obtained in TECSTES thus also allow to obtain important speedups and reductions in storage space.

Hernán Ponce de León has completed his thesis [40] reporting on the above results, and very successfully defended on Nov. 7, 2014, at ENS Cachan, before the PhD committee consisting of reviewers Rob Hierons and Alex Yakovlev, examiners Thierry Jeron, Remi Morin and Pascal Poizat, and the two supervisors.

6.4. Reachability in MDPs

Markov decision process (MDP) provide the appropriate formalism for the control of fully observable probabilistic systems. There are three kinds of methods for their analysis: linear programming, policy iteration and value iteration. However for large scale systems, only value iteration is still available as it requires less memory than the other methods. For quantitative problems like optimal control for maximizing the discounted reward of an MDP, value iteration is equipped with a stopping criterion that ensures an error bound provided by the user. Value iteration algorithms have also been proposed for the central problem of reachability. However neither stopping criterion nor convergence rate were known for such algorithms. In [37], we have solved these two problems and based on it we have also improved the bound on the number of iterations in order to adapt the value iteration for an exact computation.

6.5. Parameterized Communicating Automata

As a part of our research program on concurrent systems with variable communication topology, we studied system models where the topology is *static* but *unknown*, so that it becomes a parameter of the system. In [28], we introduced parameterized communicating automata (PCAs), where finite-state processes exchange messages via rendez-vous or through bounded FIFO channels. Unlike classical communicating automata, a given PCA can be run on any network topology of bounded degree. We presented various Büchi-Elgot-Trakhtenbrot theorems for PCAs, which roughly read as follows: Let φ be an existential MSO formula and T be any of the following topology classes: pipelines, ranked trees, grids, or rings. There is a PCA that is equivalent to φ on all topologies from T. In the case where each process executes a bounded number of contexts (each context restricting communication in a suitable way), we could show that PCAs are closed under complementation, are expressively equivalent to full MSO logic [29], and have a decidable emptiness problem [31]. The papers [29], [31] are a result of a collaboration with Akshay Kumar (IIT Kanpur) and Jana Schubert (TU Dresden).

6.6. Quantitative behaviours

Several measures have been proposed in literature for quantifying the information leaked by the public outputs of a program with secret inputs. In [32] we studied how to quantify the information leaked by a deterministic or probabilistic program when the measure of information is based on min-entropy or Shannon entropy. A direct computation of these quantities is often infeasible because of the state-explosion problem. In our paper, we model the program as a pushdown system equipped with multi-terminal decision diagrams (ADDs) and propose algorithms to compute said entropies.

The advantage of this approach is that the resulting algorithms can be easily implemented in any BDD-based model-checking tool that checks for reachability in deterministic non-recursive programs by computing program summaries. We demonstrate the validity of our approach by implementing these algorithms in a tool Moped-QLeak.

PARSIFAL Project-Team

6. New Results

6.1. Highlights of the Year

Dale Miller's 1994 LICS paper titled "A Multiple-Conclusion Meta-Logic" [67] was a co-recipient of the LICS Test of Time Award.

6.2. Modular Systems for Classical and Intuitionistic Logic

Participants: Sonia Marin, Lutz Straßburger.

Last year we have shown deductive systems for all intuitionistic modal logics in the modal S5-cube using logical rules in nested sequents [75]. This year we managed to exhibit fully modular systems. That is to say that there is a bijective correspondence between the modal axioms and the inference rules in the deductive system. This is achieved by using a combination of structural and logical rules. This result has been presented at AiML 2014 [24].

6.3. Nested Sequents for Constructive Modal Logics

Participants: Ryuta Arisaka, Anupam Das, Lutz Straßburger.

In the propositional case, "constructive" and "intuitionistic" logic are usually considered the same. However, in the presence of the modalities $\Box$ and $\diamond$ this situation changes because there are several choice of which variants of the k-axiom (which are all equivalent in the classical case) are to be included. Whereas in [75] the intuitionistic variant of the S5-cube has been studied, we studied in this years work [34] the constructive variant of the logics in the S5-cube.

6.4. Intuitionistic Logic in the Calculus of Structures

Participants: Nicolas Guenot, Lutz Straßburger.

The calculus of structures has mainly be used for "classical" logics that come with a De Morgan duality. The reason is that all normalization procedures developed so far for the calculus of structures rely on this De Morgan duality.

In this work, we give two proof systems for implication-only intuitionistic logic in the calculus of structures. The first is a direct adaptation of the standard sequent calculus to the deep inference setting. It comes with a cut elimination procedure that is similar to the one from the sequent calculus, using a non-local rewriting. The second system is the symmetric completion of the first, as normally given in deep inference for logics with a De Morgan duality: all inference rules have duals, as cut is dual to the identity axiom. For this symmetric system we prove a generalization of cut elimination, that we call symmetric normalization, where all rules dual to standard ones are permuted up in the derivation. The result is a decomposition theorem having cut elimination and interpolation as corollaries. This work has been presented at the CSL-LICS 2014 conference [22].

6.5. Free Theorems for Curry

Participant: Lutz Straßburger.

Free theorems [79] are a means of type-based reasoning and are being successfully applied for typed functional programming languages like Haskell, e.g., for program transformation and generally establishing semantic properties [53], [78]. As a simple example, for every polymorphic function $f :: [\alpha] \to [\alpha]$ from lists to lists, arbitrary types τ_1 and τ_2, and a function $g :: \tau_1 \to \tau_2$, we have $f \circ (\mathsf{map}\ g) = (\mathsf{map}\ g) \circ f$, for the standard function $\mathsf{map} :: (\alpha \to \beta) \to [\alpha] \to [\beta]$ which takes a function and a list and applies that function to every entry of the list. It would be of interest to also have such free theorems available for typed functional-logic languages like Curry.

Previous work [48] has investigated free theorems for such a language, Curry [60], phenomenologically and provides intuition for premises of free theorems as well as counterexamples. Proof of the positive claims has been elusive so far, mainly because Curry's type system fails to reflect the key feature: nondeterminism. This avoidance is convenient for programmers, as they do not have to distinguish between deterministic and nondeterministic values. However, it is a hindrance to formal reasoning: the conditions identified in [48] include a notion of determinism, and hence it is a serious weakness of the type system not to capture this.

In a joint work with colleagues at the University of Bonn, published in [25], we have developed an intermediate language, called SaLT, that allowed us to prove a *Parametricity Theorem* which could be used to derive free theorems for Curry.

This work is the result of the PHC Procope collaboration with the University of Bonn (duration 2012-2013).

6.6. A logical basis for quantum evolution and entanglement

Participant: Lutz Straßburger.

In discrete quantum causal dynamics, quantum systems are viewed as discrete structures, namely directed acyclic graphs. In such a graph, events are considered as vertices and edges depict propagation between events. Evolution is described as happening between a special family of space-like slices, which were referred to as locative slices in [41]. Such slices are not so large as to result in acausal influences, but large enough to capture nonlocal correlations. It was an open problem whether such slices can be captured by a deductive system, such that proof search corresponds to quantum evolution. In a joint work with Blute, Guglielmi, Ivanov, and Panangaden, Straßburger has shown that the logic BV with its mix of commutative and noncommutative connectives, is precisely the right logic for such analysis. More precisely, it was shown that the commutative tensor encodes (possible) entanglement, and the noncommutative *seq* encodes causal precedence. With this interpretation, the locative slices are precisely the derivable strings of formulas. Several new technical results about BV are developed as part of this analysis, which is published in [28]

6.7. On the Pigeonhole and Related Principles in Deep Inference and Monotone Systems

Participant: Anupam Das.

The size of proofs of the propositional pigeonhole principle over various systems is a topic of much interest in the proof complexity literature. In particular, it has received notable attention in recent years from the deep inference community, where its classification over the system KS appears as an open problem in numerous publications. In [21] we construct quasipolynomial-size proofs of the propositional pigeonhole principle in the deep inference system KS, addressing this question by matching the best known upper bound for the more general class of monotone proofs.

We make significant use of monotone formulae computing boolean threshold functions, an idea previously considered in works of Atserias et al. The main construction, monotone proofs witnessing the symmetry of such functions, involves an implementation of merge-sort in the design of proofs in order to tame the structural behavior of atoms, and so the complexity of normalization. Proof transformations from previous work on atomic flows are then employed to yield appropriate KS proofs.

As further results we show that our constructions can be applied to provide quasipolynomial-size KS proofs of the parity principle and the generalized pigeonhole principle. These bounds are inherited for the class of monotone proofs, and we are further able to construct $nO(\log\log n)$-size monotone proofs of the weak pigeonhole principle, thereby also improving the best known bounds for monotone proofs.

6.8. A multi-focused proof system isomorphic to expansion proofs

Participants: Kaustuv Chaudhuri, Stefan Hetzl [Vienna University of Technology, Vienna, Austria], Dale Miller.

The sequent calculus is often criticized for requiring proofs to contain large amounts of low-level syntactic details that can obscure the essence of a given proof. Because each inference rule introduces only a single connective, sequent proofs can separate closely related steps—such as instantiating a block of quantifiers—by irrelevant noise. Moreover, the sequential nature of sequent proofs forces proof steps that are syntactically non-interfering and permutable to nevertheless be written in some arbitrary order. The sequent calculus thus lacks a notion of *canonicity*: proofs that should be considered essentially the same may not have a common syntactic form. To fix this problem, many researchers have proposed replacing the sequent calculus with proof structures that are more parallel or geometric. Proof-nets, matings, and atomic flows are examples of such *revolutionary* formalisms. In [13], we propose, instead, an *evolutionary* approach to recover canonicity within the sequent calculus, which we illustrate for classical first-order logic. The essential element of our approach is the use of a *multi-focused* sequent calculus as the means for abstracting away low-level details from classical cut-free sequent proofs. We show that, among the multi-focused proofs, the *maximally multi-focused* proofs that collect together all possible parallel foci are canonical. Moreover, if we start with a certain focused sequent proof system, such proofs are isomorphic to *expansion proofs*—a well known, minimalistic, and parallel generalization of Herbrand disjunctions—for classical first-order logic. This technique appears to be a systematic way to recover the "essence of proof" from within sequent calculus proofs.

6.9. Equality and fixpoints in the calculus of structures

Participants: Kaustuv Chaudhuri, Nicolas Guenot [IT University of Copenhagen, Denmark].

The standard proof theory for logics with equality and fixpoints suffers from limitations of the sequent calculus, where reasoning is separated from computational tasks such as unification or rewriting. We propose in [20] an extension of the calculus of structures, a deep inference formalism, that supports incremental and contextual reasoning with equality and fixpoints in the setting of linear logic. This system allows deductive and computational steps to mix freely in a continuum which integrates smoothly into the usual versatile rules of multiplicative-additive linear logic in deep inference.

6.10. Automatically deriving schematic theorems for dynamic contexts

Participants: Kaustuv Chaudhuri, Olivier Savary-Bélanger [Princeton University, USA].

Hypothetical judgments go hand-in-hand with higher-order abstract syntax for meta-theoretic reasoning. Such judgments have two kinds of assumptions: those that are statically known from the specification, and the *dynamic assumptions* that result from building derivations out of the specification clauses. These dynamic assumptions often have a simple regular structure of repetitions of *blocks* of related assumptions, with each block generally involving one or several variables and their properties, that are added to the context in a single backchaining step. Reflecting on this regular structure can let us derive a number of structural properties about the elements of the context.

In [26], we present an extension of the Abella theorem prover, which is based on a simply typed intuitionistic reasoning logic supporting (co-)inductive definitions and generic quantification. Dynamic contexts are represented in Abella using lists of formulas for the assumptions and quantifier nesting for the variables, together with an inductively defined *context relation* that specifies their structure. We add a new mechanism for defining particular kinds of regular context relations, called *schemas*, and *tacticals* to derive theorems from these schemas as needed. Importantly, our extension leaves the trusted kernel of Abella unchanged. We show that these tacticals can eliminate many commonly encountered kinds of administrative lemmas that would otherwise have to be proven manually, which is a common source of complaints from Abella users.

6.11. A two-level logic approach for reasoning about typed specification languages

Participants: Kaustuv Chaudhuri, Mary Southern [University of Minnesota, USA].

The *two-level logic approach* (*2LLA*) to reasoning about computational specifications, as implemented by the Abella theorem prover, represents derivations of a *specification language* as an inductive definition in a *reasoning logic*. This approach has traditionally been formulated with the specification and reasoning logics having the *same* type system, and only the formulas being translated. However, requiring identical type systems limits the approach in two important ways: (1) every change in the specification language's type system requires a corresponding change in that of the reasoning logic, and (2) the same reasoning logic cannot be used with two specification languages at once if they have incompatible type systems. In [27], we propose a technique based on *adequate* encodings of the types and judgments of a typed specification language in terms of a simply typed higher-order logic program, which is then used for reasoning about the specification language in the usual *2LLA*. Moreover, a single specification logic implementation can be used as a basis for a number of other specification languages just by varying the encoding. We illustrate our technique with an implementation of the LF dependent type theory as a new specification language for Abella, co-existing with its current simply typed higher-order hereditary Harrop specification logic, without modifying the type system of its reasoning logic.

6.12. Undecidability of multiplicative subexponential logic

Participant: Kaustuv Chaudhuri.

Subexponential logic is a variant of linear logic with a family of exponential connectives—called *subexponentials*—that are indexed and arranged in a pre-order. Each subexponential has or lacks associated structural properties of weakening and contraction. In [18], we show that classical propositional multiplicative linear logic extended with one unrestricted and two incomparable linear subexponentials can encode the halting problem for two register Minsky machines, and is hence undecidable.

6.13. Meta-theoretic results on type isomorphisms in the presence of sums

Participant: Danko Ilik.

Type isomorphisms are a pervasive notion of Theoretical Computer Science. In functional programming, two data types being isomorphic means that we can coerce data and programs back-and-forth between two specifications without loss of information. In Constructive Mathematics, two sets are of the same cardinality exactly when they are isomorphic as types. In the proof theory of intuitionistic logic, two formulas are strongly equivalent precisely when they are isomorphic as types.

However, the theory of simple types made from functions, products, and sums, is well understood only when we do not treat functions and sums at the same time. Fiore, Di Cosmo, and Balat [50], presented a "negative" results: the theory of those type isomorphisms is not finitely axiomatizable. To establish the result, they used the work around the Tarski High School Algebra Problem from Mathematical Logic.

We showed that the picture is not so dark by presenting a positive result: the theory is recursively axiomatizable and decidable. The proofs exploit further the deep theory around Tarski's Problem. This work was presented at the Joint Meeting of the Twenty-Third EACSL Annual Conference on Computer Science Logic (CSL) and the Twenty-Ninth Annual ACM/IEEE Symposium on Logic in Computer Science (LICS) in Vienna, Austria [23].

6.14. Towards proof canonicity in presence of disjunction and induction

Participants: Hichem Chihani, Danko Ilik.

The previous work on type isomorphisms showed a way to treat the problem of identity/canonicity of proofs for intuitionistic logic with disjunction, or, equivalently, the problem of the (non-)existence of a canonical eta-long normal form for lambda calculus with if-expressions, which is a long standing open question.

One can see this from the perspective of focusing sequent calculi. The asynchronous phase of proof search is an oriented application of type isomorphisms (by the formulas-as-types correspondence). As we already know that, in the absence of disjunction (sum types), a cut-free focused derivation is eta-long and unique (when the data provided by the synchronous phase is the same), what is necessary in order to handle disjunction is to propagate isomorphisms further than what usual sequent calculus allows. This is related in spirit to deep inference, but more conservative. An implementation of a canonical normalizer and a paper on the topic is under way.

We also intend to use the method to give a proof of focused cut-elimination for the sequent calculi LJF and LKF (at least, for the Sigma-2 fragment) extended with induction. A formal proof in Agda is under development.

6.15. Interpretation of the Sigma-2-classical Axiom of Choice in System T

Participant: Danko Ilik.

Updating previous work, we showed that one can develop a realizability interpretation for the Σ^0_2-fragment of classical Analysis in System T only [36].

This is known to be possible, in principle, by a 1979 result of Schwichtenberg. However, up to day no method that avoids both bar recursion (Spector) and control operators (Krivine) has been known. In fact, we propose to treat control operators as a meta-mathematical technique, rather than to have them in the language of realizers as classical realizability does; we provide a formal proof in Agda that control operators can be completely normalized away from System T while preserving essential equations. [15]

6.16. Axiomatization of constraint systems for first-order reasoning modulo a theory

Participants: Damien Rouhling, Stéphane Graham-Lengrand, Assia Mahboubi, Jean-Marc Notin, Mahfuza Farooque.

This result is part of a work in theorem proving, whose purpose is to provide a theoretical basis for the handling of quantifiers in presence of a theory for which we have specific decision procedures. Inspired by the way first-order unifiers are generated and propagated in automated reasoning techniques such as *tableaux* methods, we sought to generalise these mechanisms to the presence of a theory: We introduced a axiomatic notion of constraint system and a sequent calculus introducing meta-variables and propagating constraints. We then identified the axioms that should be satisfied by the theory's decision procedure, in order for the sequent calculus to be sound and complete. This provides the theoretical basis for the development of Psyche 2.0. This result is submitted for publication.

6.17. Realisability models for cut-elimination in focused systems

Participant: Stéphane Graham-Lengrand.

This result is part of the effort to build meaningful semantics for classical proofs, here based on a polarisation of logical formulae: positive or negative.

Following work by Zeilberger [80], a computational interpretation of cut-elimination in the focused systems LJF and LKF can be given: proofs of positive formulae provide structured data, while proofs of negative formulae consume such data; focusing allows the description of the interaction between the two kinds of proofs as pure pattern-matching.

First, we showed this at a level of abstraction where formulae are no longer made of syntax, yet we also extended the approach so that it could treat quantifiers.

Second, we connected this interpretation to realisability semantics, more precisely orthogonality models, where positive formulae are interpreted as sets of data, and negative formulae are interpreted as their orthogonal sets.

Our construction of orthogonality models for the focused systems LKF and LJF describe the pattern-matching process of cut-elimination in terms of orthogonality. This result has been proved in the Coq proof assistant and forms the second part of [11].

6.18. Refining the FPC framework

Participants: Roberto Blanco, Zakaria Chihani, Quentin Heath, Dale Miller, Fabien Renaud.

We have continued to develop our approach to Foundational Proof Certificates (FPCs). This framework allows defining proof evidence in a general fashion. Proofs in both intuitionistic and classical logics are definable in this framework. We originally have written two different kernels for checking these results but more recently we have found that we can exploit an encoding due to Chaudhuri [43] that enables us to only implement the intuitionistic kernel and then simply encode the classical formulas so that they operator directly on the intuitionistic kernel. This encoding allows for a much more precise and simple means for encoding classical logic into intuitionistic logic than the more familiar double negation translations.

We have also started to develop the second phase of defining proof evidence that was proposed in the ProofCert proposal: the definition of proofs that require fixed points (induction / co-induction). We now have two different kernels being developed on top of the Bedwyr model checker that are checking (and in some cases, proving) theorems involving induction, reachability, and bisimulation.

6.19. Structuring a refinement engine using logic programming

Participants: Dale Miller, Claudio Sacerdoti Coen [University of Bologna], Enrico Tassi [MSR Inria Joint Lab].

The Matita theorem prover is an implementation of the Calculus of Inductive Constructions that is meant to be more accessible (as an implementation) than the Coq system. In an effort to make the Matita kernel more accessible and more flexible, the implementers of that system are experimenting with using a logic programming language similar to λProlog as the control system of the refinement mechanism. In order to use such a logic programming language in this capacity, the notion of flexible goal suspension and *when* declarations are needed. Such a λProlog re-implementation has been written and some experiments in deploying such a system are underway. Formal aspects of λProlog specifications have also been performed using the Abella theorem prover.

POSTALE Team

5. New Results

5.1. Highlights of the Year

CovTrack: Agile multi-target multi-threaded realtime tracker We have developed and highly optimized a multi-target tracking system based on covariance tracking algorithm. The complexity of the algorithm – connected to the number of features – can be tuned to fit the processor computation power (with/without SIMD). Moreover the features can be also selected from a large set of features to adapt the algorithm to the scene and the nature of tracking (indoor/outdoor, pedestrian/car,). Some software and algorithmic transforms have been also applied to accelerate the code for scalar/SIMD processors. [20]

The Light Speed Labeling (LSL) algorithm is still the world fastest connected component labeling (CCL) algorithm. We have proposed a new benchmark that performs fair comparisons for such a data-dependent algorithm (that involves Union-Find algorithm optimization combined with memory and control flow optimization). We show that thanks to its run-based approach and its line-relative labeling, LSL is intrinsically more efficient that all State-of-the-Art pixel-based algorithms, whatever the memory management.[23]

5.2. Excalibur: An Autonomic Cloud Architecture for Executing Parallel Applications

Participants: Alessandro Ferreira Leite, Claude Tadonki, Christine Eisenbeis, Tainá Raiol, Maria Emilia Walter, Alba Cristina de Melo.

IaaS providers often allow the users to specify many requirements for their applications. However, users without advanced technical knowledge usually do not provide a good specification of the cloud environment, leading to low performance and/or high monetary cost. In this context, the users face the challenges of how to scale cloud-unaware applications without re-engineering them. Therefore, in this paper, we propose and evaluate a cloud architecture, namely Excalibur, to execute applications in the cloud. In our architecture, the users provide the applications and the architecture sets up the whole environment and adjusts it at runtime accordingly. We executed a genomics workflow in our architecture, which was deployed in Amazon EC2. The experiments show that the proposed architecture dynamically scales this cloud-unaware application up to 10 instances, reducing the execution time by 73compared to the execution in the configuration specified by the user.[25]

5.3. A Fine-grained Approach for Power Consumption Analysis and Prediction

Participants: Alessandro Ferreira Leite, Claude Tadonki, Christine Eisenbeis, Alba Cristina de Melo.

Power consumption has became a critical concern in modern computing systems for various reasons including financial savings and environmental protection. With battery powered de-vices, we need to care about the available amount of energy since it is limited. For the case of supercomputers, as they imply a large aggregation of heavy CPU activities, we are exposed to a risk of overheating. As the design of current and future hardware is becoming more and more complex, energy prediction or estimation is as elusive as that of time performance. However, having a good prediction of power consumption is still an important request to the computer science community. Indeed, power consumption might become a common performance and cost metric in the near future. A good methodology for energy prediction could have a great impact on power-aware programming, compilation, or runtime monitoring. In this paper, we try to understand from measurements where and how power is consumed at the level of a computing node. We focus on a set of basic programming instructions, more precisely those related to CPU and memory. We propose an analytical prediction model based on the

hypothesis that each basic instruction has an average energy cost that can be estimated on a given architecture through a series of micro-benchmarks. The considered energy cost per operation includes both the overhead of the embedding loop and associated (hardware/software) optimizations. Using these precalculated values, we derive a linear extrapolation model to predict the energy of a given algorithm expressed by means of atomic instructions. We then use three selected appli-cations to check the accuracy of our prediction method by comparing our estimations with the corresponding measurements obtained using a multimeter. We show a 9.48% energy prediction on sorting.[27]

5.4. Automated Code Generation for Lattice Quantum Chromodynamics and beyond

Participants: Denis Barthou, Konstantin Petrov, Olivier Brand-Foissac, Olivier Pène, Gilbert Grosdidier, Michael Kruse, Romain Dolbeau, Christine Eisenbeis, Claude Tadonki.

This is ongoing work on a Domain Specific Language which aims to simplify Monte-Carlo simulations and measurements in the domain of Lattice Quantum Chromodynamics. The tool-chain, called Qiral, is used to produce high-performance OpenMP C code from LaTeX sources. We discuss conceptual issues and details of implementation and optimization. The comparison of the performance of the generated code to the well-established simulation software is also made.[17]

5.5. Switchable Scheduling for Runtime Adaptation of Optimization

Participants: Lénaïc Bagnères, Cédric Bastoul.

Parallel applications used to be executed alone until their termination on partitions of supercomputers: a very static environment for very static applications. The recent shift to multicore architectures for desktop and embedded systems as well as the emergence of cloud computing is raising the problem of the impact of the execution context on performance. The number of criteria to take into account for that pur-pose is significant: architecture, system, workload, dynamic parameters, etc. Finding the best optimization for every context at compile time is clearly out of reach. Dynamic optimization is the natural solution, but it is often costly in execution time and may offset the optimization it is en-abling. In this paper, we present a static-dynamic compiler optimization technique that generates loop-based programs with dynamic auto-tuning capabilities with very low overhead. Our strategy introduces switchable scheduling, a family of program transformations that allows to switch between optimized versions while always processing useful computation. We present both the technique to generate self-adaptive programs based on switchable scheduling and experimental evidence of their ability to sustain high-performance in a dynamic environment.[22]

5.6. Efficient distributed randomized algorithms for solving large dense symmetric indefinite linear systems

Participants: Marc Baboulin, Dulceneia Becker, George Bosilca, Anthony Danalis, Jack Dongarra.

Randomized algorithms are gaining ground in high-performance computing applications as they have the potential to outperform deterministic methods, while still providing accurate results. We propose a randomized solver for distributed multicore architectures to efficiently solve large dense symmetric indefinite linear systems that are encountered, for instance, in parameter estimation problems or electromagnetism simulations. Our contribution is to propose efficient kernels for applying random butterfly transformations (RBT) and a new distributed implementation combined with a runtime (PaRSEC) that automatically adjusts data structures, data mappings, and the scheduling as systems scale up. Both the parallel distributed solver and the supporting runtime environment are innovative. To our knowledge, the randomization approach associated with this solver has never been used in public domain software for symmetric indefinite systems. The underlying runtime framework allows seamless data mapping and task scheduling, mapping its capabilities to the underlying hardware features of heterogeneous distributed architectures. The performance of our software is similar to that obtained for symmetric positive definite systems, but requires only half the execution time and half the amount of data storage of a general dense solver. [15]

5.7. Solvers for 3D incompressible Navier-Stokes equations on hybrid CPU/GPU systems

Participants: Yushan Wang, Marc Baboulin, Karl Rupp, Olivier Le Maître, Yann Fraigneau.

We developed a hybrid multicore/GPU solver for the incompressible Navier-Stokes equations with constant coefficients, discretized by the finite difference method. By applying the prediction-projection method, the Navier-Stokes equations are transformed into a combination of Helmholtz-like and Poisson equations for which we describe efficient solvers. We propose a new implementation that takes advantage of GPU accelerators. We present numerical experiments on a current hybrid machine.

5.8. The Numerical Template toolbox: A Modern C++ Design for Scientific Computing

Participants: Pierre Esterie, Joël Falcou, Mathias Gaunard, Jean-Thierry Lapresté, Lionel Lacassagne.

The design and implementation of high level tools for parallel programming is a major challenge as the complexity of modern architectures increases. Domain Specific Languages (or DSL) have been proposed as a solution to facilitate this design but few of those DSL s actually take full advantage of said parallel architectures. In this paper, we propose a library-based solution by designing a C++ DSL s using generative programming: View the MathML source. By adapting generative programming idioms so that architecture specificities become mere parameters of the code generation process, we demonstrate that our library can deliver high performance while featuring a high level API and being easy to extend over new architectures. [18]

5.9. Boost.SIMD: generic programming for portable simdization

Participants: Pierre Esterie, Joël Falcou, Mathias Gaunard, Jean-Thierry Lapresté, Lionel Lacassagne.

Abstract SIMD extensions have been a feature of choice for processor manufacturers for a couple of decades. Designed to exploit data parallelism in applications at the instruction level, these extensions still require a high level of expertise or the use of potentially fragile compiler support or vendor-specific libraries. While a large fraction of their theoretical accelerations can be obtained using such tools, exploiting such hardware becomes tedious as soon as application portability across hardware is required. In this paper, we describe Boost.SIMD, a C++ template library that simplifies the exploitation of SIMD hardware within a standard C++programming model. Boost.SIMD provides a portable way to vectorize computation on Altivec, SSE or AVX while providing a generic way to extend the set of supported functions and hardwares. We introduce a C++standard compliant interface for the users which increases expressiveness by providing a high-level abstraction to handle SIMD operations, an extension-specific optimization pass and a set of SIMD aware standard compliant algorithms which allow to reuse classical C++ abstractions for SIMD computation. We assess Boost.SIMD performance and applicability by providing an implementation of BLAS and image processing algorithms.

5.10. Automatic Task-based Code Generation for High Performance Domain Specific Embedded Language

Participants: Antoine Tran Tan, Joël Falcou, Daniel Etiemble, Harmut Kaiser.

Providing high level tools for parallel programming while sustaining a high level of performance has been a challenge that techniques like Domain Specific Embedded Languages try to solve. In previous works, we investigated the design of such a DSEL-NT2- providing a Matlab-like syntax for parallel numerical computations inside a C++ library. In this paper, we show how NT2 has been redesigned for shared memory systems in an extensible and portable way.[28]

5.11. High Level Transforms for SIMD and low-level computer vision algorithms

Participants: Lionel Lacassagne, Daniel Etiemble, Alain Dominguez, Pascal Vezolle.

This paper presents a review of algorithmic transforms called High Level Transforms for IBM, Intel and ARM SIMD multi-core processors to accelerate the implementation of low level image processing algorithms. We show that these optimizations provide a significant acceleration. A first evaluation of 512-bit SIMD XeonPhi is also presented. We focus on the point that the combination of optimizations leading to the best execution time cannot be predicted, and thus, systematic benchmarking is mandatory. Once the best configuration is found for each architecture, a comparison of these performances is presented. The Harris points detection operator is selected as being *representative* of low level image processing and computer vision algorithms. Being composed of five convolutions, it is more complex than a simple filter and enables more opportunities to combine optimizations. The presented work can scale across a wide range of codes using 2D stencils and convolutions. Such High Level Transforms provide a speedup of x89 on a 2×4 core Intel Xeon processor versus a code that is already SIMDized and OPenMPized.[26]

5.12. What Is the World's Fastest Connected Component Labeling Algorithm?

Participants: Laurent Cabaret, Lionel Lacassagne.

Optimizing connected component labeling is cur- rently a very active research field. Some teams claim to have design the fastest algorithm ever designed. This paper presents a review of these algorithms and a enhanced benchmark that improve classical random images benchmark with a varying granularity set of random images in order to become closer to natural image behavior. Our algorithm, the Light Speed Labeling is from $\times 3.5$ up to $\times 5.3$ faster than the best State-of-the-Art competitor.[23]

5.13. Covariance tracking: architecture optimizations for embedded systems

Participants: Andrés Romero, Lionel Lacassagne, Michèle Gouiffès, Ali Hassan Zahraee.

Covariance matching techniques have recently grown in interest due to their good performances for object retrieval, detection, and tracking. By mixing color and texture information in a compact representation, it can be applied to various kinds of objects (textured or not, rigid or not). Unfortunately, the original version requires heavy computations and is difficult to execute in real time on embedded systems. This article presents a review on different versions of the algorithm and its various applications; our aim is to describe the most crucial challenges and particularities that appeared when implementing and optimizing the covariance matching algorithm on a variety of desktop processors and on low-power processors suitable for embedded systems. An application of texture classification is used to compare different versions of the region descriptor. Then a comprehensive study is made to reach a higher level of performance on multi-core CPU architectures by comparing different ways to structure the information, using single instruction, multiple data (SIMD) instructions and advanced loop transformations. The execution time is reduced significantly on two dual-core CPU architectures for embedded computing: ARM Cortex-A9 and Cortex-A15 and Intel Penryn-M U9300 and Haswell-M 4650U. According to our experiments on covariance tracking, it is possible to reach a speedup greater than 2 on both ARM and Intel architectures, when compared to the original algorithm, leading to real-time execution. [20]

SPECFUN Project-Team

6. New Results

6.1. Highlights of the Year

Two results are particularly important this year, our computer-checked proof [11] of irrationality of $\zeta(3)$ and our new algorithm [19] for the integration of multiple integrals. The former is our first success in the merger between computer algebra and formal methods, and stimulates further research in this direction around special functions and creative telescoping. The latter has made a large class of integrals possible in practice, thus allowing us to compute a challenging list of integrals related to famous Calabi–Yau various; it has also received attention by physicists.

6.2. A formal proof of the irrationality of $\zeta(3)$

We have obtained a formal proof, machine-checked by the Coq proof assistant, of the irrationality of the constant $\zeta(3)$, that is, the evaluation at 3 of the Riemann zeta function of number theory. The result has been known in mathematics since the French mathematician Apéry's work in 1978, and several alternative proofs have been given since then. Our formalized result is the first complete proof by the computer (under the single assumption of the asymptotic behavior of the least common multiple of the first n natural numbers). The core of this formal proof is based on (untrusted) computer-algebra calculations performed outside the proof assistant with the Mgfun Maple library developed by members of the team in the past. Then, we verify formally and a posteriori the desired properties of the objects computed by Maple and complete the proof of irrationality. This work [11] was formally presented at the conference on interactive theorem proving, ITP'14, and also as talks at MSC 2014 (Mathematical Structures of Computation) [0], at the meeting MAP 2014 of the community on mathematics, algorithms and proofs [0], and at JNCF'14, the meeting of the French computer-algebra community [0].

6.3. Criterion for the existence of telescopers for mixed hypergeometric terms

Creative telescoping is a process that determines a univariate recurrence satisfied by the sum of a summand described by a system of bivariate recurrences. For hypergeometric summands, that is, summands given by first-order linear recurrences, this has led to Zeilberger's algorithm in the early 1990s, since then followed by a large number of works, including a natural counterpart for integration. The history of creative-telescoping algorithms was surveyed this year in Chyzak's HDR [1]. Also this year, we presented in [6] a criterion for the existence of telescopers for mixed hypergeometric terms, which is based on additive and multiplicative decompositions. The criterion had enabled us to determine the termination of Zeilberger's algorithms for mixed hypergeometric inputs prior to any costly computations, and to verify that certain indefinite sums do not satisfy any polynomial differential equation.

6.4. Integration of rational functions

Periods of rational integrals are specific integrals, with respect to one or several variables, whose integrand is a rational function and whose domain of integration is closed. Periods with a parameter are classically known to satisfy linear differential equations of a type called Picard-Fuchs equations. As for other special-function manipulations, handling periods through those differential equations is a good way to actually compute them, and this was the topic of Pierre Lairez' PhD, defended this year [2].

[0] http://smc2014.univ-lyon1.fr/
[0] http://perso.crans.org/cohen/map2014/
[0] http://www.lifl.fr/jncf2014/

Computing multivariate integrals is one speciality of the team and our algorithms are known to treat much more general integrals than just periods of rational integrals. However, integration is still slow in practice when the number of variables goes increasing. By looking at periods of rational function, the hope is to obtain relevant complexity bounds and faster algorithms.

The goal of reaching relevant theoretical complexity bounds has been reached last year [35] but a practically fast algorithm was still missing. This year, we described a new algorithm which is efficient in practice [19], though its complexity is not known. This algorithm allows to compute quickly integrals that are too big to be computed with previous algorithms. As a challenging benchmark, we computed 210 integrals given by Batyrev and Kreuzer in their work on Calabi–Yau varieties. This achievement gave strong visibility to the paper and allowed a quick dissemination of the implementation, which is provided in Magma under a CeCILL B license. The algorithm is now used on a regular basis by several teams. We know of:

- Tom Coates' team (Dpt. of Mathematics, Imperial College, London, UK), which uses the software in their work about mirror symmetry and classification of Fano varieties;
- Duco van Straten (Institute of Mathematics, University of Mainz, Germany), who uses the software in his work in algebraic geometry;
- Gert Alkmvist (Dpt. of Mathematics, University of Lund, Sweden), who uses the software in his work of enumerating the Calabi–Yau differential equations.

6.5. Efficient algorithms for linear differential equations in positive characteristic

The p-curvature of a linear differential operator in characteristic p is a matrix that measures to what extent the space of polynomial solutions of the operator has dimension close to its order. This makes the p-curvature a useful tool in concrete applications, like in combinatorics and statistical physics, where it serves for instance as an a posteriori certification filter for differential operators obtained by guessing techniques. In [9], we designed a new algorithm for computing the characteristic polynomial of the p-curvature in sublinear time $\tilde{\mathrm{O}}(p^{0.5})$. Prior to this work, the fastest algorithms for this task, and even for the subtask of deciding nilpotency of the p-curvature, had had merely slightly subquadratic complexity $\tilde{\mathrm{O}}(p^{1.79})$. The new algorithm is also efficient in practice: it allows to test the nilpotency of the p-curvature for primes p of order 10^6, for which the p-curvature itself is impossible to compute using current algorithms.

6.6. Efficient algorithms for rational first integrals

We presented in [4] fast algorithms for computing rational first integrals with degree bounded by N of a planar polynomial vector field of degree $d \leq N$. The main novelty is that such rational first integrals are obtained by computing via systems of linear equations instead of systems of quadratic equations. This leads to a probabilistic algorithm with arithmetic complexity $\tilde{\mathrm{O}}(N^{2\omega})$ and to a deterministic algorithm for solving the problem in $\tilde{\mathrm{O}}(d^2 N^{2\omega+1})$ arithmetic operations, where ω is the exponent of linear algebra. By comparison, the best previous algorithm uses at least $d^{\omega+1} N^{4\omega+4}$ arithmetic operations. Our new algorithms are moreover very efficient in practice.

6.7. Computation of necessary integrability conditions for parametrized Hamiltonian systems

Let $V(\mathbf{q}_1, \mathbf{q}_2)$ be a homogeneous function whose coefficients depend rationally on parameters $\mathbf{a}_1, ..., \mathbf{a}_n$. In [10] we designed an algorithm to compute polynomial necessary conditions on the parameters $(\mathbf{a}_1, ..., \mathbf{a}_n)$ such that the dynamical system associated to the potential V is integrable. These conditions originate from those of the classical Morales-Ramis-Simó integrability criterion. The implementation of the algorithm allows to treat applications that were out of reach before, for instance concerning the non-integrability of polynomial potentials up to degree 9. Another striking application is the first complete proof of the non-integrability of the collinear three-body problem.

6.8. Non-D-finite excursions in the quarter plane

Counting lattice paths obeying various geometric constraints is a classical topic in combinatorics and probability theory. Many recent works deal with the enumeration of 2-dimensional walks with prescribed steps confined to the positive quadrant. A large part of the effort has been devoted to the classification of classes of walks according to the nature of equations that they satisfy (linear, polynomial, differential, etc). Equivalently, this provides properties of the classes of walks according to the algebraic nature of their enumerative series: whether rational, algebraic, D-finite, etc. The classification is now complete for walks with unit steps: the trivariate generating function of the numbers of walks with given length and prescribed ending point is D-finite if and only if a certain group associated with the step set is finite. We proved in [5] a refinement of this result: we showed that the sequence of numbers of excursions (finite paths starting and ending at the origin) in the quarter plane corresponding to a nonsingular step set with infinite group does not satisfy any nontrivial linear recurrence with polynomial coefficients. This solves an open problem in the field of lattice-path combinatorics.

6.9. A human proof of the Gessel conjecture

Gessel walks are planar walks confined to the positive quarter plane, that move by unit steps in any of the following directions: West, North-East, East and South-West. In 2001, Ira Gessel conjectured a closed-form expression for the number of Gessel walks of a given length starting and ending at the origin. In 2008, Kauers, Koutschan and Zeilberger gave a computer-aided proof of this conjecture. The same year, Bostan and Kauers showed, using again computer algebra tools, that the trivariate generating function of Gessel walks is algebraic. We propose in [17] the first "human proofs" of these results. They are derived from a new expression for the generating function of Gessel walks.

6.10. Enumeration of 3-dimensional lattice walks confined to the positive octant

We explored in [3] the classification problem for 3-dimensional walks with unit steps confined to the positive octant. The first difficulty is their number: there are 11 074 225 cases (instead of 79 in dimension 2). In our work, we focused on the 35 548 that have at most six steps. We applied to them a combined approach, first experimental and then rigorous. Among the 35 548 cases, we first found 170 cases with a finite group; in the remaining cases, our experiments suggest that the group is infinite. We then rigorously proved D-finiteness of the generating series in all the 170 cases, with the exception of 19 intriguing step sets for which the nature of the generating function still remains unclear. In two challenging cases, no human proof is currently known, and we derived computer-algebra proofs, thus constituting the first proofs for those two step sets.

6.11. Asymptotic expansions for linear homogeneous divide-and-conquer recurrences: Algebraic and analytic approaches collated

Linear divide-and-conquer recurrences are a classical topic in computer science, but they are often dealt with in an offhand way. Particularly the subtle oscillations they show are usually not emphasized. After having elaborated last year a new approach to the asymptotic study of such recurrences, we provide in [7] a comparison with an older approach based on number theoretic tools as Dirichlet series and residue computation. The most striking aspect of the linear approach is the simplicity and the ease of use. Reduction to normal Jordan form, computation of a joint spectral radius, dealing with a dilatation equation are all workable with a computer-algebra system. Moreover these concepts are better known by computer scientists than those of complex analysis and analytic number theory. So there is hope that this approach will more easily gain acceptance among computer scientists.

6.12. Asynchronous interaction with Coq

We have integrated the Coq proof assistant with the PIDE architecture [13], [12] ("prover integrated development environment"). The architecture is aimed at asynchronous, parallel interaction with proof assistants, originally aimed at the Isabelle proof assistant, and is tied in heavily with a plugin that allows the jEdit editor to work with proof assistants. We have made several generalizations to the PIDE architecture to accommodate for more provers than just Isabelle, and adapted Coq to understand the core protocol: this delivered a working system in about two man-months; further work improved the connection and added novel functionalities to the interface. The tool has also been presented informally at seminars at the University of Dundee and the Université Paris 13.

TOCCATA Project-Team

6. New Results

6.1. Highlights of the Year

- The ACM Software System Award 2013 was given, during a ceremony in June 2014 in San Francisco, to the Coq proof assistant (http://awards.acm.org/software_system/). The prestigious ACM price was previously awarded to the LLVM compiler infrastructure (2012) and to the Eclipse IDE (2011). Among the 9 recipients of the 2013 award are Christine Paulin and Jean-Christophe Filliâtre, from the Toccata team.
- The *Concours Castor informatique* (http://castor-informatique.fr/) had an even larger success than in the previous years. In November 2014, more than 228,000 teenagers from over 1500 schools participated and solved the interactive tasks of the contest. Arthur Charguéraud and Sylvie Boldo, from the Toccata team, significantly contributed to the prepation of the tasks and to the organization of the contest.

6.2. Deductive Verification

- J.-C. Filliâtre, L. Gondelman, and A. Paskevich have formalized the notion of ghost code implemented in *Why3*, in a paper *The Spirit of Ghost Code* [35] presented at CAV 2014. This is an outcome of L. Gondelman's M2 internship (spring/summer 2013).
- M. Clochard published at the POPL conference a paper presenting a work done during an internship at Rice University (Houston, TX, USA) with S. Chaudhuri and A. Solar-Lezama [29]. It is a new technique for parameter synthesis under boolean and quantitative objectives. The input to the technique is a "sketch"—a program with missing numerical parameters—and a probabilistic assumption about the program's inputs. The goal is to automatically synthesize values for the parameters such that the resulting program satisfies: (1) a boolean specification, which states that the program must meet certain assertions, and (2) a quantitative specification, which assigns a real valued rating to every program and which the synthesizer is expected to optimize.
- J.-C. Filliâtre, C. Marché, and A. Paskevich, together with F. Bobot (CEA LIST), took part in the VerifyThis program verification competition, held at the 18th FM symposium in August 2012. They used *Why3* to solve three challenges (which can be found at http://fm2012.verifythis.org/challenges/), and their solutions have been published in a special issue of the journal *Software Tools for Technology Transfer* [16].
- M. Clochard developed, using *Why3*, verified implementations of several data structures, including random-access lists and ordered maps. These are derived from a common parametric implementation of self-balancing binary trees in the style of Adelson-Velskii and Landis trees (so-called AVLs). This work appeared at the VSTTE conference [30]. Its originality relies on the genericity of the specifications and the code, and the very high level of proof automation. Such a case study is aimed at illustrating the capabilities of *Why3* for designing certified libraries. Development is available from our gallery at http://toccata.lri.fr/gallery/avl.fr.html.
- S. Conchon and A. Mebsout have extended the core algorithm of the Cubicle model checker with a mechanism for inferring invariants. This new algorithm, called BRAB, is able to automatically infer invariants strong enough to prove industrial cache coherence protocols. BRAB computes over-approximations of backward reachable states that are checked to be unreachable in a finite instance of the system. These approximations (candidate invariants) are then model-checked together with the original safety properties. Completeness of the approach is ensured by a mechanism for backtracking on spurious traces introduced by too coarse approximations. Details can be found in A. Mebsout's PhD thesis [15].

- A. Charguéraud extended his tool CFML to support, in addition to the verification of the full functional correctness of a piece of code, the verification of the asymptotic complexity of the code. Even though it had been previously established that, in theory, amortized analysis can be explained as the manipulation of *time credits*, and that time credits can be encoded as resources in Separation Logic, CFML is the first practical tool to support the formal verification of amortized analyses for arbitrarily-complex pieces of code. The *time-credit* extension to CFML was put to practice to verify dynamic arrays (Julien Grangier's internship), and to verify a *chunked sequence* data structure [26], particularly challenging due to its use of Tarjan's data structural bootstrapping technique. The latter piece of work was presented in July at the workshop *Semantics of proofs and certified mathematics*, which took place at the Institut Henri Pointcaré. A paper describing the time-credit extension to CFML is under preparation.

6.3. Floating-Point and Numerical Programs

- C. Marché published in the *Science of Computer Programming* journal [22] a detailed description of an industrial research initially conducted in the context of the U3CAT project (ended in 2012) on static analysis of critical C code. The code involves floating-point computations on quaternions that should be of norm 1. Because of the round-off errors, a drift of this norm is observed over time. In this work a bound on this drift is determined and formally proved correct, using *Frama-C*, *Jessie* and *Why3*. Proofs are done using automated provers and in a few complex cases the Coq proof assistant. The published version is up to date with the recent versions of those tools, and the development is available on our gallery at http://toccata.lri.fr/gallery/quat.en.html
- S. Boldo, C. Lelay, and G. Melquiond worked on the Coquelicot library, designed to be a user-friendly Coq library about real analysis. An easier way of writing formulas and theorem statements is achieved by relying on total functions in place of dependent types for limits, derivatives, integrals, power series, and so on. To help with the proof process, the library comes with a comprehensive set of theorems and some automation. We have exercised the library on several use cases: in an exam at university entry level, for the definitions and properties of Bessel functions, and for the solution of the one-dimensional wave equation. These results are published in the journal *Mathematics in Computer Science* [19].
- S. Boldo and G. Melquiond, with J.-H. Jourdan and X. Leroy (Gallium team, Inria Paris - Rocquencourt) extended the CompCert compiler to get the first formally verified C compiler that provably preserves the semantics of floating-point programs This work, published in the *Journal of Automated Reasoning* [18], also covers the formalization of numerous algorithms of conversion between integers and floating-point numbers.
- S. Boldo, C. Lelay, and G. Melquiond, have conducted a survey on the formalization of real arithmetic and real analysis in various proof systems. This work, published in the journal *Mathematical Structures in Computer Science* [20], details the axioms, definitions, theorems, and methods of automation, available in these systems.
- É. Martin-Dorel and G. Melquiond worked on integrating the CoqInterval and CoqApprox libraries into a single package. The CoqApprox library is dedicated to computing verified Taylor models of univariate functions so as to compute approximation errors. The CoqInterval library reuses this work to automatically prove bounds on real-valued expressions. A large formalization effort took place during this work, so as to get rid of all the holes remaining in the formal proofs of CoqInterval. It was also the chance to perform a comparison between numerous decision procedures dedicated to proving nonlinear inequalities involving elementary functions. A report is available [43].
- S. Boldo, J.-C. Filliâtre, and G. Melquiond, with F. Clément and P. Weis (POMDAPI team, Inria Paris - Rocquencourt), and M. Mayero (LIPN), completed the formal proof of a numerical analysis program: the second-order centered finite-difference scheme for the one-dimensional acoustic wave. This proof was published with a focus towards numerical analysts, in the journal *Computers and Mathematics with Applications* [17].

- P. Roux formalized the influence of double rounding on the accuracy of floating-point arithmetic operators. In particular, this includes all the corner cases that were ignored from Figueroa's original pen-and-paper proof. Results appeared in the *Journal of Formalized Reasoning* [24].
- P. Roux formalized a theory of numerical analysis for bounding the round-off errors of a floating-point algorithm. This approach was applied to the formal verification of a program for checking that a matrix is semi-definite positive. The challenge here is that testing semi-definiteness involves algebraic number computations, yet it needs to be implemented using only approximate floating-point operations. A report is available [45].

6.4. Automated Reasoning

- In the context of the BWare project, aiming at using *Why3* and Alt-Ergo for discharging proof obligations generated by Atelier B, we made progress into several directions. New drivers have been designed for *Why3*, in order to use new back-end provers Zenon modulo and iProver modulo. A notion of rewrite rule was introduced into *Why3*, and a transformation for simplifying goals before sending them to back-end provers was designed. Intermediate results obtained so far in the project were presented both at the French conference AFADL [38] and at the international conference on Abstract State Machines, Alloy, B, VDM, and Z [34].

  On the side of Alt-Ergo, recent developments have been made to efficiently discharge proof obligations generated by Atelier B. This includes a new plugin architecture to facilitate experiments with different SAT engines, new heuristics to handle quantified formulas, and important modifications in its internal data structures to boost performances of core decision procedures. Benchmarks realized on more than 10,000 proof obligations generated from industrial B projects show significant improvements [33].
- C. Dross defended her PhD thesis in April 2014 [14], on the topic of automated reasoning modulo theories, and in particular the handling of quantifiers in the SMT approach. The main results of the thesis are: (1) a formal semantics of the notion of *triggers* typically used to control quantifier instantiation in SMT solvers, (2) a general setting to show how a first-order axiomatization with triggers can be proved correct, complete, and terminating, and (3) an extended DPLL(T) algorithm to integrate a first-order axiomatization with triggers as a decision procedure for the theory it defines. Significant case studies were conducted on examples coming from SPARK programs, and on the benchmarks on B set theory constructed within the BWare project.

6.5. Certification of Languages, Tools and Systems

- M. Clochard, C. Marché, and A. Paskevich developed a general setting for developing programs involving binders, using *Why3*. This approach was successfully validated on two case studies: a verified implementation of untyped lambda-calculus and a verified tableaux-based theorem prover. This work was presented at the PLPV conference in January 2014 [32].
- M. Clochard, J.-C. Filliâtre, C. Marché, and A. Paskevich developed a case study on the formalization of semantics of programming languages using *Why3*. This case study aimed at illustrating recent improvements of *Why3* regarding the support for higher-order logic features in the input logic of *Why3*, and how these are encoded into first-order logic, so that goals can be discharged by automated provers. This case study also illustrates how reasoning by induction can be done without need for interactive proofs, via the use of *lemma functions*. This works was presented at the VSTTE conference [31].
- M. Clochard and L. Gondelman developed a formalization of a simple compiler in *Why3*. It compiles a simple imperative language into assembler instructions for a stack machine. This case study was inspired by a similar example developed using Coq and interactive theorem proving. The aim is to improve significantly the degree of automation in the proofs. This is achieved by the formalization of a Hoare logic and a Weakest Precondition Calculus on assembly programs, so that the correctness of compilation is seen as a formal specification of the assembly instructions generated. This work conducted in 2014 will be presented at the JFLA conference in January 2015 [75].

- S. Dumbrava and É. Contejean, with V. Benzaken (VALS team, at LRI) proposed a *Coq* formalization of the relational data model which underlies relational database systems. More precisely, they have presented and formalized the data definition part of the model including integrity constraints. They have modelled two different query language formalisms: relational algebra and conjunctive queries. They also present logical query optimization and prove the main "database theorems": algebraic equivalences, the homomorphism theorem and conjunctive query minimization. This work has been published at ESOP 2014 [27].
- A. Charguéraud, together with the other members of the *JsCert* team have developed this year the first complete formalization of the semantics of the JavaScript programming language. This project is joint work with Philippa Gardner, Sergio Maffeis, Gareth Smith, Daniele Filaretti and Daiva Naudziuniene from Imperial College, and Alan Schmitt and Martin Bodin from Inria Rennes (see http://jscert.org). The formalization consists of a set of inductive rules translating the prose from the *ECMAScript Language Specification, version 5*, using the pretty-big-step semantics [74]. These rules can be used to formally reason about program behaviors or to establish the correctness of program transformations. In addition to the inductive rules, a reference interpreter has been proved correct. This interpreter may be used to run actual JavaScript program following the rules of the formal semantics. It has been used in particular to validate the formal semantics against official JavaScript test suites. The formalization of JavaScript has been published at POPL [28].

6.6. Miscellaneous

A. Charguéraud worked together with Umut Acar and Mike Rainey, as part of the ERC project *DeepSea*, on the developement of efficient data structures and algorithms targeting modern, shared memory multicore architectures. Two major results were obtained this year.

The first result is a sequence data structure that provides amortized constant-time access at the two ends, and logarithmic time concatenation and splitting at arbitrary positions. These operations are essential for programming efficient computation in the fork-join model. Compared with prior work, this novel sequence data structure achieves excellent constant factors, allowing it to be used as a replacement for traditional, non-splittable sequence data structures. This data structure, called *chunked sequence* due to its use of chunks (fixed-capacity arrays), has been implemented both in C++ and in OCaml. It is described in a paper published at ESA [26].

Another result by A. Charguéraud and his co-authors is the development of fast and robust parallel graph traversal algorithms, more precisely for parallel BFS and parallel DFS. The new algorithms leverage the aforementioned sequence data structure for representing the set of edges remaining to be visited. In particular, it uses the split operation for balancing the edges among the several processors involved in the computation. Compared with prior work, these new algorithms are designed to be efficient not just for particular classes of graphs, but for all input graphs. This work has not yet been published, however it is described in details in a technical report [40]. Note that these two graph algorithms, which involve nontrivial use of concurrent data structures, will be very interesting targets for formal verification.

COMMANDS Project-Team

6. New Results

6.1. Highlights of the Year

6.1.1. Optimization of running strategies based on anaerobic energy and variations of velocity

Participant: Frédéric Bonnans.

The paper [10] about running strategies proves Keller's conjecture. It was highlighted in SIAM Connect,see http://connect.siam.org/insightful-mathematics-for-an-optimal-run/

6.1.2. Research and transfer collaboration in aeronautics with the startup Safety Line

Participants: Frédéric Bonnans, Daphné Giorgi, Stéphan Maindrault, Pierre Martinon.

Following the meeting with the startup Safety Line at Imatch "Optimisation and Control" in october 2013, we conducted a first collaboration of six months on optimizing the fuel consumption of civil airliners. This first step successfully established the proof of concept and was validated by actual test flights in June 2014, leading to a shared patent and the development of a specific module of our software 'Bocop', included in the tool 'OptiClimb' developed at Safety Line. Future prospects include improving the numerical robustness of the current tool, as well as expanding the optimization to the cruise flight in addition to the climb phase.

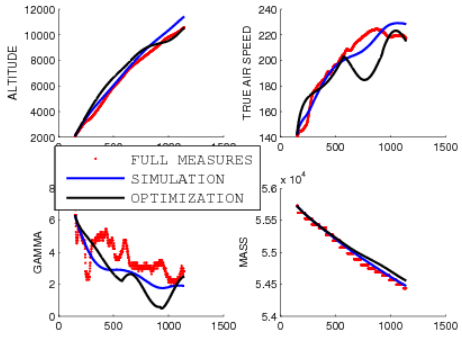

Figure 2. Plane climb phase (Boeing 737)

6.2. Second order analysis of deterministic optimal control problems

Participant: Frédéric Bonnans.

F. Bonnans, with M.S. Aronna (IMPA, Rio de Janeiro) and B.-S. Goh (Curtin U., Miri, Sarawak, Malaysia) obtained in [32] new second order necessary and sufficient optimality conditions for a class of control-affine problems with a scalar control and a scalar state constraint. These optimality conditions extend to the constrained state framework the Goh transform, which is the classical tool for obtaining an extension of the Legendre condition. We propose a shooting algorithm to solve numerically this class of problems and we provide a sufficient condition for its local convergence. We provide examples to illustrate the theory. An article by F. Bonnans, X. Dupuis (Ceremade, U. Dauphine) and L. Pfeiffer (U. Graz) has been published in the SIAM J. Control Optim. on "Second-order necessary conditions in Pontryagin form for optimal control problems" [16].

6.3. Stochastic optimization

6.3.1. Stochastic control

Participant: Frédéric Bonnans.

With J. Gianatti (U. Rosario) and F. Silva (U. Limoges) we obtained an extension of the Sakawa-Shindo algorithm (for computing a solution of the optimality system of a deterministic optimal control problem) to stochastic control problems. The paper is in progress.

6.3.2. Stochastic programming

Participants: Frédéric Bonnans, Nicolas Grebille, Faisal Wahid.

In the framework of the thesis of Nicolas Grébille, we continued our study of decomposition algorithms for a stochastic model of optimal electricity energy production.The energy production is divided in a number of zones. The idea is to constrain the energy flows between these zones, by linear feedback to the demand (wich is a random variable). The coefficients of the feedback are to be optimized. Then the problem is decomposed for each zone (and can then be solved easily by a SDDP type algorithm). We obtained encouraging preliminary numerical results in a three zones problem.
Faisal Wahid developed a mixed integer program model for hydro-power producers participating in the future intra-day French Electricity Balancing Market. He has also formulated the mixed integer stochastic dynamic program model for the more general hydro- bidding under uncertainty. The objective of this model is to produce optimal offer policies in the form of supply curves under a time inhomogeneous Markov process of electricity market clearing prices.

6.3.3. Dynamic programming and error estimates for stochastic control problems with maximum cost

Participants: Athena Picarelli, Hasnaa Zidani.

The paper [14] is concerned with stochastic optimal control for a running maximum cost. A direct approach based on dynamic programming techniques is studied leading to the characterization of the value function as the unique viscosity solution of a second order Hamilton- Jacobi-Bellman (HJB) equation with an oblique derivative boundary condition. A general numerical scheme is proposed and a convergence result is provided. Error estimates are obtained for the semi-Lagrangian scheme. These results can apply to the case of lookback options in finance. Moreover, optimal control problems with maximum cost arise in the characterization of the reachable sets for a system of controlled stochastic differential equations. Some numerical simulations on examples of reachable analysis are included to illustrate our approach.

6.4. Hamilton Jacobi Bellman approach

6.4.1. Optimal feedback control of undamped wave equations by solving a HJB equation

Participants: Hasnaa Zidani, Axel Kröner.

An optimal finite-time horizon feedback control problem for (semi linear) wave equations is studied in [25]. The feedback law can be derived from the dynamic programming principle and requires to solve the evolutionary Hamilton-Jacobi-Bellman (HJB) equation. Classical discretization methods based on finite elements lead to approximated problems governed by ODEs in high dimensional space which makes infeasible the numerical resolution by HJB approach. In the present paper, an approximation based on spectral elements is used to discretize the wave equation. The effect of noise is considered and numerical simulations are presented to show the relevance of the approach

6.4.2. Transmission conditions on interfaces for Hamilton-Jacobi-Bellman equations

Participant: Hasnaa Zidani.

The works [27], [91] deal with deterministic control problems where the dynamic and the running cost can be completely different in two (or more) complementary domains of the space $I\!R^N$. As a consequence, the dynamics and running cost present discontinuities at the interfaces of these domains. This leads to a complex interplay that has to be analyzed among transmission conditions to "glue" the propagation of the value function on the interfaces. Several questions arise: how to define properly the value function(s) and what is (are) the right Bellman Equation(s) associated with this problem?. In the case of a simple geometry (namely when the space $I\!R^N$ is partitioned into two subdomains separated with an interface which is assumed to be a regular hypersurface without any connectedness requirement), [27] discuss different conditions on the hyperplane where the dynamic and the running cost are discontinuous, and the uniqueness properties of the Bellman problem are studied. In this paper we use a dynamical approach, namely instead of working with test functions, the accent is put on invariance properties of an augmented dynamics related to the integrated control system. The comparison principle is accordingly based, rather than on (semi)continuity of the Hamiltonian appearing in the Hamilton–Jacobi–Bellman equation, on some weak separation properties of this dynamics with respect to the stratification.

6.4.3. Control Problems on Stratifiable state-constraints Sets

Participants: Cristopher Hermosilla, Hasnaa Zidani.

This work deals with a state-constrained control problem. It is well known that, unless some compatibility condition between constraints and dynamics holds, the value function has not enough regularity, or can fail to be the unique constrained viscosity solution of a Hamilton-Jacobi-Bellman (HJB) equation. Here, we consider the case of a set of constraints having a strati

ed structure. Under this circumstance, the interior of this set may be empty or disconnected, and the admissible trajectories may have the only option to stay on the boundary without possible approximation in the interior of the constraints. In such situations, the classical pointing quali

cation hypothesis are not relevant. The discontinuous Value Function is then characterized by means of a system of HJB equations on each stratum that composes the state-constraints. This result is obtained under a local controllability assumption which is required only on the strata where some chattering phenomena could occur.

6.4.4. Constrained optimization problems in finite and infinite dimensional spaces

Participant: Cristopher Hermosilla.

We investigate in [39] convex constrained nonlinear optimization problems and optimal control with convex state constraints. For this purpose we endow the interior of constraints set with the structure of Riemannian manifold. In particular, we consider a class of Riemannian metric induced by the squared Hessian of a Legendre functions. We describe in details the geodesic curves on this manifolds and we propose a gradient-like algorithm for constrained optimization based on linear search along geodesics. We also use the Legendre change of coordinates to study the Value Function of a Mayer problem with state constraints. We provide a characterization of the Value Function for this problem as the unique viscosity solution of the Hamilton-Jacobi-Bellman equation.

6.5. Robustness of discontinuous Feedbacks

Participant: Cristopher Hermosilla.

In the paper [40] we study state-constrained discontinuous ordinary differential equations for which the corresponding vector field has a set of singularities that forms a stratification of the state domain. Existence of solutions and robustness with respect to external perturbations of the righthand term are investigated. Moreover, notions of regularity for stratifications are discussed.

6.6. Optimal control of PDEs

6.6.1. Closed-loop optimal control of PDEs

Participant: Axel Kröner.

Stabilization of Burgers equation to nonstationary trajectories A. Kröner and Sérgio S. Rodrigues (RICAM, Linz, Austria) considered in [82] using infinite-dimensional internal controls. Estimates for the dimension of the controller are derived; in the particular case of no constraint in the support of the control a better estimate is derived and the possibility of getting an analogous estimate for the general case is discussed. Numerical examples are presented illustrating the stabilizing effect of the feedback control, and suggesting that the existence of an estimate in the general case analogous to that in the particular one is plausible. In [81] the problem was consided for a finite number of internal piecewise constant controls.

Reduced-order minimum time control of advection-reaction -diffusion systems via dynamic programming Dante Kalise (RICAM, Linz, Austria) and A. Kröner considered in [79]. The authors use balanced truncation for the model reduction part and include a Luenberger observer.

A semi-Lagrangian scheme for L^p-penalized minimum time problems was considered by M. Falcone (Sapienza-Università di Roma, Italy), D. Kalise (RICAM, Austria) and A. Kröner in [78].

6.6.2. Open-loop optimal control of PDEs

Participant: Axel Kröner.

The minimum effort problem for the wave equation K. Kunisch (University of Graz, Austria) and A. Kröner considered in [80]. The problem involves L^∞-control costs which lead to non-differentiability. Uniqueness of the solution of a regularized problem is proven and the convergence of the regularized solutions is analyzed. Further, a semi-smooth Newton method is formulated to solve the regularized problems and its superlinear convergence is shown. Numerical examples confirm the theoretical results.

6.7. Applications in deterministic optimal control

6.7.1. Contrast imaging problem in nuclear magnetic resonance

Participant: Pierre Martinon.

In collaboration with team McTAO (Sophia), we started in 2013 to study the contrast imaging problem in nuclear magnetic resonance, modeled as Mayer problem in optimal control ([58]). Using tools from the Maximum Principle and geometric control, we obtained a first synthesis of locally optimal solutions is given in the single-input case, as well as preliminary results in the bi-input case. This analysis was supported by comprehensive numerical investigations using a combination of indirect shooting (HAMPATH software) and direct method (BOCOP), with a moment-based (LMI) technique to estimate the global optimum.

These results have been extended in 2014, on the theoretical side with the classification of singular extremals ([35]), and on the numerical side with the study of a large number of spins particles subject to spatial inhomogeneities in the magnetic field.

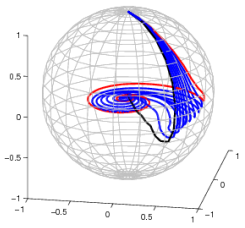

Figure 3. Contrast in quantum control for NMR - Spatial inhomogeneities

6.7.2. Optimal strokes and design for N-link microswimmer

Participant: Pierre Martinon.

Following [71] , we pursued the study of the N-link swimmer, a generalization of the classical Purcell swimmer. We use the model of the Resistive Force Theory to derive the motion equation for the swimmer in a fluid with a low Reynolds number. This allows use to study and solve the optimal swimming problem in the framework of optimal control. We extend our previous study of the optimal strokes by moving to the optimal design of the swimmer. In [72] we provide an estimate of the optimal link ratio for maximal displacement, based on an expansion for small amplitudes. This theoretical result is supported by numerical simulations, that also give some insight on the type of optimal strokes depending on the constraints on the amplitude and deformation speed.

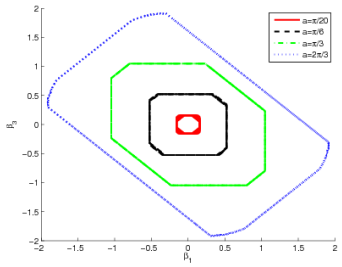

Figure 4. Phase portrait of the optimal stroke w.r.t maximal amplitude

6.7.3. Energy management for a micro-grid

Participants: Frédéric Bonnans, Daphné Giorgi, Benjamin Heymann, Stéphan Maindrault, Pierre Martinon.

We study the energy management problem for a microgrid including a diesel generator and a photovoltaic plant with a battery storage system. The objective is to minimize the total operational cost over a certain timeframe, primarily the diesel consumption, while satisfying a prescribed power load. After reformulation, the decision variables can be reduced to the charging/discharging power for the battery system. We take into account the switching cost for the diesel generator, the non-convex objective, and the long-term aging of the batteries. We solve this problem using a continuous optimal control framework, with both a direct transcription method (time discretization) and a Dynamic Programming method (Hamilton Jacobi Bellman). This project is a collaboration between team COMMANDS (Inria Saclay, France) and Centro de Energia (Universidad de Chile, Chile). A first paper is currently in preparation, while ongoing studies include comparison with the existing MILP approach, more refined battery aging models, and modeling the stochastic nature of the photovoltaic power and power load.

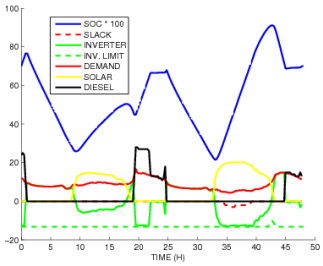

Figure 5. Microgrid management - Winter day sample case

DEFI Project-Team

6. New Results

6.1. Qualitative methods for inverse scattering problems

6.1.1. Identifying defects in an unknown background using differential measurements

Participants: Lorenzo Audibert, Houssem Haddar.

With Alexandre Girard, we developed a new qualitative imaging method capable of selecting defects in complex and unknown background from differential measurements of farfield operators: i.e. far measurements of scattered waves in the cases with and without defects. Indeed, the main difficulty is that the background physical properties are unknown. Our approach is based on a new exact characterization of a scatterer domain in terms of the far field operator range and the link with solutions to so-called interior transmission problems. We present the theoretical foundations of the method and some validating numerical experiments in a two dimensional setting [10]. This work is based on the generalized formulation of the Linear Sampling Method with exact characterization of targets in terms of farfield measurements that has been introduced in [1].

6.1.2. The Factorization Method for a Cavity in an Inhomogeneous Medium

Participants: Houssem Haddar, Shixu Meng.

With F. Cakoni we considered the inverse scattering problem for a cavity that is bounded by a penetrable anisotropic inhomogeneous medium of compact support where on is interested in determining the shape of the cavity from internal measurements on a curve or surface inside the cavity. We derived a factorization method which provides a rigorous characterization of the support of the cavity in terms of the range of an operator which is computable from the measured data. The support of the cavity is determined without a-priori knowledge of the constitutive parameters of the surrounding anisotropic medium provided they satisfy appropriate physical as well as mathematical assumptions imposed by our analysis. Numerical examples were given showing the viability of our method [7].

6.1.3. Asymptotic analysis of the transmission eigenvalue problem for a Dirichlet obstacle coated by a thin layer of non-absorbing media

Participant: Houssem Haddar.

With F. Cakoni and N. Chaulet we considered the transmission eigenvalue problem for an impenetrable obstacle with Dirichlet boundary condition surrounded by a thin layer of non-absorbing inhomogeneous material. We derived a rigorous asymptotic expansion for the first transmission eigenvalue with respect to the thickness of the thin layer. Our convergence analysis is based on a Max–Min principle and an iterative approach which involves estimates on the corresponding eigenfunctions. We provided explicit expressions for the terms in the asymptotic expansion up to order 3 [3].

6.1.4. Boundary Integral Equations for the Transmission Eigenvalue Problem for Maxwell's Equations

Participants: Houssem Haddar, Shixu Meng.

In this work, we considered the transmission eigenvalue problem for Maxwell's equations corresponding to non-magnetic inhomogeneities with contrast in electric permittivity that changes sign inside its support. Following the approach developed by Cossonnière-Haddar in the scalar case, we formulate the transmission eigenvalue problem as an equivalent homogeneous system of boundary integral equation and prove that assuming that the contrast is constant near the boundary of the support of the inhomogeneity, the operator associated with this system is Fredholm of index zero and depends analytically on the wave number. Then we show the existence of wave numbers that are not transmission eigenvalues which by an application of the analytic Fredholm theory implies that the set of transmission eigenvalues is discrete with positive infinity as the only accumulation point. This is a joint work with F. Cakoni.

6.1.5. Invisibility in scattering theory

Participant: Lucas Chesnel.

We investigated a time harmonic acoustic scattering problem by a penetrable inclusion with compact support embedded in the free space. We considered cases where an observer can produce incident plane waves and measure the far field pattern of the resulting scattered field only in a finite set of directions. In this context, we say that a wavenumber is a non-scattering wavenumber if the associated relative scattering matrix has a non trivial kernel. Under certain assumptions on the physical coefficients of the inclusion, we showed that the non-scattering wavenumbers form a (possibly empty) discrete set. This result is important in the justification of certain reconstruction techniques like the Linear Sampling Method in practical applications.

In a second step, for a given real wavenumber and a given domain D, we developed a constructive technique to prove that there exist inclusions supported in D for which the corresponding relative scattering matrix is null. These inclusions have the important property to be impossible to detect from far field measurements. The approach leads to a numerical algorithm which allows to provide examples of (approximated) invisible inclusions. This is a joint work with A.-S. Bonnet-Ben Dhia and S.A. Nazarov [11].

6.1.6. Invisibility in electrical impedance tomography

Participant: Lucas Chesnel.

We adapted the technique to construct invisible isotropic conductivities in for the point electrode model in electrical impedance tomography. Again, the theoretical approach, based on solving a fixed point problem, is constructive and allows the implementation of an algorithm for approximating the invisible perturbations. We demonstrated the functionality of the method via numerical examples. This a joint work with N. Hyvönen and S. Staboulis [13].

6.1.7. A quasi-backscattering problem for inverse acoustic scattering in the Born regime

Participants: Houssem Haddar, Jacob Rezac.

In this work we propose a data collection geometry in which to frame the inverse scattering problem of locating unknown obstacles from far-field measurements of time-harmonic scattering data. The measurement geometry, which we call the quasi-backscattering set-up, is configurated such that one device acts as a transmitter and a line of receivers extends in one-dimension a small distance from the transmitter. We demonstrate that the data collected can be used to locate inhomogeneities whose physical properties are such that the Born approximation applies. In particular, we are able to image a two-dimensional projection of the location of an obstacle by checking if a test function which corresponds to a point in $\mathbb{R}^2$ belongs to the range of a measurable operator. The reconstruction algorithm is based on the MUSIC (Multiple SIgnal Classification) algorithm.

6.2. Iterative Methods for Non-linear Inverse Problems

6.2.1. Inverse medium problem for axisymmetric eddy current models

Participants: Houssem Haddar, Zixian Jiang, Mohamed Kamel Riahi.

We continued our developments of shape optimization methods for inclusion detection in an axisymmetric eddy current model. This problem is motivated by non-destructive testing methodologies for steam generators. We finalized a joint work with A. Lechleiter on numerical methods for the solution of the direct problem in weighted Sobolev spaces using approriate Dirichlet-to-Neumann mappings to bound the computational domain. We are also finalized jointly with M. El Guedri the work on inverse solver using a regularized steepest descent method for the problem of identifying a magnetite deposits using axial eddy current probe. We addressed two issues:

- We developed asymptotic models to identify thin highly conducting deposits. We derived three possible asymptotic models that can be exploited in the inverse problem. We are about to finalize a preprint on this topic.
- We extended the inverse scheeme to 3D configurations with axisymmetry at infinity: this includes exact charactherization of the shape derivative for a mixed formulation of eddy current problems and a parametric inversion scheme based on a pre-defined discrete grid for deposit location [14].

6.2.2. The conformal mapping method and free boundary problems

Participant: Houssem Haddar.

Together with R. Kress we employed a conformal mapping technique for the inverse problem to reconstruct a perfectly conducting inclusion in a homogeneous background medium from Cauchy data for electrostatic imaging, that is, for solving an inverse boundary value problem for the Laplace equation. In a recent work we proposed an extension of this approach to inverse obstacle scattering for time-harmonic waves, that is, to the solution of an inverse boundary value problem for the Helmholtz equation. The main idea is to use the conformal mapping algorithm in an iterative procedure to obtain Cauchy data for a Laplace problem from the given Cauchy data for the Helmholtz problem. We presented the foundations of the method together with a convergence result and exhibit the feasibility of the method via numerical examples. We are currently investigating the extention of this method to solve free boundary value problems.

6.2.3. A steepest descent method for inverse electromagnetic scattering problems

Participant: Houssem Haddar.

Together with N. Chaulet, we proposed the application of a non linear optimization techniques to solve the inverse scattering problems for the $3D$ Maxwell's equations with generalized impedance boundary conditions. We characterized the shape derivative in the case where the GIBC is defined by a second order surface operator. We then applied a boundary variation method based on a regularized steepest descent to solve the 3-D inverse problem with partial farfield data. The obtained numerical results demonstrated the possibility of identifying the shape of coated objects as well as the parameters of the coating in the $3D$ Maxwell case [4].

6.2.4. A posteriori error estimates: Application to Electrical Impedance Tomography

Participants: Olivier Pantz, Matteo Giacomini.

One of the main problem in shape optimization problems is due to the fact that the gradient is never computed exactly. When the current solution is far from a local optimum, this is not a problem: even a rough approximation of the gradient enable us to exhibit a descent direction. On the contrary, when close to a local optimal, a very precise computation of the gradient is needed. Together with Karim Trabelsi, we propose to use a-posteriori error estimates to evaluate the error made on the computation of the gradient. This enables us to ensure that at each step, a genuine descent direction is used in the gradient method. Our method has been applied to the minimization of the Kohn-Vogelius functional in the context of electrical impedance tomography. An article is currently in preparation.

6.2.5. A robust stopping rule for EM algorithm with applications to SAXS measurements

Participants: Federico Benvenuto, Houssem Haddar.

The aim of this work was to develop a fully automatic method for the reconstruction of the volume distribution of diluted polydisperse non-interacting nanoparticles with identical shape from Small Angle X-ray Scattering measurements. The described method solves a maximum likelihood problem with a positivity constraint on the solution by means of an Expectation Maximization iterative scheme coupled with an innovative type of regularization. Such a regularization, together with the positivity constraint results in high fidelity quantitative reconstructions of particle volume distributions making the method particularly effective in real applications. The performance of the method on synthetic data in the case of uni- and bi-modal particle volume distributions are shown. Moreover, the reliability of the method is tested when applied to real data provided by a Xenocs device prototype. Finally, the method can be extended to the analysis of the particle distribution for different types of nano-structures.

6.3. Shape and topology optimization

6.3.1. Stacking sequence and shape optimization of laminated composite plates

Participant: Grégoire Allaire.

We consider the optimal design of composite laminates by allowing a variable stacking sequence and in-plane shape of each ply. In order to optimize both variables we rely on a decomposition technique which aggregates the constraints into one unique constraint margin function. Thanks to this approach, a rigorous equivalent bi-level optimization problem is established. This problem is made up of an inner level represented by the combinatorial optimization of the stacking sequence and an outer level represented by the topology and geometry optimization of each ply. We propose for the stacking sequence optimization an outer approximation method which iteratively solves a set of mixed integer linear problems associated to the evaluation of the constraint margin function. For the topology optimization of each ply, we lean on the level set method for the description of the interfaces and the Hadamard method for boundary variations by means of the computation of the shape gradient. An aeronautic test case is exhibited subject to different constraints, namely compliance, reserve factor and first buckling load. This is joint work with G. Delgado.

6.3.2. Thickness control in structural optimization via a level set method

Participant: Grégoire Allaire.

In the context of structural optimization via a level-set method we propose a framework to handle geometric constraints related to a notion of local thickness. The local thickness is calculated using the signed distance function to the shape. We formulate global constraints using integral functionals and compute their shape derivatives. We discuss different strategies and possible approximations to handle the geometric constraints. We implement our approach in two and three space dimensions for a model of linearized elasticity. As can be expected, the resulting optimized shapes are strongly dependent on the initial guesses and on the specific treatment of the constraints since, in particular, some topological changes may be prevented by those constraints. This is a joint work with G. Michailidis

6.4. Asymptotic Analysis

6.4.1. Effective boundary conditions for thin periodic coatings

Participants: Mathieu Chamaillard, Houssem Haddar.

This topic is the object of a collaboration with Patrick Joly and is a continuation of our earlier work on interface conditions done in the framework of the PhD thesis of Berangère Delourme. The goal here is to derive effective conditions that model scattering from thin periodic coatings where the thickness and the periodicity are of the same length but very small compared to the wavelength. The originality of our work, compared to abundant literature is to consider the case of arbitrary geometry (2-D or 3-D) and to consider higher order approximate models. We formally derived third order effective conditions after exhibiting the full asymptotic expansion of the solution in terms of the periodicity length.

6.4.2. Homogenization of the transmission eigenvalue problem with applications to inverse problems

Participant: Houssem Haddar.

In a joint work with F. Cakoni and I. Harris, we consider the interior transmission problem associated with the scattering by an inhomogeneous (possibly anisotropic) highly oscillating peri-odic media. We show that, under appropriate assumptions, the solution of the interior transmission problem converges to the solution of a homogenized problem as the period goes to zero. Furthermore, we prove that the associ-ated real transmission eigenvalues converge to transmission eigenvalues of the homogenized problem. Finally we show how to use the first transmission ei-genvalue of the period media, which is measurable from the scattering data, to obtain information about constant effective material properties of the periodic media. The obtained convergence results are not optimal. Such results with rate of convergence involve the analysis of the boundary correction and will be subject of a forthcoming paper.

6.4.3. Homogenization of electrokinetic models in porous media

Participant: Grégoire Allaire.

With R. Brizzi, J.-F. Dufrêche, A. Mikelic and A. Piatnitski, are interested in the homogenization (or upscaling) of a system of partial differential equations describing the non-ideal transport of a N-component electrolyte in a dilute Newtonian solvent through a rigid porous medium. Our recent work has focused on the so-called non-ideal case. Namely we consider the mean spherical approximation (MSA) model which takes into account finite size ions and screening effects. On the one hand we established a rigorous homogenized transport model starting from this microscopic model. On the other hand we did numerical simulations to ompute the corresponding effective parameters and make systematic comparisons between the idea model and the MSA model.

6.4.4. Modeling and Simulation of the Mechanical behavior of Vesicles and Red Blood Cells

Participant: Olivier Pantz.

6.4.4.1. Highly anisotropic thin shells

With K. Trabelsi (IPSA), we have proposed a new justification of various non linear highly anisotropic elastic shell models. Among others, we do derive the so called Helfrich functional, that describe the behavior of the lipid bilayer of the vesicle and red blood cells. Our results will soon be published in MEMOCS (Mathematics and Mechanics Complex Systems).

6.4.4.2. Minimization of the Helfrich functional

Our work with K. Trabelsi established that the mechanical behavior of vesicles and red blood cells can be approximated by thin non linear anisotropic elastic shells. Minimizing directly the Helfrich functional is not an easy task from the numerical point of view. Most methods require the use of high order finite elements and stabilization techniques so to prevent mesh degeneration. Instead, we propose to approximate the two dimensional membrane of a vesicle (or red blood cell) by a three dimensional non linear elastic body of small thickness. Firstly, this enable us to use standard finite elements and discretization (basically Lagrange of degree 2). Secondly, the discretized formulation is intrinsically stable, so no stabilization is needed. Finally, even if it leads us to solve a three dimensional problem (instead of the two dimensional initial one), it is no more costly than a direct two dimensional approach as the scale of the mesh can be chosen to be of the same order than the "thickness" of the shell. We have already obtained encouraging results for vesicles. We plan to extend them to the case of vesicles with spontaneous curvature and to red blood cells. Moreover, we are considering different strategies to minimize the computational cost (that is already quite satisfying compared with some other methods).

6.4.5. Modeling of Damage and Fracture

Participant: Olivier Pantz.

6.4.5.1. Fracture as limit of Damage

With Leila Azem (PhD Student), we use a model introduced by G. Allaire, F. Jouve and N. Van Goethem in a previous work to simulate the propagation of fracture. The main idea is to approximate the fracture as a damage material and to compute the evolution of the path of the crack using a shape gradient analysis. Our main contribution consists to propose to use a material derivative approach to compute the shape gradient. The advantage is that it drastically simplify the evaluation of the shape gradient, as no regularization is needed and no jump terms as to be computed on the interface bewteen the healthy and damaged areras. An article is currently in preparation to present our results.

6.4.5.2. Fracture with penalization of the jump

With Leila Azem, we propose to approximate a model of fracture with penalization of the jump of the displacement as a limit of a damage model. This is achieved by a specific choice of the softness of the damage material with respect to the cost to turn material from a healthy to a damaged state. We have carried out a formal analysis to justify our approach and have already obtained several numerical results.

6.5. Diffusion MRI

Participants: Jing-Rebecca Li, Houssem Haddar, Simona Schiavi, Khieu Van Nguyen, Gabrielle Fournet, Dang Van Nguyen.

Diffusion Magnetic Resonance Imaging (DMRI) is a promising tool to obtain useful information on microscopic structure and has been extensively applied to biological tissues. In particular, we would like to focus on two applications:

- Inferring from DMRI measurements changes in the cellular volume fraction occurring upon various physiological or pathological conditions. This application is one of the first to show the promise of DMRI because it can detect acute cerebral ischemia (cell swelling) on the basis of lower than normal apparent diffusion coefficient a few minutes after stroke;
- Estimating the average cell size in the case of tumor imaging This application is useful as a diagnostic tool as well as a tool for the evaluation of tumor treatments;

For both of the above applications we approach the problem via the following steps:

- Construct reduced models of the multiple-compartment Bloch-Torrey partial differential equation (PDE) using homogenization methods.
- Invert the resulting reduced models for the biological parameters of interest: the cellular volume fraction in the first case, and the average distance between neighboring cells in the second case.

We obtained the following results.

- We derived using homogenization techniques an asymptotic model of the diffusion MRI signal for finite pulse magnetic field gradient sequences in the long diffusion time regime and numerically verified it using a Finite Element method for both isotropic and anisotropic diffusion configurations in three dimensions. This resulted in 2 publications.
- We derived a new asymptotic model of the diffusion MRI signal for low gradient strengths that is valid for a wide range of diffusion time scales. An article describing our results is under preparation.
- We performed a numerical study of a cylinder model of the diffusion MRI signal for neuronal dendrite trees. This resulted in 1 publication.
- We implemented a compressed sensing method for obtaining T2-weighted images in shorter scanning time and this method was used to segment nerve cells of the Aplysia Californica at the MRI center Neurospin. An article describing our results is under preparation.
- We participated in the characterization of glioma microcirculation and tissue features in a rat brain model using diffusion-encoding magnetic field gradient pulses sequences, working along with collaborators at the high field brain MRI center Neurospin. This resulted in 1 publication.
- We performed Monte-Carlo simulation of blood flow in micro-vessels in the brain with the goal of using the results to explain the MRI signal drop due to incoherent flow in the micro-vessels. This is an ongoing project.

DISCO Project-Team

6. New Results

6.1. Zero-parameter mono and multi objective methods for the tuning of controllers

The synthesis of controllers for any kind of system is the main point in Automatic Control. The traditional approach is to use a simplified model of the system to control and/or use some reformulations of the specifications to tune an often efficient but suboptimal controller. In a more and more competitive industrial context, the design of high performances controllers has emerged as a crucial point to enhance the global productivity. However, the design of optimal controllers supposes the solution of non-convex and non-differentiable optimization problems, for which deterministic and (often) local search algorithm fail in the solution. In this work, Particle Swarm Optimization is used to solve the problem, and tested to define some controllers for a magnetic levitation. The use of standard settings and penalization terms leads to a zero-parameter and reformulation free method. Results are much than satisfactory and show that Evolutionary Computation could be of great interest in the Automatic Control field.

6.2. Fixed-structure H_∞ synthesis for multiple plants

This work proposes an efficient evolutionary approach to the fixed-order and structured H_∞ control design problem extended to the multiple plants case. By testing it on the classical example of a flexible plant, this evolutionary approach proves to be very efficient compared with other recent tools, especially in the case of a high number of plants; it can then be considered as an interesting alternative for such problems.

6.3. Fixed-Structure Mu-Synthesis

This work proposes to shed a new light on the Mu-synthesis problem using the differential evolution algorithm. This algorithm allows optimizing simultaneously the structured controller and the dynamic (or static) D-scalings, which leads to robust performance controllers. This method has been applied successfully to a classical flexible plant control problem. After a comparison between the evolutionary approach and the non-smooth optimization one has envisaged proving the high potential of the proposed method.

6.4. Algebraic Analysis Approach to Linear Functional Systems

6.4.1. Serre's reduction problem

The purpose of this work is to study the connections existing between Serre's reduction of linear functional systems - which aims at finding an equivalent system defined by fewer equations and fewer unknowns - and the decomposition problem - which aims at finding an equivalent system having a diagonal block structure - in which one of the diagonal blocks is assumed to be the identity matrix. In order to do that, in [62], we further develop results on Serre's reduction problem and on the decomposition problem. Finally, we show how these techniques can be used to analyze the decomposability problem of standard linear systems of partial differential equations studied in hydrodynamics such as Stokes equations, Oseen equations and the movement of an incompressible fluid rotating with a small velocity around a vertical axis.

6.4.2. A spectral sequence central in the behaviour approach

Within the algebraic analysis approach to multidimensional systems, the behavioural approach developed by J. C. Willems can be understood as a dual approach to the module-theoretic approach. This duality is exact when the signal space is an injective cogenerator module over the ring of differential operators. In particular, the obstruction to the existence of a parametrization of a multidimensional system is characterized by the existence of autonomous elements of the multidimensional system. In [52], we consider the case of a general signal space and investigate the connection between the algebraic properties of the differential module defining the multidimensional system and the obstruction to the existence of parametrizations of the multidimensional system. To do so, we investigate a certain Grothendieck spectral sequence connecting the obstructions to the existence of parametrizations to the obstructions to the differential module - defining the multidimensional system - to be torsion-free, reflexive ...projective.

6.4.3. Restrictions of n-D systems and inverse images of D-modules

The problem of characterizing the restriction of the solutions of a n-D system to a subvector space of $\mathbb{R}^n$ has recently been investigated in the literature of multidimensional systems theory. For instance, this problem plays an important role in the stability analysis and in stabilization problems of multidimensional systems. In this work, we characterize the restriction of a n-D behaviour to an algebraic or analytic submanifold of $\mathbb{R}^n$. In [51], within the algebraic analysis approach to multidimensional systems, we show that the restriction of a n-D behaviour to an algebraic or analytic submanifold can be characterized in terms of the inverse image of the differential module defining the behaviour. Explicit characterization of inverse images of differential modules is investigated. Finally, we explain Kashiwara's extension of the Cauchy-Kowalevsky theorem for general n-D behaviours and non-characteristic algebraic or analytic submanifolds.

6.4.4. Artstein's transformation of linear time-delay systems

Artstein's classical results show that a linear first-order differential time-delay system with delays in the input is equivalent to a linear first-order differential system without delays thanks to an invertible transform which includes integral and delay operators. Within a constructive algebraic approach, we show how Artstein's reduction can be found again and generalized as a particular isomorphism problem between the finitely presented modules defined by the two above linear systems over the ring of integro-differential time-delay operators. Moreover, we show that Artstein's reduction can be obtained in an automatic way by means of symbolic computation, and thus can be implemented in computer algebra systems.

6.5. New Techniques for Robust Control of Linear Infinite-Dimensional Systems

6.5.1. Robust stabilization of a flexible rod moving in rotation and translation

We develop a hierarchy of models for a flexible rod moving in rotation and translation from a nonlinear partial differential model (generalization of the Euler-Bernoulli equation) to a linear partial differential equation and finite-dimensional models via approximations. We study the stability of those models as well as their robust stabilizations. This work is an extension of the results obtained in [61]. This work will be pursued within the framework of a CIFRE PhD thesis developed in collaboration with SAGEM (2015).

6.5.2. Noncommutative geometric approach to infinite-dimensional systems theory:

This new field of research aims at showing that noncommutative geometric structures such as connections and curvatures exist on internally stabilizable infinite-dimensional linear systems and on their stabilizing controllers. To see this new geometry, using the noncommutative geometry developed by Connes, we have to replace the standard differential calculus by the quantized differential calculus and classical vector bundles by projective modules. In [50], we give an explicit description of the connections on an internally stabilizable system and on its stabilizing controllers in terms of the projectors of the closed-loop system classically used in robust control. Their curvatures are explicitly computed. These connections aim at studying the variations

of the signals in the closed-loop system in response to a disturbance or a change of the reference. The study of these connections are useful to understand how techniques of (noncommutative) differential geometry can be used in the study of H^∞ control theory.

6.5.3. A fractional ideal approach to the robust regulation problem

We show how fractional ideal techniques developed in [8] can be used to obtain a general formulation of the internal model principle for stabilizable infinite-dimensional SISO plants which do not necessarily admit coprime factorization. This result is then used to obtain necessary and sufficient conditions for the robust regulation problem. In particular, we find again all the standard results obtained in the literature.

6.5.4. Robust control as an application to the homological perturbation lemma:

Within the lattice approach to transfer matrices developed in [8], we have recently shown how standard results on robust control can be obtained in a unified way and generalized when interpreted as a particular case of the so-called Homological Perturbation Lemma. This lemma plays a significant role in algebraic topology, homological algebra, algebraic and differential geometry, computer algebra, ... Our results show that it is also central to robust control theory for infinite-dimensional linear systems.

6.6. Set invariance for discrete-time delay systems

We studied the existence of positively invariant sets for linear delay-difference equations. In particular, we regarded two strong stability notions: robust (with respect to delay parameter) asymptotic stability for the discrete-time case and delay-independent stability for the continuous-time case. The correlation between these stability concepts is also considered. Furthermore, for the delay-difference equations with two delay parameters, we provided a computationally efficient numerical routine which is necessary to guarantee the existence of contractive sets of Lyapunov–Razumikhin type. This condition also appears to be necessary and sufficient for the delay-independent stability and sufficient for the robust asymptotic stability. The results are published in [25].

We proposed a new perspective on the structural properties of invaraint sets for time delay systems via set factorization. This novel perspective describes, in a unified framework, different existing notions of invariant sets [60]. Additionally, it is shown that the (possible non-minimal) state space representation is a key element in the description of low complexity invariant sets.

6.7. Low complexity constrained control

On one side, we proposed an explicit (piecewise affine feedback) control obtained via interpolation for constrained linear systems [23]. On another side, we studied the Linear Constrained Regulation problem for Continuous-Time Systems in the presence of non-convex constraints [32]. This might prove to be useful for the multi-agent dynamical systems operating under collision avoidance constraints.

6.8. Fault detection based on set theoretic methods and connexions with fault tolerant control

We proposed a set-theoretic fault detection mechanism for multisensor systems with a classification of possible functioning according to the use in the feedback mechanism. The healthy, faulty and under-recovery class are characterized via set descriptions in the residual space and as such can be monitored via on-line mechanisms [26]. Furthermore, the robust detection has been enhanced with an interval observer mechanism for the monitorig during the transients [28].

6.9. Interval Observer

We made several progresses in the domain of the construction of state estimators called interval observers.

1) In [16], we have shown how interval observers can be constructed for nonlinear (and not Lipschitz) systems possessing a special triangular system. These systems are not cooperative and not globally Lipschitz and have a rather general structure which may result from a change of coordinates or an output injection. Besides, under additional assumptions, input to state stability (ISS) properties are derived. We illustrated the constructions by designing a framer and an ISS interval observer for two models of bioreactors.

2) The contributions [17] and [18] present major results for the design of interval observers for discete-time systems. In [18], coordinate transformations which change an arbitrary linear discrete-time system into a positive one and general nonlinear designs of interval observers for nonlinear systems (satisfying a restricitive stability assumption) are proposed. In [17], it is explained how two classical Luenberger observers can be used, (even in the absence of the positivity property of the studied system or the error equations) as interval observer, provided two appropriate outputs, which compose the lower and the upper bound of the interval observer, are selected.

3) In [33], we present a new type of interval observers for nonlinear systems that are affine in the unmeasured part of the state.They are composed of two copies of classical observers and upper and lower bounds which are designed by taking advantage of positivity properties of the error equations when written in appropriate coordinates.

6.10. Reduction model approach: new advances

We solved several distinct problems entailing to the celebrated reduction model approach. Let us recall that this technique makes it possible to stabilize systems with arbitrarily large pointwise or distributed delays in the input.

1) In [46], solutions to the problem of globally exponentially stabilizing linear systems with an arbitrarily long pointwise delay with sampled feedbacks are presented. The main result of a contribution by F. Mazenc and D. Normand-Cyrot is recalled and compared with other results available in the literature.

2) We considered in [41] a stabilization problem for continuous-time linear systems with discrete-time measurements and a sampled input with a pointwise constant delay. In a first step, we constructed a continuous-discrete observer which converges when the maximum time interval between two consecutive measurements is sufficiently small. We also constructed a dynamic output feedback through a technique which is strongly reminiscent of the reduction model approach. It stabilizes the system when the maximal time between two consecutive sampling instants is sufficiently small. No limitation on the size of the delay was imposed.

3) In [43], we studied a general class of nonlinear systems with input delays of arbitrary size. We adapted the reduction model approach to prove local asymptotic stability of the closed loop input delayed systems, using feedbacks that may be nonlinear. We determined estimates of the basins of attraction for the closed loop systems using Lyapunov-Krasovskii functionals.

4) The contribution [21] is devoted to stabilization problems for time-varying linear systems with constant input delays. The reduction model approach we proposed ensures a robustness property (input-to-state stability) with respect to additive uncertainties, under arbitrarily long delays. It applies to rapidly time-varying systems, and gives a lower bound on the admissible rapidness parameters. We also covered slowly time-varying systems, including upper bounds on the allowable slowness parameters. We illustrated our work using a pendulum model.

6.11. Nonlinear systems with delay

1) In [45], we developed a new technique for stability analysis for nonlinear dynamical systems with delays and possible discontinuities. In contrast with Lyapunov based approaches, the trajectory based approach we proposed involves verifying certain inequalities along solutions of auxiliary systems. It applies to a wide range of systems, notably time-varying systems with time-varying delay, ODE coupled with difference equations, and networked control systems with delay. It relies on the input-to-state stability notion, and yields input-to-state stability with respect to uncertainty.

2) In [39], to address various types of delays including the neutral-type arising in dynamical networks, we dealt with coupled delay differential and continuous-time difference equations and proposed stability and robustness criteria. In these criteria, differential equation parts do not necessarily exhibit unbounded dissipation rate. Subsystems described by differential equations are not required to be input-to-state stable either. No assumptions on network topology are made. To handle such a general case, we construct explicit Lyapunov-type functionals. We established stability and robustness of the overall networks.

3) In [22], [42] and [44], stability results for several families of systems with delay are established. The key ingredient of these contributions is the use of comparison systems of a new type, the theory of the positive systems and linear Lyapunov functionals. We provided robustness of the stability with respect to multiplicative uncertainty in the vector fields. We allowed cases where the delay may be unknown, and where the vector fields defining the systems are not necessarily bounded. We illustrate our work using a chain of integrators and other examples.

6.12. Strictification

In [40], the problem of stabilizing rigid-body attitude dynamics in the presence of pointwise time-delay for the input torque is considered. A quaternion-based linear state feedback controller is shown to achieve local stability in addition to the characterization of sufficient condition that depends only on the magnitude of the initial angular rates. More specifically, no restrictions are imposed on the body initial orientation which is a significant contrast with other results from recent literature that adopt three-dimensional representations for the attitude kinematics. Using the quaternion-based linear feedback structure, the closed-loop system is shown to never admit the possibility for finite-time escapes. While the actual magnitude of the time-delay can be unknown, an upper bound on the delay is assumed to be known. The proof relies on the construction of a functional which does not belong to the family of the strict Lyapunov-Krasovskii functionals, but shares important features with the functionals of this family. The stability conditions and results are illustrated through numerical simulations.

6.13. Stability analysis of fractional and classical neutral systems with commensurate delays

Fractional and classical neutral systems with commensurate delays have chains of poles asymptotic to vertical lines (see [66] for classical systems). The delicate case where system have some chains of poles asymptotic to the imaginary axis is interesting as the absence of poles in the open left half-plane does not guarantee the H_∞-stability of the system.

Stability analysis of classical or fractional neutral systems with one single chain of poles asymptotic to the imaginary axis has been investigated in [88], [70], [2], [69], where the asymptotic location of poles of neutral chains was given and necessary and sufficient conditions for H_∞-stability were derived.

We have performed a full analysis of classical and fractional systems with multiple chains of poles approaching a set of points on the imaginary axis. Moreover, a unified method to analyze the stability of fractional and classical systems has been derived.

6.14. Stabilization of fractional neutral systems with commensurate delays

We consider strictly proper fractional neutral systems with one delay and one chain of poles asymptotic to the imaginary axis including the case where this chain may approach the axis from the right side. Thus the system may possess infinitely many poles in the right half-plane. For these systems, a Youla-Kučera parametrization regarding H_∞-stability of all stabilizing controllers has been obtained in [59]. Having in mind the robustness of the closed-loop relative to parameter uncertainties, we wish to find controllers which are able to provide a closed-loop free of chain of poles asymptotic to the imaginary axis. However, we prove that a large class of realizable stabilizing controllers cannot achieve this. [47].

6.15. Stabilization of MISO fractional systems with I/O delays

In order to yield the set of all the stabilizing controllers of a class of MISO fractional systems with delays by mean of Youla-Kučera parametrization regarding H_∞-stability, we are interested in determining coprime factorizations of the transfer function. Explicit expressions of left coprime factorizations and left Bézout factors have been derived in [85]. Explicit expressions of right coprime factorizations and right Bézout factors for some classes of systems have also been derived in [86]. Recently, we obtain right factors for a more general class of systems. Furthermore, we present these right factors in the minimal form, i.e. factors with the minimal number of coefficients to be determined and with the lowest degree. We also obtain left factors in the minimal form.

6.16. Modeling and control of Acute Myeloid Leukemia

Starting from a PDE model of hematopoiesis given in [64], we have derived several models of healthy or cancer cell dynamics in hematopoiesis and performed several stability analyses.

We have proposed in [58] a new mathematical model of the cell dynamics in Acute Myeloid Leukemia (AML) which takes into account the four different phases of the proliferating compartment as well the fast self-renewal phenomenon frequently observed in AML. As was the case in [58] this model is transformed into a distributed delay system and was analyzed here with input-output techniques. Local stability conditions for an equilibrium point of interest are derived in terms of a set of inequalities involving the parameters of the mathematical model.

We have also studied a coupled delay model for healthy and cancer cell dynamics in Acute Myeloid Leukemia consisting of two stages of maturation for cancer cells and three stages of maturation for healthy cells. For a particular healthy equilibrium point, locally stability conditions involving the parameters of the mathematical model have been obtained [30], [31].

We have performed in [29] a stability analysis of both the PDE model of healthy hematopoiesis and a coupled PDE model of healthy and cancer cell dynamics. The stability conditions obtained here in the time domain strengthen the idea that fast self-renewal plays an important role in AML.

A time-domain stability analysis by means of Lyapunov-Krasovskii functionals has been performed on the delay system modeling healthy hematopoiesis for a strictly positive equilibrium point of interest.

6.17. Algebraic geometry techniques for polynomial systems

6.17.1. Testing the structural stability of N-d discrete linear systems

The goal of this work is to propose new computer algebra based methods for testing the structural stability of N-d discrete linear systems. Recall that a discrete linear system given by its transfer function $G(z_1, ..., z_n) = N(z_1, ..., z_n)/D(z_1, ..., z_n)$ is said to be stable if and only if the denominator $D(z_1, ..., z_n)$ is devoid from zero inside the unit complex poly-disc. This fundamental problem in the analysis of N-d systems has been extensively studied these last decades. At the end of the seventies, DeCarlo et al [77] show that testing the previous condition is equivalent to testing the existence of complex zeros on each face of the poly-disc i.e. $D(1, ..., z_i, ..., 1)$ for $i = 1...n$ as well as testing the existence of complex zero on the poly-circle i.e. the zeros of $D(z_1, ..., z_n)$ when $|z_1| = ... = |z_n| = 1$.

Starting from the conditions of DeCarlo et al, we propose a new approach that transform the last condition, that is, the non-existence of complex zeros on the unit poly-circle to a condition on the existence of real solutions inside a region of R^n. More precisely we propose two type of transformations. The first one reduces the problem to looking for real solutions inside the unit box while the second one reduces the problem to looking for real solutions in the whole space R^n. In order to check the existence of real solutions, we use classical computer algebra algorithms for solving systems of polynomial equations. In the case of one or two variables, the appearing systems are generally zero-dimensional. To count or locate the real solutions of such systems, we compute a rational univariate representation [95], that is a one to one mapping between the solutions of the

system and the roots of a univariate polynomial, thus the problem is reduced to a univariate problem. When the number of variables is larger than two, the systems that stem from the conditions above are no longer zero-dimensional. In such case, we use critical points method that allow to compute solutions in each real connected component of the zeros of the systems [65].

We implemented the previous approach on maple using the external library *R*aglib [63] which provides routines for testing the existence of real solutions of an algebraic system. Preliminary tests show the relevance of our approach.

This work is supported by the ANR MSDOS grant.

6.17.2. Efficient algorihtms for solving bivariate algebraic systems

This work addresses the problem of solving a bivariate algebraic system (i.e computing certified numerical approximation of the solutions) via the computation or a rational univariate representation. Such a representation is useful since it allows to turn many queries on the system into queries on univariate polynomials. Given two coprime polynomials P and Q in $Z[x, y]$ of degree bounded by d and bitsize bounded by τ we present new algorithms for computing rational univariate representation of the system $\{P, Q\}$ and from this representation, isolating the real solutions of $\{P, Q\}$. The cost analysis of these algorithms show that they have a worst-case bit complexity in $sOB(d^6 + d^5\tau)$ which improves by a factor d the state-of-the-art complexity.

GECO Project-Team

6. New Results

6.1. Highlights of the Year

We organized a thematic trimester on "Geometry, analysis and dynamics on sub-Riemannian manifolds" at the Institut Henri Poincaré (IHP), including 4 workshops, 4 research courses, 8 thematic days, several seminars. We also organized an associated school at CIRM with 4 introductory courses. The web pages of the events are:
http://www.cmap.polytechnique.fr/subriemannian/
http://www.cmap.polytechnique.fr/subriemannian/cirm/

6.2. New results: geometric control

Let us list some new results in sub-Riemannian geometry and hypoelliptic diffusion obtained by GECO's members.

- The article [14] presents simple controls that generate motion in the direction of high order Lie brackets. Whereas the naive use of piecewise constant controls requires the number of switchings to grow exponentially with the length of the bracket, we show that such motion is possible with sinusoidal controls whose sum of frequencies equals the length of the bracket. This work is closely related and motivated by the study of the complexity of sub-Riemannian geodesics for generic regular distributions, i.e., whose derived flag has maximal growth vector. Of particular interest is the approximation of curves transversal to the distribution by admissible curves. We also present a surprising example that shows that it is possible to simultaneously kill higher moments without increasing the number of self-intersections of the base curve.
- The curvature discussed in [18] is a rather far going generalization of the Riemann sectional curvature. We define it for a wide class of optimal control problems: a unified framework including geometric structures such as Riemannian, sub-Riemannian, Finsler and sub-Finsler structures; a special attention is paid to the sub-Riemannian (or Carnot-Caratheodory) metric spaces. Our construction of the curvature is direct and naive, and it is similar to the original approach by Riemann. Surprisingly, it works in a very general setting and, in particular, for all sub-Riemannian spaces.
- In [19] we prove sectional and Ricci-type comparison theorems for the existence of conjugate points along sub-Riemannian geodesics. In order to do that, we regard sub-Riemannian structures as a special kind of variational problems. In this setting, we identify a class of models, namely linear quadratic optimal control systems, that play the role of the constant curvature spaces. As an application, we prove a version of sub-Riemannian Bonnet–Myers theorem and we obtain some new results on conjugate points for 3D left-invariant sub-Riemannian structures.
- In the study of conjugate times in sub-Riemannian geometry, linear quadratic optimal control problems show up as model cases. In [1] we consider a dynamical system with a constant, quadratic Hamiltonian h, and we characterize the number of conjugate times in terms of the spectrum of the Hamiltonian vector field H. We prove the following dichotomy: the number of conjugate times is identically zero or grows to infinity. The latter case occurs if and only if H has at least one Jordan block of odd dimension corresponding to a purely imaginary eigenvalue. As a byproduct, we obtain bounds from below on the number of conjugate times contained in an interval in terms of the spectrum of H.

- A 3D almost-Riemannian manifold is a generalized Riemannian manifold defined locally by 3 vector fields that play the role of an orthonormal frame, but could become collinear on some set called the singular set. Under the Hormander condition, a 3D almost-Riemannian structure still has a metric space structure, whose topology is compatible with the original topology of the manifold. Almost-Riemannian manifolds were deeply studied in dimension 2. In [21] we start the study of the 3D case which appear to be reacher with respect to the 2D case, due to the presence of abnormal extremals which define a field of directions on the singular set. We study the type of singularities of the metric that could appear generically, we construct local normal forms and we study abnormal extremals. We then study the nilpotent approximation and the structure of the corresponding small spheres. We finally give some preliminary results about heat diffusion on such manifolds.
- In [22] we study spectral properties of the Laplace-Beltrami operator on two relevant almost-Riemannian manifolds, namely the Grushin structures on the cylinder and on the sphere. As for general almost-Riemannian structures (under certain technical hypothesis), the singular set acts as a barrier for the evolution of the heat and of a quantum particle, although geodesics can cross it. This is a consequence of the self-adjointness of the Laplace-Beltrami operator on each connected component of the manifolds without the singular set. We get explicit descriptions of the spectrum, of the eigenfunctions and their properties. In particular in both cases we get a Weyl law with dominant term $E \log E$. We then study the effect of an Aharonov-Bohm non-apophantic magnetic potential that has a drastic effect on the spectral properties. Other generalized Riemannian structures including conic and anti-conic type manifolds are also studied. In this case, the Aharonov-Bohm magnetic potential may affect the self-adjointness of the Laplace-Beltrami operator.
- In [28] we investigate the number of geodesics between two points p and q on a contact sub-Riemannian manifold M. We show that the count of geodesics on M is controlled by the count on its nilpotent approximation at p (a contact Carnot group). For contact Carnot groups we give sharp bounds for a generic point q. Removing the genericity condition for q, geodesics might appear in families and we prove a similar statement for their topology. We study these families, and in particular we focus on the unexpected appearance of isometrically non-equivalent geodesics: families on which the action of isometries is not transitive. We apply the previous study to contact sub-Riemannian manifolds: we prove that for any given point $p \in M$ there is a sequence of points p_n such that $p_n \to p$ and that the number of geodesics between p and p_n grows unbounded (moreover these geodesics have the property of being contained in a small neighborhood of p).

New results on automatic control and motion planning for various type of applicative domains are the following.

- [8] is devoted to the problem of model-based prognostics for a Waste Water Treatment Plant (WWTP). Our aim is to predict degradation of certain parameters in the process, in order to anticipate malfunctions and to schedule maintenance. It turns out that a WWTP, together with the possible malfunction, has a specific structure: mostly, the malfunction appears in the model as an unknown input function. The process is observable whatever this unknown input is, and the unknown input can itself be identified through the observations. Due to this property, our method does not require any assumption of the type “slow dynamics degradation", as is usually assumed in ordinary prognostic methods. Our system being unknown-input observable, standard observer-based methods are enough to solve prognostic problems. Simulation results are shown for a typical WWTP.
- In [9] we study the problem of controlling an unmanned aerial vehicle (UAV) to provide a target supervision and/or to provide convoy protection to ground vehicles. We first present a control strategy based upon a Lyapunov-LaSalle stabilization method to provide supervision of a stationary target. The UAV is expected to join a predesigned admissible circular trajectory around the target which is itself a fixed point in the space. Our strategy is presented for both high altitude long endurance (HALE) and medium altitude long endurance (MALE) types of UAVs.
- In [12] we study how a particular spatial structure with a buffer impacts the number of equilibria and their stability in the chemostat model. We show that the occurrence of a buffer can allow a

species to setup or on the opposite to go to extinction, depending on the characteristics of the buffer. For non-monotonic response function, we characterize the buffered configurations that make the chemostat dynamics globally asymptotically stable, while this is not possible with single, serial or parallel vessels of the same total volume and input flow. These results are illustrated with the Haldane kinetic function.

- In [15] and [25] we present new results on the path planning problem in the case study of the car with trailers. We formulate the problem in the framework of optimal nonholonomic interpolation and we use standard techniques of nonlinear optimal control theory for deriving hyperelliptic signals as controls for driving the system in an optimal way. The hyperelliptic curves contain as many loops as the number of nonzero Lie brackets generated by the system. We compare the hyperelliptic signals with the ordinary Lissajous-like signals that appear in the literature, we conclude that the former have better performance.
- In [27] we consider affine-control systems, i.e., systems in the form $\dot{q}(t) = f_0(q(t)) + \sum_{i=1}^{m} u_i(t) f_i(q(t))$. Here, the point q belongs to a smooth manifold M, the f_i's are smooth vector fields on M. This type of system appears in many applications for mechanical systems, quantum control, microswimmers, neuro-geometry of vision...

We conclude the section by mentioning the book [17] that we edited, collecting some papers in honour of Andrei A. Agrachev for his 60th birthday. The book contains new results on sub-Riemannian geometry and more generally on the geometric theory of control.

6.3. New results: quantum control

New results have been obtained for the control of the bilinear Schrödinger equation.

- In [2] we present a sufficient condition for approximate controllability of the bilinear discrete-spectrum Schrödinger equation in the multi-input case. The controllability result extends to simultaneous controllability, approximate controllability in H^s, and tracking in modulus. The sufficient condition is more general than those present in the literature even in the single-input case and allows the spectrum of the uncontrolled operator to be very degenerate (e.g. to have multiple eigenvalues or equal gaps among different pairs of eigenvalues). We apply the general result to a rotating polar linear molecule, driven by three orthogonal external fields. A remarkable property of this model is the presence of infinitely many degeneracies and resonances in the spectrum.
- In [5] we consider the minimum time population transfer problem for a two level quantum system driven by two external fields with bounded amplitude. The controls are modeled as real functions and we do not use the Rotating Wave Approximation. After projection on the Bloch sphere, we treat the time-optimal control problem with techniques of optimal synthesis on 2D manifolds. Based on the Pontryagin Maximum Principle, we characterize a restricted set of candidate optimal trajectories. Properties on this set, crucial for complete optimal synthesis, are illustrated by numerical simulations. Furthermore, when the two controls have the same bound and this bound is small with respect to the difference of the two energy levels, we get a complete optimal synthesis up to a small neighborhood of the antipodal point of the initial condition.
- In [11] we investigate the controllability of quantum electrons trapped in a two-dimensional device, typically a metal oxide semiconductor (MOS) field-effect transistor. The problem is modeled by the Schrödinger equation in a bounded domain coupled to the Poisson equation for the electrical potential. The controller acts on the system through the boundary condition on the potential, on a part of the boundary modeling the gate. We prove that, generically with respect to the shape of the domain and boundary conditions on the gate, the device is controllable. We also consider control properties of a more realistic nonlinear version of the device, taking into account the self-consistent electrostatic Poisson potential.
- In [29] we prove the approximate controllability of a bilinear Schrödinger equation modelling a two trapped ions system. A new spectral decoupling technique is introduced, which allows to analyze the controllability of the infinite-dimensional system through finite-dimensional considerations.

6.4. New results: neurophysiology

- [3] presents a semidiscrete alternative to the theory of neurogeometry of vision, due to Citti, Petitot, and Sarti. We propose a new ingredient, namely, working on the group of translations and discrete rotations $SE(2,N)$. The theoretical side of our study relates the stochastic nature of the problem with the Moore group structure of $SE(2,N)$. Harmonic analysis over this group leads to very simple finite dimensional reductions. We then apply these ideas to the inpainting problem which is reduced to the integration of a completely parallelizable finite set of Mathieu-type diffusions (indexed by the dual of $SE(2,N)$ in place of the points of the Fourier plane, which is a drastic reduction). The integration of the the Mathieu equations can be performed by standard numerical methods for elliptic diffusions and leads to a very simple and efficient class of inpainting algorithms. We illustrate the performances of the method on a series of deeply corrupted images.
- In [4] and [7] we consider the problem of minimizing $\int_0^l \sqrt{\xi^2 + K(s)^2}ds$ for a planar curve having fixed initial and final positions and directions. The total length l is free. Here s is the arclength parameter, $K(s)$ is the curvature of the curve and $\xi > 0$ is a fixed constant. This problem comes from a model of geometry of vision due to Petitot, Citti and Sarti. We study existence of local and global minimizers for this problem. In [7] we characterize sub-Riemannian geodesics and the range of the exponential map. In [4] we prove that if for a certain choice of boundary conditions there is no global minimizer, then there is neither a local minimizer nor a geodesic. We finally give properties of the set of boundary conditions for which there exists a solution to the problem.

6.5. New results: switched systems

- In [6] we consider a family of linear control systems $\dot{x} = Ax + \alpha Bu$ on $\mathbb{R}^d$, where α belongs to a given class of persistently exciting signals. We seek maximal α-uniform stabilization and destabilization by means of linear feedbacks $u = Kx$. We extend previous results obtained for bidimensional single-input linear control systems to the general case as follows: if there exists at least one K such that the Lie algebra generated by A and BK is equal to the set of all $d \times d$ matrices, then the maximal rate of convergence of (A, B) is equal to the maximal rate of divergence of (A, B). We also provide more precise results in the general single-input case, where the above result is obtained under the simpler assumption of controllability of the pair (A, B).
- The paper [10] considers the stabilization to the origin of a persistently excited linear system by means of a linear state feedback, where we suppose that the feedback law is not applied instantaneously, but after a certain positive delay (not necessarily constant). The main result is that, under certain spectral hypotheses on the linear system, stabilization by means of a linear delayed feedback is indeed possible, generalizing a previous result already known for non-delayed feedback laws.
- In [16] and [26] we give a collection of converse Lyapunov–Krasovskii theorems for uncertain retarded differential equations. We show that the existence of a weakly degenerate Lyapunov–Krasovskii functional is a necessary and sufficient condition for the global exponential stability of the linear retarded functional differential equations. This is carried out using the switched system transformation approach.
- Consider a continuous-time linear switched system on $\mathbb{R}^n$ associated with a compact convex set of matrices. When it is irreducible and its largest Lyapunov exponent is zero there always exists a Barabanov norm associated with the system. In [23] we deal with two types of issues: (a) properties of Barabanov norms such as uniqueness up to homogeneity and strict convexity; (b) asymptotic behaviour of the extremal solutions of the linear switched system. Regarding Issue (a), we provide partial answers and propose four related open problems. As for Issue (b), we establish, when $n = 3$, a Poincaré-Bendixson theorem under a regularity assumption on the set of matrices. We then revisit a noteworthy result of N.E. Barabanov describing the asymptotic behaviour of linear switched system on $\mathbb{R}^3$ associated with a pair of Hurwitz matrices $\{A, A + bc^T\}$. After pointing out a fatal gap in Barabanov's proof we partially recover his result by alternative arguments.

- In [24] we address the exponential stability of a system of transport equations with intermittent damping on a network of $N \geq 2$ circles intersecting at a single point O. The N equations are coupled through a linear mixing of their values at O, described by a matrix M. The activity of the intermittent damping is determined by persistently exciting signals, all belonging to a fixed class. The main result is that, under suitable hypotheses on M and on the rationality of the ratios between the lengths of the circles, such a system is exponentially stable, uniformly with respect to the persistently exciting signals. The proof relies on an explicit formula for the solutions of this system, which allows one to track down the effects of the intermittent damping.

Maxplus Project-Team

6. New Results

6.1. Highlights of the Year

Nous avons donné un contre exemple inattendu à l'analogue continu de la conjecture de Hirsch, proposé par Deza, Terlaky et Zinchenko, voir Section 6.4.4 .

English version

We gave a somehow unexpected counter example to the continuous analogue of the Hirsch conjecture proposed by Deza, Terlaky and Zinchenko, see Section 6.4.4 .

6.2. Théorie spectrale max-plus et géométrie métrique/Max-plus spectral theory and metric geometry

6.2.1. Introduction

Participants: Marianne Akian, Stéphane Gaubert, Cormac Walsh.

Étant donné un noyau $a : S \times S \to \mathbb{R} \cup \{-\infty\}$, on peut lui associer le problème spectral max-plus

$$\sup_{y \in S} a(x,y) + u(y) = \lambda + u(x), \quad \forall x \in S, \tag{2}$$

dans lequel on cherche le vecteur propre $u : S \to \mathbb{R} \cup \{-\infty\}$ et la valeur propre correspondante $\lambda \in \mathbb{R} \cup \{-\infty\}$. Comme nous l'avons rappelé dans les §3.2 et 3.3 , le problème spectral (9) intervient en contrôle ergodique: l'ensemble S est l'espace des états, et l'application $a(x,y)$ fournit le gain associé à la transition $x \to y$. Le cas où S est fini est classique, l'on a alors un résultat précis de représentation de l'espace propre, à l'aide d'un certain graphe, dit graphe critique. Des résultats existent également lorsque S est compact et que le noyau vérifie certaines propriétés de régularité.

Dans [64], nous avons considéré le cas où S est non compact. Lorsque $\lambda = 0$, l'espace propre est analogue à l'espace des fonctions harmoniques défini en théorie (classique ou probabiliste) du potentiel. En introduisant l'analogue max-plus de la frontière de Martin, nous avons obtenu un analogue de la formule de représentation de Poisson des fonctions harmoniques : toute solution u de (9) peut être représentée sous la forme :

$$u = \sup_{w \in \mathcal{M}_m} w + \mu_u(w) \ , \tag{3}$$

où $\mathcal{M}_m \subset (\mathbb{R} \cup \{-\infty\})^S$ est l'analogue max-plus de la frontière de Martin minimale (l'ensemble des fonctions harmoniques extrémales normalisées), et où μ_u joue le rôle de la mesure spectrale. Nous avons montré aussi que les éléments de l'espace de Martin minimal peuvent être caractérisés comme les limites de "quasi-géodésiques". La frontière de Martin max-plus généralise dans une certaine mesure la frontière d'un espace métrique construite à partir des horo-fonctions (fonctions de Busemann généralisées), ou horo-frontière. Ces résultats inspirent les travaux des sections suivantes, qui portent sur des cas remarquables d'espaces métriques (§6.2.3) ou sur des applications en théorie des jeux (§6.2.2).

English version

Let the kernel $a : S \times S \to \mathbb{R} \cup \{-\infty\}$ be given. One may associate the max-plus spectral equation (9), where the eigenvector $u : S \to \mathbb{R} \cup \{-\infty\}$ and the eigenvalue $\lambda \in \mathbb{R} \cup \{-\infty\}$ are unknown. As we recalled in §3.2 and refmonotone, this spectral problem arises in ergodic optimal control: the set S is the *state space*, and the map $a(x, y)$ is the *transition reward*. The case when S is finite is classical, a precise spectral theorem is known, with a characterisation of the eigenspace in terms of a critical graph. Some results have been shown when S is compact, assuming that the kernel a satisfies some regularity properties.

In [64], we considered the case where S is non-compact. When $\lambda = 0$, the eigenspace is analoguous to the set of harmonic functions defined in classical or probabilistic potential theory. By introducing a max-plus analogue of the classical Martin boundary, we obtained an analogue of the Poisson representation of harmonic functions, showing that any solution u of (9) may be represented as in (10) where $\mathcal{M}_m \subset (\mathbb{R} \cup \{-\infty\})^S$ is a max-plus analogue of the minimal Martin boundary (the set of normalised extremal harmonic functions), and μ_u plays the role of the spectral measure. We also showed that the elements of the minimal Martin boundary can be characterised as limits of certain "almost-geodesics". The max-plus Martin boundary generalises to some extent the boundary of metric spaces defined in terms of horofunctions (generalised Busemann functions), or horoboundary. These results have inspired the work of the next sections, which deal either with interesting examples of metric spaces (§6.2.3) or with applications to zero-sum games (§6.2.2).

6.2.2. *Asymptotiques d'itérées d'applications contractantes au sens large et jeux à somme nulle en horizon long/Asymptotics of iterates of nonexpansive mappings and zero-sum games*

Participants: Jérôme Bolte, Stéphane Gaubert, Guillaume Vigeral.

On s'est intéressé ici à l'existence du paiement moyen pour les jeux répétés, et plus généralement, à l'existence du vecteur de "taux de fuite" $\lim_k f^k(x)/k$ où f est une application de $\mathbb{R}^n$ dans lui même, nonexpansive pour une norme quelconque. Dans le cas particulier des jeux, f est un opérateur de Shapley, qui est nonexpansif pour la norme sup. On a montré dans [15] que la limite existe si l'application f est définissable dans une structure o-minimale. Ceci généralise des résultats de Bewley, Kohlberg, et Neyman, qui montraient que la limite existe si f est semi-algébrique. L'extension au cas o-minimal permet notamment de traiter des opérateurs de type "log-exp" apparaissant en contrôle sensible au risque.

English version

We studied the question of the existence of the mean payoff for repeated games, and more generally, the existence of a vector of "escape rates", $\lim_k f^k(x)/k$, where f is a self-map of $\mathbb{R}^n$, non-expansive in some norm. In the special case of zero-sum games, f is a Shapley operator, and it is sup-norm nonexpansive. We showed in [15] that this limit does exist as soon as the map f is definable in an o-minimal structure. This generalizes results of Bewley, Kohlberg, and Neyman, who showed that this limit exists if f is semi-algebraic. The extension to the case of o-minimal structures allows one in particular to deal with log-exp type operators arising in risk sensitive control.

6.2.3. *Isométries de la géométrie de Hilbert/Isometries of the Hilbert geometry*

Participants: Cormac Walsh, Bas Lemmens [Kent University, UK].

Dans nos travaux précédents, nous avons étudié la géométrie de Hilbert (d'un ensemble convexe) en dimension finie, en particulier son horo-frontière et son groupe des isométries. Le chapitre de livre [44] donne une vue d'ensemble de ces travaux. Le cas de la dimension infinie est aussi intéressant, et a été utilisé depuis de nombreuses années en analyse non linéaire. Malgré cela, la géométrie de ces espaces est très peu connue en dimension infinie. Nous collaborons sur ce sujet avec Bas Lemmens de l'Université de Kent. Nous étudions par exemple le problème suivant. En dimension finie, il est connu que la géométrie de Hilbert est isométrique à un espace normé si et seulement si le convexe est un simplexe. Nous essayons de montrer plus généralement que la géométrie de Hilbert est isométrique à un espace de Banach si et seulement si le convexe est le cône des fonctions positives continues sur un espace topologique compact. Pour cela, nous étudions l'horo-frontière en dimension infinie.

English version

Previously, we have been studying the Hilbert geometry in finite dimensions, especially its horofunction boundary and isometry group. The book chapter [44] contains a survey of this work. However, the infinite dimensional case is also interesting, and has been used as a tool for many years in non-linear analysis. Despite this, very little is known about the geometry of these spaces when the dimension is infinite. We are collaborating on this topic with Bas Lemmens of the University of Kent. An example of a problem we are working on is the following. In finite dimension it is known that a Hilbert geometry is isometric to a normed space if and only if it is a simplex. We are attempting to show that, more generally, a Hilbert geometry is isometric to a Banach space if and only if it is the cross-section of a positive cone, that is, the cone of positive continuous functions on some compact topological space. To tackle this problem we are finding it useful to study the horofunction boundary in the infinite-dimensional case.

6.2.4. Croissance des boules dans la géométrie de Hilbert/Volume growth in the Hilbert geometry

Participants: Cormac Walsh, Constantin Vernicos [Université Montpellier 2].

Avec Constantin Vernicos de l'Université Montpellier 2, nous étudions la croissance du volume de la boule d'une géométrie de Hilbert (d'un ensemble convexe) en fonction du rayon. En particulier, nous étudions l'entropie volumique:

$$\lim_{r\to\infty} \frac{\log \mathrm{Vol}\, B(x,r)}{r}, \tag{4}$$

où $B(x,r)$ désigne la boule de centre x et de rayon r, et Vol est une notion de volume particulière, telle que celle définie par Holmes–Thompson ou celle de Busemann. L'entropie ne dépend pas du choix particulier de x, ni de celui du volume. Il est connu que pour l'espace hyperbolique, ou toute géométrie de Hilbert dont la frontière est C^2 et de courbure strictement positive, l'entropie est égale à $n-1$ lorsque la dimension de l'espace est n, et il a été conjecturé que ceci correspond aussi à l'entropie maximale d'une géométrie de Hilbert en dimension n. Afin de prouver cette conjecture, nous cherchons d'abord à étudier le lien entre l'entropie et l'approximabilité du convexe par des polytopes, et ensuite à borner cette approximabilité. La première étape nécessite d'étudier la croissance du volume dans le cas de polytopes. Dans ce cas, la croissance est polynomiale de degré n, plutôt qu'exponentielle, et il est important de comprendre le lien entre le coefficient dominant du polynôme exprimant le volume et la complexité du polytope. Nous avons obtenu une formule pour ce coefficient, laquelle dépend de la structure combinatoire du polytope.

English version

In a collaboration with Constantin Vernicos of Université Montpellier 2, we are investigating how the volume of a ball in a Hilbert geometry grows as its radius increases. Specifically, we are studing the volume entropy (11) where $B(x,r)$ is the ball with center x and radius r, and Vol denotes some notion of volume, for example, the Holmes–Thompson or Busemann definitions. Note that the entropy does not depend on the particular choice of x, nor on the choice of the volume. It is known that the hyperbolic space, or indeed any Hilbert geometry with a C^2-smooth boundary of stricty positive curvature, has entropy $n-1$, where n is the dimension, and it has been conjectured this is the maximal entropy possible for Hilbert geometries of the given dimension. Our approach to this conjecture is to first relate the entropy to the approximability of the convex domain by polytopes, and then bound this approximability. The first of these steps requires us study the volume growth in the polytopal case. Here the growth is polynomial rather than exponential, of degree n, and it is important to know how the constant on front of the highest term depends on the complexity of the polytope. We have a formula for this constant in terms of the combinatorial structure of the polytope.

6.2.5. Consensus non-commutatif et contraction d'opérateurs de Kraus/Noncommutative consensus and contraction of Kraus maps

Participants: Stéphane Gaubert, Zheng Qu.

Dans le travail [17], on s'est intéressé à la vitesse de convergence vers l'équilibre d'une itération de la forme $x^{k+1} = T(x^k)$, $x^k \in X$, où T est une application linéaire préservant un cône dans un espace de Banach X, telle que $T(e) = e$, pour un certain vecteur e dans l'interieur du cône. On s'intéresse aussi à l'itération dans l'espace dual, $y^{k+1} = T^*(y^k)$, $y^k \in X^*$, lorsque $\langle y^0, e\rangle = 1$.

Le cas classique est celui où $T(x) = Px$ est un opérateur de Markov. L'itération primale traduit alors la convergence vers le "consensus", et l'itération duale traduit la convergence de la distribution de probabilité en temps k vers l'état stationnaire. Dans ce cas, le taux de contraction (en un coup) $\kappa(P)$ d'une itération primale, pour la semi-norme de Hilbert $\|z\|_H := \max_i z_i - \min_j z_j$, ainsi que le taux de contraction d'une itération duale, pour la métrique en variation totale, coïncident et sont caractérisés par une formule dûe à Doeblin et Dobrushin (coefficient d'ergodicité),

$$\kappa(P) := 1 - \min_{i,j} \sum_{s=1}^{n} \min\left(P_{is}, P_{js}\right).$$

On a donné ici une généralisation de cette formule au cas d'opérateurs abstraits, qui s'applique en particulier aux opérateurs de Kraus qui interviennent en information quantique. Ces derniers opérent sur l'espace des matrices symmétriques, et sont de la forme

$$T(x) = \sum_k a_k x a_k^* \qquad \text{avec} \qquad \sum_k a_k a_k^* = I \ .$$

Dans [53], nous avons étudié des questions de complexité pour les applications de Kraus, montrant en particulier qu'il est NP-dur de vérifier qu'une application de Kraus envoie le cone dans son interieur.

English version

In [17], we studied the speed of convergence to equilibrium of an iteration of the form $x^{k+1} = T(x^k)$, $x^k \in X$, where T is a linear map preserving a cone in a Banach space X, such that $T(e) = e$, for some vector e in the interior of the cone. We also considered the iteration in the dual space X^*, $y^{k+1} = T^*(y^k)$, $y^k \in X^*$, where $\langle y^0, e\rangle = 1$.

The classical application arises when $T(x) = Px$ is a Markov operator. Then, the primal iteration represents the dynamics of consensus, whereas the dual iteration represents the evolution of the probability distribution as a function of time. Then, the (one-shot) contraction rate $\kappa(P)$ of the primal iteration, with respect to Hilbert's seminorm $\|z\|_H := \max_i z_i - \min_j z_j$, and the contraction rate of the dual iteration, with respect to the total variation metric, coincide, and are characterized by a formula of Doeblin and Dobrushin (ergodicity coefficient),

$$\kappa(P) := 1 - \min_{i,j} \sum_{s=1}^{n} \min\left(P_{is}, P_{js}\right).$$

We gave here a generalization of this formula to an abstract operators on a cone. This covers in particular the Kraus maps arising in quantum information theory. The latter maps act on the space of symmetric matrices. They can be written as

$$T(x) = \sum_k a_k x a_k^* \qquad \text{with} \qquad \sum_k a_k a_k^* = I \ .$$

In [53], we studied complexity issues related to Kraus maps, and showed in particular that checking whether a Kraus map sends the cone to its interior is NP-hard.

6.3. Algèbre linéaire max-plus, convexité tropicale et jeux à somme nulle/Max-plus linear algebra, tropical convity and zero-sum games

6.3.1. Polyèdres tropicaux/Tropical polyhedra

Participants: Xavier Allamigeon, Stéphane Gaubert, Eric Goubault [CEA], Ricardo Katz [Conicet, Argentine].

On étudie les analogues max-plus ou tropicaux des ensembles convexes. Ceux-ci sont utiles en particulier pour représenter de manière effective les ensembles d'états accessibles de systèmes à événements discrets [9], ils sont aussi apparus récemment en géométrie tropicale, dans toute une série de travaux à la suite de Sturmfels et Develin [108]. Les polyèdres max-plus peuvent aussi être vus comme des limites de déformations de polyèdres classiques, sur lesquels ils donnent un éclairage de nature combinatoire. Toutes ces motivations ont inspiré la recherche d'analogues des résultats fondamentaux d'analyse convexe classique: séparation, projection, points extrémaux, à la suite en particulier de [8].

Dans un travail de X. Allamigeon, S. Gaubert, et E. Goubault [71], [72], on a mis en évidence un critère combinatoire pour la caractérisation des sommets des polyèdres tropicalement convexes. Celui-ci s'exprime à l'aide d'hypergraphes orientés, et de leurs composantes fortement connexes. Ce critère possède la propriété d'être vérifiable en un temps presque linéaire en la taille de l'hypergraphe.

On en déduit un analogue tropical de la méthode de la double description [72] (méthode très utilisée sur les polyèdres classiques, et dûe à Motzkin *et al.* [160]). Cet algorithme permet de calculer les sommets d'un polyèdre défini de façon externe (intersection de demi-espaces ou d'hyperplans tropicaux). Grâce au critère combinatoire précédent, l'algorithme améliore de plusieurs ordres de grandeur les techniques connues jusqu'alors. Ceci est confirmé par de nombreuses expérimentations. Ce travail est motivé par des applications à l'analyse statique [70] et aux systèmes à événements discrets [113], dans lesquelles la manipulation de tels polyèdres est le goulot d'étranglement.

Il est connu qu'un polyèdre tropical peut être représenté comme l'enveloppe convexe d'un ensemble minimal de points et rayons, donnés par ses sommets et ses rayons extrêmes [124]. Dans un travail réalisé par X. Allamigeon et R. Katz [75], et effectué en partie lors de visites de R. Katz à Inria, on étudie la question duale de la caractérisation des représentations minimales par demi-espaces. On montre qu'un polyèdre tropical possède *essentiellement* une unique représentation minimale par demi-espaces, lorsque leurs apex appartiennent au polyèdre. On montre que les apex de ces demi-espaces non-redondants correspondent à certains sommets du complexe tropical introduit par Develin et Sturmfels [108]. On introduit également un critère combinatoire pour l'élimination de demi-espaces redondants à l'aide d'hypergraphes orientés.

Dans un travail de X. Allamigeon et R. Katz [52], nous étudions la tropicalisation des représentations par demi-espaces des polyèdres convexes sur le corps des séries de Puiseux. Nous démontrons ainsi une conjecture de Develin et Yu [109]. Celle-ci assure qu'étant donné un polytope tropical pur, il existe un polytope *relevé* sur les séries de Puiseux, dont les demi-espaces associés aux faces se "tropicalisent" en une représentation par demi-espaces du polytope tropical initial.

Des applications de ces travaux à l'algorithmique, concernant en particulier les jeux répétés, sont discutées dans la Section 6.5.2 . Une application aux systèmes temps réel est discutée dans la Section 6.6.4 .

English version

We study the max-plus or tropical analogues of convex sets. These have been used in particular to represent effectively the accessible sets of certain discrete event systems [9]. They also appeared in tropical geometry, following the work of Sturmfels and Develin [108]. Max-plus polyhedra can be thought of as limits of deformations of classical polyhedra, on which they give a combinatorial insight. These motivations have inspired the investigation of analogues of basic results of classical convex analysis: separation, projection, representation by extreme points, following [8].

In a work of X. Allamigeon, S. Gaubert, and E. Goubault [72], we introduce a combinatorial criterion for the characterization of the vertices of tropically convex polyhedra. It is expressed in terms of directed hypergraphs and their strongly connected components. This criterion can be verified in almost linear time in the size of the hypergraph.

This allows to develop a tropical analogue of the double description method [72] (this method is widely used for classical convex polyhedra, and is due to Motzkin *et al.* [160]). This algorithm is able to determine all the vertices of a polyhedron defined externally (intersection of tropical half-spaces of hyperplanes). Thanks to the combinatorial criterion mentioned above, the algorithm improves the existing methods by several orders of magnitude. This is confirmed by several experiments. This is motivated by applications to static analysis [70] and discrete event systems [113], in which computing such polyhedra turns out to be the bottleneck.

It is well-known that a tropical polyhedron can be represented as the convex hull of a minimal set of points and rays, provided by its vertices and extreme rays [124]. In a work of X. Allamigeon and R. Katz [75], partly done during visits of R. Katz at Inria, the dual problem of characterizing the minimal representations by half-spaces is studied. We show that a tropical polyhedron admits *essentially* a unique minimal external representation by half-spaces, provided that their apices belong to the polyhedron. We prove that the apices of these half-spaces correspond to certain vertices of the tropical complex introduced by Develin and Sturmfels [108]. We also establish a combinatorial criterion allowing to eliminate redundant half-spaces using directed hypergraphs.

In a work of X. Allamigeon and R. Katz [52], we study the tropicalization of the representation by half-spaces of convex polyhedra over the field of Puiseux series. In particular, we prove a conjecture of Develin and Yu [109]. It states that, given a pure tropical polytope, there exists a lifting polytope over Puiseux series, such that the facet-defining half-spaces are "tropicalized" into a representation by half-spaces of the initial polytope.

Some algorithmic applications of this work concerning in particular mean payoff games, will be discussed in Section 6.5.2 . Applications to real time systems will be discussed in Section 6.6.4 .

6.3.2. *Systèmes linéaires max-plus/Max-plus linear systems*

Participants: Marianne Akian, Stéphane Gaubert, Alexander Guterman [Moscow State University].

Dans [42], on montre des formules de Cramer pour des systèmes linéaires sur diverses extensions du semi-anneau max-plus. Les éléments de ces extensions sont des nombres tropicaux enrichis d'une information de multiplicité, de signe ou d'angle par exemple. On obtient ainsi des résultats d'existence et d'unicité qui généralisent plusieurs résultats de [133], [164], [120], [172], [139]. De plus, pour certaines extensions du semi-anneau max-plus, les preuves fournissent des algorithmes de type Jacobi ou Gauss-Seidel pour résoudre les systèmes linéaires.

Nous nous intéressons maintenant à la complexité de la solution de systèmes linéaires tropicaux signés, i.e. de systèmes sur l'extension du semi-anneau max-plus avec signes, ou d'hyperplans sur ce semi-anneau.

English version

In [42], we prove general Cramer type theorems for linear systems over various extensions of the tropical semiring, in which tropical numbers are enriched with an information of multiplicity, sign, or argument. We obtain existence or uniqueness results, which extend or refine earlier results in [133], [164], [120], [172], [139]. Moreover, some of our proofs lead to Jacobi and Gauss-Seidel type algorithms to solve linear systems in suitably extended tropical semirings.

We study now the complexity of the solution of signed tropical linear systems, that is systems on the extension of the tropical semiring with signs, or the one of the nonemptyness of hyperplanes over this semiring.

6.3.3. *Convexes tropicaux et théorème de Choquet/Tropical convex sets and Choquet theorem*

Participants: Marianne Akian, Stéphane Gaubert, Paul Poncet.

La thèse de Paul Poncet [165] concernait essentiellement ce que l'on appelle l'analyse idempotente, c'est-à dire l'étude des espaces fonctionnels ou linéaires de dimension infinie sur l'algèbre tropicale, ou tout autre semi-anneau idempotent. Paul Poncet a développé pour cela un point de vue treillis continus comme dans [1], ou plus généralement domaines. Depuis la soutenance, plusieurs articles issus du manuscrit de thèse sont publiés [21], [20] ou en cours de soumission, et d'autres travaux pousuivant ceux de la thèse sont en cours avec les membres de l'équipe.

En particulier avec ce point de vue domaines, Paul Poncet a pu établir des théorèmes de type Krein-Milman, réciproque de Milman, et représentation de Choquet dans les semi-treillis [20] ou l'algèbre max-plus [38].

On sait que les résultats sur les convexes tropicaux de dimension infinie de [165] permettent de retrouver partiellement les résultats sur la frontière de Martin max-plus décrits dans la section 6.2.1 . Dans un travail commun avec Klaus Keimel (TU-Darmstadt), nous essayons d'obtenir l'extension du théorème de représentation de Choquet tropical dans le cas d'ensembles ordonnés qui ne sont pas forcément des treillis tels que le cône des matrices symmetriques positives muni de l'ordre de Loewner.

English version

The PhD thesis work of Paul Poncet [165] concerned essentially what is called idempotent analysis, that is the study of infinite dimensional functional or linear spaces over tropical algebra, or any other idempotent semiring. For this aim, Paul Poncet developped the point of view of continuous lattices, as in [1], or more generally of domains. Since the defense of his thesis, several papers derived from the thesis manuscript have been published [21], [20] or up to be submitted. Some other works pursuing the thesis work are done with team members.

In particular, using the point of view of domains, Paul Poncet showed results such as a Krein-Milman type theorem, a Milman converse type theorem, and a Choquet representation type theorem over semilattices [20] or over max-plus algebra [38].

We know that the results on infinite dimensional tropical convex sets of [165] allow one to recover at least partially the results on max-plus Martin boundaries described in Section 6.2.1 . In a joint work with Klaus Keimel (TU-Darmstadt), we try to obtain the extension of the max-plus Choquet representation theorem in the case of ordered sets that are not necessarily semilattices, such as the cone of nonnegative symmetric matrices endowed with the Loewner order.

6.3.4. Points fixes d'applications monotones homogènes et jeux à somme nulle/Fixed points of order preserving homogeneous maps and zero-sum games

Participants: Marianne Akian, Stéphane Gaubert, Antoine Hochart.

Pour les jeux répétés à somme nulle, un problème de base est de savoir si le paiement moyen par unité de temps est indépendant de l'état initial. Ici, on définit le paiement moyen directement au moyen de l'opérateur de Shapley (ou de la programmation dynamique) du jeu, lequel préserve l'ordre et commute avec l'addition d'une constante. Dans le cas particulier des jeux à zero joueur, i.e. de chaînes de Markov avec fonctionnelle additive, la solution du problème ci-dessus est fournie par le théorème ergodique. Dans [46], on généralise ce résultat au cas des jeux répétés à espace d'états fini. Cette généralisation est basée sur l'étude de la sous-classe d'opérateurs de Shapley *sans-paiement* (le paiement a lieu seulement le dernier jour), lesquels commutent avec la multiplication par une constante positive. L'intérêt de cette sous-classe est qu'elle inclue la fonction de récession d'un opérateur de Shapley, lorsqu'elle existe. Nous montrons que le paiement moyen est indépendant de l'état initial pour toutes les perturbations des paiements instantannés dépendantes de l'état si, et seulement si, une condition d'ergodicité est vérifiée. Cette dernière est caractérisée par l'unicité, à constante additive près, du point fixe de la fonction de récession de l'opérateur de Shapley, ou, dans le cas particulier des jeux stochastiques à nombre fini d'actions et information parfaite, par une condition d'accessibilité dans un hypergraphe orienté, entre deux sous-ensembles conjugués d'états. On montre aussi que l'ergodicité d'un jeu ne dépend que de la probabilité de transition et qu'elle peut être vérifiée en temps polynomial lorsque le nombre d'états est fixé.

Lorsque un jeu est ergodique au sens ci-dessus, son paiement moyen est indépendant de l'état initial, et il coincide avec la valeur propre non linéaire de l'opérateur de Shapley. De plus, le vecteur propre associé, appelé biais, permet de déterminer les stratégies stationnaires optimales. Un autre problème est alors de comprendre pour quelles classes de jeux, le biais est unique (à constante additive près). Dans [25], on considère des jeux avec un nombre fini d'états et d'actions, de paiements instantannés variables, mais de probabilités de transition fixées. On montre que le vecteur de biais, considéré comme une fonction des paiements instantannés, est unique génériquement (à constante additive près).

English version

A basic question for zero-sum repeated games consists in determining whether the mean payoff per time unit is independent of the initial state. Here the mean payoff is defined in terms of the Shapley operator (dynamic programming operator) of the game, which is an order preserving map commuting with the addition of a constant. In the special case of "zero-player" games, i.e., of Markov chains equipped with additive functionals, the answer to the above question is provided by the mean ergodic theorem. In [46], we generalize this result to repeated games with a finite state space. This generalization is based on the study of the subclass of *payment-free* Shapley operators (the payment only occurs when the game stops), which are commuting with the multiplication by a positive constant, and which include the recession function of any Shapley operator, when it exists. We show that the mean payoff is independent of the initial state for all state-dependent perturbations of the rewards if and only if an ergodicity condition is satisfied. The latter is characterized by the uniqueness modulo additive constants of the fixed point of the recession function of the Shapley operator, or, in the special case of stochastic games with finite action spaces and perfect information, by a reachability condition involving conjugate subsets of states in directed hypergraphs. We show that the ergodicity condition for games only depends on the support of the transition probability and that it can be checked in polynomial time when the number of states is fixed.

Under the above ergodicity condition, the mean payoff of the game is independent of the initial state, and it is characterized as the nonlinear eigenvalue of the Shapley operator. Moreover, the associated eigenvector, also called the bias, allows one to determine optimal stationary strategies. Then, another basic question is to understand for which classes of games the bias vector is unique (up to an additive constant). In [25], we consider games with finite state and action spaces, thinking of the transition payments as variable parameters, transition probabilities being fixed. We show that the bias vector, thought of as a function of the transition payments, is generically unique (up to an additive constant).

6.4. Algèbre max-plus, déformations et asymptotiques /Max-plus algebra, deformations and asymptotic analysis

6.4.1. Introduction

Comme indiqué dans le §3.7 , l'algèbre max-plus est la limite d'une déformation de l'algèbre classique, ou plutôt du semi-corps des réels positifs. Elle peut aussi fournir des estimations de ces déformations, puisque

$$\max(a,b) \leq \epsilon \log\left(e^{a/\epsilon} + e^{b/\epsilon}\right) \leq \epsilon \log(2) + \max(a,b) \ . \tag{5}$$

L'utilisation de ces propriétés a déjà conduit dans le passé aux travaux sur les perturbations de valeurs propres [57], [56], [55], ou sur les grandes déviations [1], [61]. Dans les travaux qui suivent, nous exploitons ces propriétés dans des contextes reliés ou similaires à ceux de nos travaux précédents.

English version

As detailled in §3.7 , max-plus algebra is the limit of a deformation of classical algebra, or more precisely of the semi-field of usual real positive numbers. It can also give estimations for these deformations using for instance (12). By using these properties, we already obtained some works on singular perturbations of matrix eigenvalues [57], [56], [55], or on large deviations [1], [61]. In the works described below, we are exploiting again these properties in contexts that are related or similar to those of our earlier works.

6.4.2. *Méthodes tropicales de localisation de valeurs propres de matrices/Tropical methods for the localisation of matrix eigenvalues*

Participants: Marianne Akian, Stéphane Gaubert, Andrea Marchesini.

Dans un travail avec Meisam Sharify [63], on a comparé les modules des valeurs propres d'un polynôme matriciel au moyen des racines tropicales du polynôme obtenu en appliquant une norme donnée aux coefficients. En particulier, on a obtenu des inégalités de type majorisation qui généralisent les bornes obtenues par Polya et Ostrowski dans le cas de polynômes scalaires.

Dans [12], on montre des inégalités de type majorisation entre les modules des valeurs propres d'une matrice et les valeurs propres tropicales de la matrice de ses modules. En particulier, les majorations généralisent l'inégalité de Friedland [119] concernant le rayon spectral.

Nous avons amélioré et généralisé ces inégalités [37], en appliquant différents changements de variables diagonaux à la matrice complexe initiale, lesquels sont construits à partir des variables duales du problème d'affectation optimale paramètrique construit à partir d'une matrice tropicale associée à la matrice complexe. En particulier, lorsqu'on les applique à une matrice companion par blocs, ces inégalités sont similaires à celles de [63].

English version

In a work with Meisam Sharify [63], we compared the moduli of the eigenvalues of a matrix polynomial to the tropical roots of a polynomial obtained by applying a norm to the coefficients of the original matrix polynomial. In particular, we obtained majorization type inequalities which generalize the bounds of Polya and Ostrowski available for scalar polynomials.

In [12], we show majorization type inequalities between the moduli of the eigenvalues of a complex matrix and the tropical eigenvalues of the matrix obtained by applying the modulus entrywise. In particular, the upper bounds generalize the inequality of Friedland [119] concerning the spectral radius. The above inequalities were obtained by using the permanental and tropical analogues of the exterior power of a matrix and by showing (combinatorially) properties of their eigenvalues similar to the ones of usual exterior powers.

We recently improved and generalized these inequalities, see [37], by applying to the original complex matrix, different diagonal scalings constructed from the dual variables of the parametric optimal assignment constructed from an associated tropical matrix. In particular, when applied to a block companion matrix, our inequalities are similar to the ones in [63].

6.4.3. *Méthodes tropicales pour le calcul numérique de valeurs propres de matrices/Tropical methods for the numerical computation of matrix eigenvalues*

Participants: Marianne Akian, Stéphane Gaubert, Andrea Marchesini.

Un des buts de la thèse d'Andrea Marchesini est d'utiliser les résultats de localisation de valeurs propres tels que ceux obtenus ci-dessus pour améliorer la précision des algorithmes de calcul numérique de valeurs propres de matrices ou de polynômes matriciels, en particulier en construisant des changements d'échelle exploitant les calculs tropicaux, à effectuer préalablement à l'appel d'algorithmes classiques comme QZ. Le "changement d'échelle tropical" introduit par Stéphane Gaubert et Meisam Sharify [127] dans le cas de polynôme matriciels quadratiques consiste en un changement de variable multiplicatif de la variable scalaire du polynôme matriciel. Dans un travail en collaboration avec Françoise Tisseur et James Hook de l'Université de Manchester [36], on considère aussi un changement de variables diagonal du polynôme matriciel construit à partir des variables duales du problème d'affectation optimale paramètrique construit dans l'esprit de [55]. On montre l'intérêt de ces changements d'échelle en terme de conditionnement des valeurs propres, et la supériorité du changement de variables diagonal par rapport au changement d'échelle tropical.

English version

One of goals of the PhD thesis of Andrea Marchesini is to use results on the localisation of eigenvalues like the above ones, to improve the accuracy of the numerical compution of the eigenvalues of a complex matrix or matrix polynomial, in particular by applying scaling methods using tropical techniques, which may be used before calling usual algorithms as QZ. The "tropical scaling" introduced by Stéphane Gaubert and Meisam Sharify [127] in the case of quadratic matrix polynomials consists in a multiplicative scaling of the scalar variable of the matrix polynomial. In a work with Françoise Tisseur and James Hook from Manchester University [36], we also consider a diagonal scaling of the matrix polynomial constructed from the dual variables of the parametric optimal assignment constructed in the same spirit as in [55]. We show the interest of these scaling methods on the eigenvalue conditioning, and the superiority of the diagonal scaling with respect to the tropical scaling.

6.4.4. *Tropicalisation du chemin central, et application à la courbure/Tropicalization of the central path and application to the curvature*

Participants: Xavier Allamigeon, Pascal Benchimol, Stéphane Gaubert, Michael Joswig [TU Berlin].

En optimisation, une classe importante d'algorithmes, dits *de points intérieurs*, consiste à suivre une courbe appelée *chemin central* jusqu'à atteindre la solution optimale. Le chemin central d'un programme linéaire $\mathrm{LP}(A,b,c) \equiv \min\{c\cdot x \mid Ax \leq b,\ x \geq 0\}$ est défini comme l'ensemble des solutions optimales (x^μ, w^μ) des problèmes à barrière logarithmique:

$$\begin{aligned} \text{minimiser} \quad & c\cdot x - \mu(\sum_{j=1}^{n} \log x_j + \sum_{i=1}^{m} \log w_i) \\ \text{sous les contraintes} \quad & Ax + w = b,\ x > 0,\ w > 0 \end{aligned}$$

Les performances d'un algorithme de point intérieur sont intimement liées à la forme du chemin central. En particulier, la courbure mesure de combien un chemin diffère d'une ligne droite. Intuitivement, un chemin central à forte courbure devrait être plus difficile à approximer par des segments de droites, ce qui suggère davantage d'itérations des algorithmes de points intérieurs. La courbure totale du chemin central a été étudiée par Dedieu, Malajovich et Shub [105] à travers le théorème de Bezout dans le cas multihomogène, et par De Loera, Sturmfels and Vinzant [104] à l'aide de la théorie des matroïdes. Ces deux travaux fournissent une borne supérieure en $O(n)$ sur la courbure totale moyenne sur l'ensemble des régions formées par l'arrangement d'hyperplans en dimension n. Le cube de Klee-Minty redondant de [111] et le "serpent" de [110] sont des instances qui montrent que la courbure totale peut être de l'ordre de $\Omega(m)$ pour un polytope défini par m inégalités.

Dans un travail de X. Allamigeon, P. Benchimol, S. Gaubert, and M. Joswig, nous avons étudié la tropicalisation du chemin central. Le *chemin central tropical* est défini comme la limite logarithmique des chemins centraux d'une famille paramétrique de programmes linéaires $\mathrm{LP}(A(t),b(t),c(t))$, où les entrées $A_{ij}(t)$, $b_i(t)$ and $c_j(t)$ sont des fonctions définissables dans une structure o-minimale appelée *corps de Hardy*.

Une première contribution a été de fournir une caractérisation entièrement géométrique du chemin central tropical. Nous avons montré que le centre analytique est donné par le plus grand élément de l'ensemble des points tropicaux admissibles. De plus, tout point du chemin central tropical coincide avec le plus grand élément de l'ensemble admissible tropical intersecté avec un ensemble de sous-niveau de la fonction de coût tropicale.

Grâce à cette caractérisation, nous avons réfuté l'analogue continu de la conjecture de Hirsch proposé par Deza, Terlaky et Zinchenko. l'analogue continu de la conjecture de Hirsch proposé par Deza, Terlaky et Zinchenko. Ainsi, nous avons construit une famille de programmes linéaires définis par $3r+4$ inequalities in dimension $2r+2$, où le chemin central a une courbure totale en $\Omega(2^r/r)$. Cette famille est obtenue en relevant des programmes linéaires tropicaux introduits par Bezem, Nieuwenhuis et Rodríguez-Carbonell [83] pour montrer qu'un algorithme de Butkovič and Zimmermann [88] a une complexité exponentielle. Leur chemin central tropical a une forme de courbe en escalier avec $\Omega(2^r)$ marches.

Ces résultats sont rassemblés dans le document [50]. Ils ont été présentés à la conférence [41].

English version

In optimization, path-following interior point methods are driven to an optimal solution along a trajectory called the central path. The *central path* of a linear program $\mathrm{LP}(A,b,c) \equiv \min\{c\cdot x \mid Ax \leq b,\ x \geq 0\}$ is defined as the set of the optimal solutions (x^μ, w^μ) of the barrier problems:

$$\begin{array}{ll} \text{minimize} & c\cdot x - \mu(\sum_{j=1}^{n} \log x_j + \sum_{i=1}^{m} \log w_i) \\ \text{subject to} & Ax + w = b,\ x > 0,\ w > 0 \end{array}$$

The performance of an interior point method is tightly linked to the shape of its central path. In particular, the curvature measures how far a path differs from a straight line. Intuitively, a central path with high curvature should be harder to approximate with line segments, and thus this suggests more iterations of the interior point methods. The total curvature of the central path has been studied by Dedieu, Malajovich and Shub [105] via the multihomogeneous Bézout Theorem and by De Loera, Sturmfels and Vinzant [104] using matroid theory. These two papers provide an upper bound of $O(n)$ on the total curvature averaged over all regions of an arrangement of hyperplanes in dimension n. The redundant Klee-Minty cube of [111] and the "snake" in [110] are instances which show that that the total curvature can be in $\Omega(m)$ for a polytope described by m inequalities. By analogy with the classical Hirsch conjecture, Deza, Terlaky and Zichencko [110] conjectured that $O(m)$ is also an upper bound for the total curvature.

In a work of X. Allamigeon, P. Benchimol, S. Gaubert, and M. Joswig, we have studied the tropicalization of the central path. The *tropical central path* is defined as the logarithmic limit of the central paths of a parametric family of linear programs $\mathrm{LP}(A(t),b(t),c(t))$, where the entries $A_{ij}(t)$, $b_i(t)$ and $c_j(t)$ are definable functions in an o-minimal structure called the *Hardy field*.

A first contribution is to provide a purely geometric characterization of the tropical central path. We have shown that the tropical analytic center is the greatest element of the tropical feasible set. Moreover, any point of the tropical central path is the greatest element of the tropical feasible set intersected with a sublevel set of the tropical objective function.

Thanks to this characterization, we disprove the continuous analog of the Hirsch conjecture proposed by Deza, Terlaky and Zinchenko, by constructing a family of linear programs with $3r+4$ inequalities in dimension $2r+2$ where the central path has a total curvature in $\Omega(2^r/r)$. This family arises by lifting tropical linear programs introduced by Bezem, Nieuwenhuis and Rodríguez-Carbonell [83] to show that an algorithm of Butkovič and Zimmermann [88] has exponential running time. The tropical central path looks like a staircase shape with $\Omega(2^r)$ steps.

These results are gathered in the preprint [50]. They have been presented in the conference [41].

6.5. Algorithmes/Algorithms

6.5.1. Itération sur les politiques pour le contrôle stochastique et les jeux répétés à somme nulle/Policy iterations for stochastic control and repeated zero sum games

Participants: Marianne Akian, Stéphane Gaubert.

L'algorithme d'itération sur les politiques est bien connu pour résoudre efficacement les équations de la programmation dynamique associées à des problèmes de contrôle stochastique avec critère à horizon infini (Howard) ou ergodique (Howard, et Denardo et Fox). Il a aussi été développé dans le cas de jeux à deux joueurs et somme nulle actualisés (Denardo) ou ergodiques (Hoffman et Karp).

Des résultats récents de Ye ainsi que Hansen, Miltersen et Zwick montrent que l'algorithme d'itération sur les politiques, restreint à la classe des jeux à somme nulle (à 1 ou 2 joueurs) actualisés de facteur d'actualisation donné, est fortement polynomial. Dans [58], on montre que ceci est le cas aussi pour l'algorithme d'itération sur les politiques pour les jeux à somme nulle et paiement moyen, restreint à la classe des jeux qui ont un temps moyen de retour ou d'arrivée à un état donné borné. La preuve utilise des techniques de théorie de Perron-Frobenius non-linéaire, permettant de ramener le problème à paiement moyen à un problème actualisé (de facteur d'actualisation dépendant de l'état et des actions). La même technique permet aussi de traiter le cas de jeux à somme nulle actualisés dont le facteur d'actualisation peut dépendre de l'état et des actions et prendre éventuellement des valeurs supérieures à 1. Récemment, on a montré que la borne pour le cas des jeux à somme nulle et paiement moyen s'applique aussi au cas des jeux actualisés de facteur d'actualisation constant [31], [32], [45]. Ce dernier résultat est inspiré par des résultats récents de Post et Ye et de Scherrer concernant les algorithmes du simplexe et d'itération sur les politiques pour les problèmes de contrôle optimal (ou jeux à 1 joueur).

English version

Policy iteration is a powerful and well known algorithm to solve the dynamic programming equation associated to stochatic control (one player game) problems with infinite horizon criterion (Howard) or ergodic criterion (Howard and Denardo and Fox). It has also be developped in the case of zero-sum two player games, either in discounted case (Denardo) or the ergodic one (Hoffman et Karp).

Recent results of Ye and Hansen, Miltersen and Zwick show that policy iteration for one or two player (perfect information) zero-sum stochastic games, restricted to instances with a fixed discount rate, is strongly polynomial. In [58], we show that policy iteration for mean-payoff zero-sum stochastic games is also strongly polynomial when restricted to instances with bounded first mean return time to a given state. The proof is based on methods of nonlinear Perron-Frobenius theory, allowing us to reduce the mean-payoff problem to a discounted problem with state dependent discount rate. Our analysis also shows that policy iteration remains strongly polynomial for discounted problems in which the discount rate can be state dependent (and even negative) at certain states, provided that the spectral radii of the nonnegative matrices associated to all strategies are bounded from above by a fixed constant strictly less than 1. Recently, we have proved that the bound for the case of mean-payoff zero-sum stochastic two-player games also holds for discounted games with a constant discount factor [31], [32], [45]. The latter result was inspired by recent results of Post and Ye, and Scherrer, concerning simplex and policy iteration algorithms for Markov decision processes (1 player games).

6.5.2. Algorithmique des polyèdres tropicaux/Algorithmics of tropical polyhedra

Participants: Xavier Allamigeon, Pascal Benchimol, Stéphane Gaubert, Eric Goubault [CEA], Michael Joswig [TU Berlin].

X. Allamigeon, S. Gaubert, et E. Goubault, ont développé dans [70], [72] plusieurs algorithmes permettant de manipuler des polyèdres tropicaux. Ceux-ci correspondent aux travaux décrits dans §6.3.1 . Ils permettent notamment de déterminer les sommets et rayons extrêmes d'un polyèdre tropical défini comme intersection de demi-espaces, ou inversement, de calculer une représentation externe à partir d'un ensemble de générateurs. Ces algorithmes sont implémentés la bibliothèque TPLib (voir §5.3).

Dans un travail de X. Allamigeon, P. Benchimol, S. Gaubert et M. Joswig [51], nous avons défini un analogue tropical de l'algorithme du simplexe qui permet de résoudre les problèmes de *programmation linéaire tropicale*, *i.e.*

$$\begin{aligned} \text{minimiser} \quad & \max_{1\le j\le n} c_j + x_j \\ \text{sous les contraintes} \quad & \max\big(\max_{1\le j\le n}(a_{ij}^+ + x_j), b_i^+\big) \ge \max\big(\max_{1\le j\le n}(a_{ij}^- + x_j), b_i^-\big), \quad i = 1,\cdots,m \\ & x \in \left(\mathbb{R}\cup\{-\infty\}\right)^n \end{aligned} \tag{6}$$

où les entrées du programme $a_{ij}^{\pm}$, $b_i^{\pm}$, c_j sont à valeur dans $\mathbb{R} \cup \{-\infty\}$. Ces problèmes sont intimement liés à la résolution de jeux répétés à somme nulle, puisque résoudre un jeux à paiement moyen déterministe est équivalent à déterminer si un problème de programmation linéaire admet un point réalisable [59].

Comme son homologue usuel, le simplexe tropical pivote entre des points de base (tropicaux), jusqu'à atteindre l'optimum du programme linéaire. La différence fondamentale avec l'algorithme du simplexe classique est que le pivotage est réalisé de manière purement combinatoire, en s'appuyant sur des descriptions locales du polyèdre tropical défini par les contraintes à l'aide d'(hyper)graphes orientés. Ceci nous a permis de prouver que *l'étape de pivotage (incluant le calcul des coûts réduits) a la même complexité en temps que dans l'algorithme classique, i.e.* $O(n(m+n))$. Ceci est d'autant plus inattendu que la structure des arêtes tropicales entre deux points de base sont géométriquement plus complexes (elles sont constituées de plusieurs segments de droite, jusqu'à n).

Le simplexe tropical a la propriété d'être fortement corrélé avec l'algorithme du simplexe classique. Grâce au principe de Tarski, le simplexe usuel peut être transposé tel quel sur des programmes linéaires dont les coefficients en entrée sont non plus des réels, mais sur le corps $\mathbb{R}\{\{t\}\}$ des séries de Puiseux généralisées en une certaine indéterminée t, *i.e.* des objets de la forme :

$$c_{\alpha_1} t^{\alpha_1} + c_{\alpha_2} t^{\alpha_2} + \cdots \tag{7}$$

où les α_i sont des réels, les coefficients c_{α_i} sont des réels non-nuls, et où la séquence des $\alpha_1, \alpha_2, \cdots$ est strictement croissante et soit finie, soit non-bornée. L'opposé du plus petit exposant de la série, $-\alpha_1$, est appelé *valuation* de la série. Un programme linéaire tropical est dit *relevé* en un problème linéaire sur $\mathbb{R}\{\{t\}\}$, si la valuation des coefficients en entrée de ce dernier sont égaux aux coefficients du problème tropical. Dans nos travaux, nous avons établi la correspondance suivante entre le simplexe usuel et le simplexe tropical : *pour tout programme linéaire tropical générique, l'algorithme du simplexe tropical trace l'image par la valuation du chemin sur l'algorithme du simplexe usuel sur n'importe quel relèvement du programme tropical dans* $\mathbb{R}\{\{t\}\}$.

Les résultats présentés ci-dessus sont rassemblés dans l'article [51]. Ils ont fait l'objet de plusieurs présentations en conférence [67], [68][27].

Ces résultats ouvrent la possibilité de relier la complexité du l'algorithme du simplexe usuel avec celles des jeux déterministes. Pour ces derniers, on sait seulement que leur résolution est dans la classe de complexité $\mathsf{NP} \cap \mathsf{coNP}$, et on ignore s'il existe un algorithme de complexité polynomiale. De façon similaire, on ne sait pas caractériser de façon précise la complexité de l'algorithme du simplexe usuel. Celle-ci dépend fortement de la règle de pivotage utilisée, et il existe des problèmes sur lesquelles de nombreuses règles de pivotage ont une complexité exponentielle. L'existence d'une règle de pivotage qui permettrait au simplexe de terminer en temps polynomial sur n'importe quelle instance est encore aujourd'hui une question ouverte.

Dans un deuxième travail, nous avons relié les deux problèmes ouverts précédents, grâce à l'algorithme du simplexe tropical. Nous avons en effet exhibé une classe de règles de pivotage, dites *combinatoires*, et avons montré qu'elles satisfont la propriété suivante : *s'il existe une règle de pivotage combinatoire qui permet de résoudre tout problème de programmation linéaire usuel en temps polynomial, alors on peut résoudre les jeux à paiement moyen en temps (fortement) polynomial.* Le terme *combinatoire* fait référence au fait que la règle est définie en fonction du signe des mineurs de la matrice des coefficients du problème linéaire. Ce résultat est décrit dans l'article [49], et a été présenté dans plusieurs conférences [39], [40].

Enfin, dans un travail de X. Allamigeon, P. Benchimol et S. Gaubert [26], nous avons étendu les résultats aux règles de pivotage *semi-algébriques*, classe incluant la règle dite du *shadow-vertex*. Celle-ci est connue pour avoir fourni plusieurs bornes de complexité moyenne et lisse sur l'algorithme du simplexe. Nous avons donc tropicalisé l'algorithme du simplexe shadow-vertex, et nous avons montré que cet algorithme permet de résoudre les jeux à paiement moyen en temps polynomial en moyenne.

English version

X. Allamigeon, S. Gaubert, and E. Goubault, have developed in [70], [72] algorithms allowing one to manipulate tropical polyhedra. They correspond to the contributions described in §6.3.1 . In particular, they can be used to determine the vertices and extreme rays of a tropical polyhedron defined as the intersection of half-spaces, or inversely, to compute an external description from a set of generators. These algorithms are implemented in the library TPLib (see §5.3).

In an ongoing work of X. Allamigeon, P. Benchimol, S. Gaubert and M. Joswig, we introduced a tropical analogue of the simplex algorithm, allowing one to solve problems of *tropical linear programming*, which are of the form (13), where the coefficients of the program, $a_{ij}^{\pm}$, $b_i^{\pm}$, c_j take their values in the max-plus semiring $\mathbb{R} \cup \{-\infty\}$. These problems are closely related to mean payoff games, as solving a game of this kind is equivalent to determine whether a tropical linear program admits a feasible point [59].

Like the classical simplex algorithm, the tropical simplex algorithm performs pivoting operations between basis points, until it reaches the optimum. The main discrepancy with the classical algorithm is that the pivoting is now a purely combinatorial operation, which is performed by using a local description of the polyhedron by a directed hypergraph. This allowed us to show that *a tropical pivoting step (including computing reduced costs) has the same complexity as in the classical simplex algorithm, i.e.* $O(n(m+n))$. This is all the more surprising as the tropical edge between two given points has a geometrically more complex structure in the tropical case (it is constituted of up to n ordinary line segments).

The tropical simplex algorithm turns out to be closely related to the classical one. Thanks to Tarski's principle, the latter is also valid for linear programs over the field $\mathbb{R}\{\{t\}\}$ of generalized Puiseux series in an indeterminate t. These series are of the form (14), where the α_i are real numbers, the coefficients c_{α_i} are non-zero reals, and the sequence $\alpha_1, \alpha_2, \cdots$ is strictly increasing and either finite or unbounded. The opposite of the smallest exponent of the series, $-\alpha_1$, is called *valuation*. A tropical linear program is said to be *lifted* to a linear program over $\mathbb{R}\{\{t\}\}$ if the valuation of the coefficients of the latter are sent to the coefficients of the former by the valuation. We showed the following relation between the classical simplex algorithm and its tropical analogue: *for all generic tropical linear program, the tropical simplex algorithm computes the image by the valuation of the path of the classical simplex algorithm, applied to any lift in* $\mathbb{R}\{\{t\}\}$ *of the original program*.

These results are gathered in the article [51]. They have been presented in several conferences [67], [68][27].

They allow one to relate the complexity of the classical simplex algorithm with the complexity of mean payoff games. The latter is unsettled, these games are known to be in the class $\mathsf{NP} \cap \mathsf{coNP}$ but it is not known whether they can be solved in polynomial time. Basic complexity issues regarding the classical simplex algorithm are also unsettled: its execution time depends on the pivoting rule, and many pivoting rules have been shown to have exponential worst case behaviors. The existence of a pivoting rule leading the simplex to terminate in polynomial time is still an open question. . In a second work, we related these two open questions, via the tropical simplex algorithm. We identified a class of pivoting rules, which are said to be *combinatorial*, and show that they have the following property: *if there is a combinatorial pivoting rule allowing one to solve every classical linear programming problem in polynomial time, then, mean payoff games can be solved in (strongly) polynomial time.* By *combinatorial*, we mean that the rule depends only of the coefficients of the system through the signs of minors of the coefficients matrix. This result is given in the article [49]. It has been presented to the conferences [39], [40].

Finally, in a work of X. Allamigeon, P. Benchimol and S. Gaubert [26], we extended the latter results to *semi-algebraic* pivoting rules, which include the so-called *shadow-vertex* rule. This rule has been exploited in the literature to establish several average-case and smooth complexity bounds on the simplex algorithm. We tropicalized the shadow-vertex simplex algorithm, and showed that it solves mean payoff games in polynomial time on average.

6.5.3. Problèmes d'accessibilité dans les hypergraphes orientés et leur complexité/Reachability problems in directed hypergraphs and their complexity

Participant: Xavier Allamigeon.

Les hypergraphes orientés sont une généralisation des graphes orientés, dans lesquelles chaque arc relie un ensemble de sommets à un autre. Ils jouent un rôle important dans les travaux récents sur la convexité tropicale (voir §6.3.1), puisqu'ils offrent une représentation naturelle des cônes définis sur le sous-semi-anneau booléen $\mathbb{B} = \{-\infty, 0\}$.

Dans un travail de X. Allamigeon [66], on étudie la complexité de problèmes d'accessibilité sur les hypergraphes orientés. Nous introduisons un algorithme de complexité presque linéaire permettant de déterminer les composantes fortement connexes terminales (qui n'accèdent à aucune autre composante si ce n'est elles-mêmes) d'un hypergraphe.

Nous établissons également une borne inférieure sur-linéaire sur la taille de la réduction transitive de la relation d'accessibilité dans les hypergraphes. Cela indique que la relation d'accessibilité dans les hypergraphes orientés est combinatoirement plus complexe que celle des graphes orientés. Cela suggère aussi que des problèmes comme le calcul des composantes fortement connexes est plus difficile sur les hypergraphes que sur les graphes. Nous mettons d'ailleurs en évidence une réduction en temps linéaire du problème du calcul des ensembles minimaux dans une famille d'ensembles donnée, vers le problème du calcul de toutes les composantes fortement connexes d'un hypergraphe. Le problème du calcul des ensembles minimaux a été largement étudié dans la littérature [166], [185], [184], [167], [168], [169], [115], [80], et aucune algorithme en temps linéaire n'est connu à ce jour.

English version

Directed hypergraphs are a generalization of directed graphs, in which the tail and the head of the arcs are sets of vertices. It appears that they play an important role in the recent works on tropical convexity (see §6.3.1), since they offer a natural representation of cones defined over the boolean sub-semiring $\mathbb{B} = \{-\infty, 0\}$.

In a work of X. Allamigeon [66], we study the complexity of reachability problems on directed hypergraphs. We introduce an almost linear-time algorithm allowing to determine the terminal strongly connected components (a component is said to be *terminal* when no other component is reachable from it).

We also establish a super-linear lower bound over the size of the transitive reduction of the reachability relation in directed hypergraphs. This indicates that the reachability relation is combinatorially more complex in directed hypergraphs than in directed graphs. This also suggests that reachability problems such as computing all strongly connected components are likely to be harder in hypergraphs than in graphs. Besides, we show that the minimal set problem can be reduced in linear time to the problem of computing all strongly connected components in hypergraphs. The former problem consists in finding all minimal sets among a given family of sets. It has been well studied in the literature [166], [185], [184], [167], [168], [169], [115], [80], and no linear time algorithm is known.

6.5.4. Approximation max-plus de fonctions valeurs et équations de Riccati généralisées/Max-plus approximation of value functions and generalized Riccati equations

Participants: Stéphane Gaubert, Zheng Qu.

Les méthodes d'approximation max-plus conduisent à approcher la fonction valeur d'un problème de contrôle ou de jeux par un supremum d'un nombre fini de formes quadratiques, voir notamment [126]. On s'intéresse ici à l'analyse théorique (complexité) ainsi qu'à l'amélioration de ces méthodes. Dans certains cas, ces formes quadratiques sont propagées par des flots d'équations de Riccati généralisées. Afin d'effectuer des analyses d'erreur, on exploite les propriétés de contraction du flot de Riccati pour certaines métriques connues sur le cône des matrices positives, et en particulier pour la métrique de Thompson. Celle-ci n'est rien d'autre que $d_T(A, B) = \| \log \operatorname{spec}(A^{-1}B) \|_\infty$, où spec désigne la suite des valeurs propres d'une matrice, et log s'entend composante par composante.

Ceci nous a amené à étudier le problème général du calcul du taux de contraction d'un flot monotone sur un cône, pour la métrique de Thompson. En effet, les propriétés de contraction de l'équation de Riccati standard sont connues (résultats de Bougerol pour la métrique Riemanienne invariante, et de Wojtowski pour

la métrique de Thompson), mais les techniques de preuve employées dans ce cadre (semigroupes de matrices symplectiques) ne s'étendent pas aux équations généralisées.

On donne dans [16] une formule explicite générale pour le taux de contraction pour la métrique de Thompson d'un flot monotone, faisant seulement intervenir le générateur du flot et sa dérivée. On a notamment appliqué ce résultat à une équation de Riccati généralisée associé à des problèmes de contrôle stochastique avec critère quadratique, dans lesquels la dynamique comporte un terme bilinéaire en le contrôle et le bruit. On a montré dans ce cas que la métrique de Thompson est la seule métrique de Finsler invariante pour laquelle le flot est nonexpansif, et l'on a caractérisé la constante de contraction locale.

Une application de ces résultats de contraction à l'analyse d'une méthode de réduction de la malédiction de la dimension, dûe à McEneaney, a été donnée dans [22].

Une nouvelle méthode numérique maxplus, de nature randomisée, a été introduite dans [30], elle fait apparaître de très fortes accélérations par rapport aux méthodes précédentes.

La question de l'émondage des représentations max-plus a été abordée dans [29], où il est montré qu'une classe de relaxations convexes introduites par Sridharan et al. pour traiter numériquement un problème de contrôle quantique sont en fait exactes (pas de saut de relaxation).

English version

The max-plus methods lead to approach the value function of an optimal control or zero-sum game problem by a supremum of a finite number of quadratic forms, see in particular [126]. We are interested here in the theoretical analysis (complexity) of this class of methods, as well as of their improvement. In certain cases, the quadratic forms are propagated by the flows of generalized Riccati equations. In order to perform an error analysis, we need to use some contraction properties of the Riccati flow, for certain known metrics on the space of positive matrices, like Thompson's metric. The latter is nothing but $d_T(A,B) = \|\log \mathrm{spec}\,(A^{-1}B)\|_\infty$, where spec denotes the sequence of eigenvalues of a matrix, and $\log$ is understood entrywise.

This led us to study the general problem of computing the contraction rate of an order-preserving flow on a cone, with respect to Thompson's metric. Indeed, the contraction properties of the standard Riccati flow are known (theorem of Bougerol for the invariant Riemanian metric, of Wojtowski for the Thompson's metric), but the proof of these properties (based on symplectic semigroups) does not carry over to generalized Riccati equations.

We gave in [16] a general explicit formula for the contraction rate with respect to Thompson's metric of an order-preserving flow, involving only the generator of the flow and its derivative. We applied in particular this result to a generalized Riccati equation, associated to stochastic optimal control problems with a quadratic cost and a bilinear dynamics (presence of a bilinear term between the control and the noise). We showed that in this case, the Thompson's metric is the only invariant Finsler metric in which the generalized Riccati flow is nonexpansive, and we characterized the local contraction rate of this flow.

Z. Qu has applied these results in [22] to the analysis of a method of reduction of the curse of dimensionality, introduced by McEneaney.

A new max-plus numerical method, of a randomized nature, has been introduced in [30]. It shows an important speedup by by comparison with earlier methods.

The question of trimming max-plus representations was dealt with in [29]. It is shown there that a class of convex relaxations introduced Sridharan et al. to solve numerically some quantum control problem is exact.

6.5.5. Approximation probabiliste d'équations d'Hamilton-Jacobi-Bellman et itération sur les politiques

Participants: Marianne Akian, Eric Fodjo.

La thèse d'Eric Fodjo traite de problèmes de contrôle stochastique (de diffusions) avec critère à horizon infini actualisé ou arrêté, ou moyen en temps long, issus en particulier de problèmes de gestion de portefeuille avec coûts de transaction. La programmation dynamique conduit à une équation aux dérivées partielles d'Hamilton-Jacobi-Bellman, sur un espace de dimension au moins égale au nombre d'actifs risqués. La malédiction de la dimension ne permet pas de traiter numériquement ces équations en dimension grande (supérieure à 5). On se propose d'aborder ces problèmes avec des méthodes numériques associant itération sur les politiques, discrétisations probabilistes, et discrétisations max-plus, afin d'essayer de monter plus en dimension. Une autre piste est de remplacer l'itération sur les politiques par une approximation par des problèmes avec commutations optimales. Ces méthodes devraient aussi s'appliquer au cas de problèmes à horizon fini.

English version

The PhD thesis of Eric Fodjo concerns stochastic control problems with long term discounted or stopped payoff, or with mean-payoff in time, obtained in particular in the modelisation of portfolio selection with transaction costs. The dynamic programming method leads to a Hamilton-Jacobi-Bellman partial differential equation, on a space with a dimension at least equal to the number of risky assets. Curse of dimensionality does not allow one to solve numerically these equations for a large dimension (greater to 5). We propose to tackle these problems with numerical methods combining policy iterations, probabilistic discretisations, max-plus discretisations, in order to increase the possible dimension. Another solution is to replace policy iterations by an approximation with optimal switching problems. These methods should also be useful for finite horizon problems.

6.6. Applications

6.6.1. Introduction

Nous présentons maintenant plusieurs travaux de nature appliquée, touchant à des domaines variés, dans lesquels nous exploitons certaines des techniques mathématiques présentées précédemment, et particulièrement celles qui relèvent de la théorie de Perron-Frobenius non-linéaire et de la convexité tropicale. Ces applications utilisent aussi des techniques d'algèbre linéaire ou d'optimisation convexe.

English version

In this section, we describe several applied works in which we use some of the theoretical tools developed by the team, including non-linear Perron-Frobenius theory and tropical convexity. Some of these applications also make an intensive use of linear algebraic and convex programming methods.

6.6.2. Optimisation de la croissance de populations/Optimizing population growth

Participants: Vincent Calvez [ENS Lyon et Inria, NUMED], Pierre Gabriel [UVSQ], Stéphane Gaubert.

On s'intéresse dans [28] à l'optimisation du taux de croissance d'une population, représentée par un système dynamique $\dot{x}(t) = M(t)x(t)$, où la matrice $M(t)$ appartient à un ensemble compact de matrices de Metzler irréductibles. Ceci est motivé par un problème de biologie mathématique (modélisation de processus de croissance-fragmentation et protocole PMCA). Nous montrons que le taux de croissance est donné par la valeur propre non-linéaire d'un analogue max-plus de l'opérateur de Ruelle-Perron-Frobenius, ou de manière équivalente, par la constante ergodique d'une EDP d'Hamilton-Jacobi, dont les solutions et sous-solutions fournissent respectivement des normes de Barabanov et des normes extrémales. Nous exploitons les propriétés de contraction des flots monotones, relativement à la métrique projective de Hilbert, pour démontrer que le vecteur propre non-linéaire, qui correspond à une solution "KAM faible" de l'équation d'Hamilton-Jacobi, a bien une solution. Des exemples en petite dimension sont discutés, montrant en particulier que le contrôle optimal peut produire un cycle limite.

English version

We study in [28] a growth maximization problem for a continuous time positive linear system with switches. More precisely, we consider a dynamical system $\dot{x}(t) = M(t)x(t)$, where the matrix $M(t)$ must be chosen in a compact set of irreducible Metzler matrices. This is motivated by a problem of mathematical biology (modeling growth-fragmentation processes and the PMCA protocol). We show that the growth rate is determined by the non-linear eigenvalue of a max-plus analogue of the Ruelle-Perron-Frobenius operator, or equivalently, by the ergodic constant of a Hamilton-Jacobi (HJ) partial differential equation, the solutions or subsolutions of which yield Barabanov and extremal norms, respectively. We exploit contraction properties of order preserving flows, with respect to Hilbert's projective metric, to show that the non-linear eigenvector of the operator, or the "weak KAM" solution of the HJ equation, does exist. Low dimensional examples are presented, showing that the optimal control can lead to a limit cycle.

6.6.3. Preuve formelle d'inégalités non-linéaires/Formal proofs of non-linear inequalities

Participants: Xavier Allamigeon, Stéphane Gaubert, Victor Magron, Benjamin Werner [LIX].

La thèse de Victor Magron [153], dirigée par Benjamin Werner, codirigée par Stéphane Gaubert et Xavier Allamigeon, a porté sur la certification de bornes inférieures de fonctions multivariées à valeurs réelles, définies par des expressions semi-algébriques ou transcendantes, et sur la preuve de validité de celles-ci au moyen de certificats dans l'assistant de preuves Coq.

De nombreuses inégalités de cette nature apparaissent notamment dans la preuve par Thomas Hales de la conjecture de Kepler. Voici un exemple typique d'inégalité à prouver.

LEMME 9922699028 FLYSPECK. *Soit K, $\Delta\mathbf{x}$, l, t et f définis comme suit:*

$$\begin{aligned}
K &:= [4, 6.3504]^3 \times [6.3504, 8] \times [4, 6.3504]^2 \ , \\
\Delta\mathbf{x} &:= x_1x_4(-x_1+x_2+x_3-x_4+x_5+x_6) \\
&\quad +x_2x_5(x_1-x_2+x_3+x_4-x_5+x_6) \\
&\quad +x_3x_6(x_1+x_2-x_3+x_4+x_5-x_6) \\
&\quad -x_2x_3x_4-x_1x_3x_5-x_1x_2x_6-x_4x_5x_6 \ , \\
l(\mathbf{x}) &:= -\pi/2+1.6294-0.2213(\sqrt{x_2}+\sqrt{x_3}+\sqrt{x_5}+\sqrt{x_6}-8.0) \\
&\quad +0.913(\sqrt{x_4}-2.52)+0.728(\sqrt{x_1}-2.0) \ , \\
t(\mathbf{x}) &:= \arctan \tfrac{\partial_4\Delta\mathbf{x}}{\sqrt{4x_1\Delta\mathbf{x}}} \ , \\
f(\mathbf{x}) &:= l(\mathbf{x})+t(\mathbf{x}) \ .
\end{aligned}$$

Alors, $\forall \mathbf{x} \in K, f(\mathbf{x}) \geq 0$.

On s'est donc intéressé à des fonctions non-linéaires, faisant intervenir des opérations semi-algébriques ainsi que des fonctions transcendantes univariées ($\cos$, $\arctan$, $\exp$, etc).

De manière classique, on peut approcher les fonctions transcendantes qui interviennent de la sorte par des polynômes, ce qui permet de se ramener à des problèmes d'optimisation semi-algébriques, que l'on peut résoudre par des techniques de sommes de carrés creuses conduisant à des problèmes SDP. Cependant, en pratique, cette approche est limitée par la taille des SDP à résoudre, qui croît rapidement avec le degré des approximations polynomiales.

Dans ce travail de thèse, on a développé une méthode alternative, qui consiste a borner certains des constituants de la fonction non-linéaire par des suprema de formes quadratiques dont les Hessiens sont judicieusement choisis. On reprend donc ici l'idée des approximations "max-plus" initialement introduites en contrôle optimal, en s'appuyant sur des techniques d'interprétation abstraite (généralisation non-linéaire de la méthode des gabarits de Manna et al.). Ainsi, on obtient une nouvelle technique d'optimisation globale, basée sur les gabarits, qui exploite à la fois la precision des sommes de carrés et la capacité de passage à l'échelle des méthodes d'abstraction.

L'implémentation de ces méthodes d'approximation a abouti à un outil logiciel : `NLCertify`. Cet outil génère des certificats à partir d'approximations semi-algébriques et de sommes de carrés. Son interface avec Coq permet de bénéficier de l'arithmétique certifiée disponible dans l'assistant de preuves, et ainsi d'obtenir des estimateurs et des bornes valides pour chaque approximation.

Les performances de cet outil de certification ont été démontrées sur divers problèmes d'optimisation globale ainsi que sur des inégalités essentiellement serrées qui interviennent dans la preuve de Hales (projet Flyspeck).

Ce travail est exposé dans [73], [74] et [18], [19].

English version

The PhD work of Victor Magron [153], supervised by Benjamin Werner, and cosupervised by Stéphane Gaubert and Xavier Allamigeon, dealt with the certification of lower bounds for multivariate functions, defined by semi-algebraic or transcendental expressions, and their correctness proof through certificates checked in the Coq proof assistant.

Many inequalities of this kind appear in particular in the proof by Thomas Hales of Kepler's conjecture. Here is a typical example of inequality.

LEMMA 9922699028 FLYSPECK. *Let K, $\Delta\mathbf{x}$, l, t and f be defined as follows:*

$$\begin{aligned}
K &:= [4, 6.3504]^3 \times [6.3504, 8] \times [4, 6.3504]^2 \ , \\
\Delta\mathbf{x} &:= x_1x_4(-x_1+x_2+x_3-x_4+x_5+x_6) \\
&\quad +x_2x_5(x_1-x_2+x_3+x_4-x_5+x_6) \\
&\quad +x_3x_6(x_1+x_2-x_3+x_4+x_5-x_6) \\
&\quad -x_2x_3x_4-x_1x_3x_5-x_1x_2x_6-x_4x_5x_6 \ , \\
l(\mathbf{x}) &:= -\pi/2+1.6294-0.2213(\sqrt{x_2}+\sqrt{x_3}+\sqrt{x_5}+\sqrt{x_6}-8.0) \\
&\quad +0.913(\sqrt{x_4}-2.52)+0.728(\sqrt{x_1}-2.0) \ , \\
t(\mathbf{x}) &:= \arctan \tfrac{\partial_4\Delta\mathbf{x}}{\sqrt{4x_1\Delta\mathbf{x}}} \ , \\
f(\mathbf{x}) &:= l(\mathbf{x})+t(\mathbf{x}) \ .
\end{aligned}$$

Then, $\forall \mathbf{x} \in K, f(\mathbf{x}) \geq 0$.

Thus, we considered non-linear functions, defined in terms of semi-algebraic operations and univariate transcendental functions ($\cos$, $\arctan$, $\exp$, etc).

Such transcendental functions can be classically approximated by polynomials, which leads to semi-algebraic optimization problems, which can be solved by sparse sum of squares techniques leading to SDP formulations. However, in practice, this approach is limited by the growth of the size of the SDP instances to be solved, whichs grows quickly with the degree of polynomial approximations.

In this PhD, we developed an alternative method, which consists in bounding some constituents of the non-linear function to be optimized by suprema of quadratic forms with well chosen Hessians. This is based on the idea of "maxplus approximation" initially introduced in optimal control, and also, on abstract interpretation (the template method introduced by Manna et al. in static analysis). In this way, we end up with a new global optimization technique, which takes advantage of the precision of sum of squares and of the scalability of abstraction methods.

These methods have been implemented in a software tool: `NLCertify`. This tool generates certificates from semi-algebraic and sum of square certificates. Its interface with Coq allows one to take benefit of the certified arithmetics available in this proof assistant, and so, to obtain estimators and valid bounds for each approximation.

The performances of this certification tool have been shown on several global optimization problems from the literature, as well as on essentially tight inequalities taken from Hales' proof (Flyspeck project).

This work is presented in [73], [74] and [18], [19].

6.6.4. Vérification de systèmes temps-réels/Verification of real-time systems

Participants: Xavier Allamigeon, Uli Fahrenberg [IRISA], Stéphane Gaubert, Ricardo Katz [Conicet], Axel Legay [IRISA].

Dans [152], Lu, Madsen, Milata, Ravn, Fahrenberg et Larsen ont montré que les polyèdres tropicaux peuvent être utilisés dans le cadre de l'analyse d'accessibilité d'automates temporisés. En effet, les polyèdres tropicaux expriment naturellement des invariants non-convexes, qui sont en fait des disjonctions d'invariants fournis par des DBM (*difference bound matrices*). A ce titre, les polyèdres tropicaux devraient permettre de réduire le nombre de disjonctions réalisées pendant l'analyse d'automates temporisés. Une limitation importante de cette approche est cependant que les polyèdres tropicaux sont topologiquement fermés, et qu'ils ne peuvent donc pas exprimer de contraintes d'inégalités strictes. Ces dernières sont néanmoins fondamentales dans l'analyse de systèmes temps-réels.

Nous avons donc développé dans [69] une généralisation des polyèdres tropicaux permettant d'exprimer des contraintes mixtes, *i.e.* strictes ou larges. Notre approche repose sur l'utilisation d'inégalités tropicales linéaires à coefficients dans un (quotient du) semi-anneau de germes affines. Afin de réaliser des opérations sur cette nouvelle classe de polyèdres tropicaux, nous avons défini deux nouveaux algorithmes. Le premier est un analogue tropical de l'élimination de Fourier-Motzkin. Celle-ci s'applique plus généralement à des systèmes d'inégalités linéaires sur des semi-anneaux idempotents et totalement ordonnés. Le second algorithme permet de tester si un système de contraintes mixtes admet une solution. Nous montrons en effet que ce problème est équivalent en temps polynomial à la résolution d'un problème de jeux déterministes à somme nulle. Ces deux contributions nous permettent de définir les primitives requises pour l'analyse d'accessibilité d'automates temporisés.

English version

Lu, Madsen, Milata, Ravn, Fahrenberg and Larsen have shown in [152] that tropical polyhedra can be applied to the reachability analysis of timed automata. Indeed, tropical polyhedra naturally express non-convex invariants, which correspond to disjunctions of invariants provided by DBM (*difference bound matrices*). Consequently, tropical polyhedra should allow to reduce the number of disjunctions arising during the analysis of timed automata. An important limitation of this approach is that tropical polyhedra are topologically closed, and thus they cannot express strict inequality constraints. However, such constraints plays an important role in the analysis of real-time systems.

As a result, we have developed in [69] a generalization of tropical polyhedra, in order to express mixed constraints, *i.e.* strict or loose ones. Our approach relies on tropical linear inequalities with coefficients in a (quotient of) the semiring of affine germs. In order to perform operations on this new class of polyhedra, we have introduced two new algorithms. The first one is a tropical analog of Fourier-Moztkin elimination. In fact, it applies more generally to systems of linear inequalities over totally ordered and idempotent semirings. The second algorithm allows to test the feasability of a mixed constraint system. We indeed show that this problem is polynomial-time equivalent to solving mean payoff games. These two contributions allow to define the primitives required by the reachability analysis of timed automata.

6.6.5. Géométrie de l'ordre de Loewner et application au calcul d'invariants quadratiques en analyse statique de programme/Geometry of the Loewner order and appliction to the synthesis of quadratic invariants in static analysis of program

Participants: Xavier Allamigeon, Stéphane Gaubert, Éric Goubault [LIX], Sylvie Putot [LIX], Nikolas Stott.

Le stage de recherche de l'École des Mines de Nikolas Stott a porté sur la caractérisation de l'ensemble des majorants minimaux de deux matrices symmétriques, relativement à l'ordre de Loewner, et sur l'application de cette caractérisation à la synthèse d'invariants quadratiques en analyse statique de programme.

English version

The research internship of "École des Mines" made by Nikolas Stott dealt with the characterization of the set of minimal upper bounds of two matrices with respect to Loewner order, motivated by the generation of quadratic invariants in static analysis of programs.

6.6.6. Optimisation de l'affectation temps réel des moyens de secours des pompiers/Optimization of the real time assignment of firemen vehicles

Participants: Marianne Akian, Xavier Allamigeon, Vianney Boeuf, Stéphane Gaubert, Stéphane Raclot [BSPP].

La thèse de Vianney Boeuf, qui a démarré en Septembre, est effectuée en partenariat avec la Brigade des Sapeurs Pompiers de Paris (BSPP). Elle est motivée par l'optimisation des moyens de secours, en incluant les questions de dimensionnement et d'affectation temps réel des moyens. On s'intéresse en particulier à l'affectation des engins et véhicules de secours, éventuellement empruntés à différentes casernes. Ce travail intervient en complément du travail de l'équipe au sein du projet ANR Democrite, qui porte sur l'évaluation du risque en milieu urbain.

English version

The PhD work of Vianney Boeuf started in September. It is carried out with the Brigade of Paris Firemen (BSPP). It is motivated by the issue of optimization of emergency resources, including the real time dynamic assignment of engines or emergency vehicles. This work is carried out in complement to the ANR project Democrite, dealing with risk evaluation in urban environment.

POEMS Project-Team

6. New Results

6.1. Wave propagation in non classical media

6.1.1. Plasmonic black-hole waves at corners of metals

Participants: Anne-Sophie Bonnet-Ben Dhia, Camille Carvalho, Patrick Ciarlet.

This work, which is a part of the PhD of Camille Carvalho, is done in collaboration with Lucas Chesnel from CMAP at Ecole Polytechnique. We study the scattering of time-harmonic electromagnetic waves by a metallic obstacle in a 2D setting, at frequencies such that the dielectric permittivity of the metal has a negative real part and a small imaginary part. When the obstacle has corners, due to the sign-changing real part of the permittivity, unusual strong singularities for the electromagnetic field can appear. If the material dissipation is neglected, it can be necessary to consider a new functional framework, containing these singularities, to derive a well-posed problem. In this new framework, everything happens like if plasmonic waves were propagating to the corners, and a part of the energy is trapped by the corner, even if the material has been supposed non-dissipative. We have implemented an original numerical method consisting in using Perfectly Matched Layers at the corners to capture these black-hole waves. We have also proposed a new rule to mesh the corner in order to achieve convergence of classical finite elements in the simpler case where the problem is still well-posed in the classical framework. Finally, in collaboration with André Nicolet and Frédéric Zolla from Institut Fresnel in Marseille, we are now considering realistic dissipative metals. We show that there is still a significant effect of the black-hole phenomenon, which results in an unsual energy leakage in some frequency range.

6.1.2. Limiting amplitude principle for a two-layered dielectric/metamaterial medium

Participants: Maxence Cassier, Christophe Hazard, Patrick Joly.

This work has been a part of the PhD of Maxence Cassier and has allowed to initiate a collaboration with Boris Gralak from Institut Fresnel. For wave propagation phenomena, the limiting amplitude principle holds if the time-harmonic regime represents the large time asymptotic behavior of the solution of the evolution problem with a time-harmonic excitation. Considering a two-layered medium composed of a dielectric material and a Drude metamaterial separated by a plane interface, we prove that the limiting amplitude principle holds except for a critical situation related to a surface resonance phenomenon. Then the solution can either converge to the superposition of two time-periodic fields, or blow up linearly in time.

6.1.3. Perfectly Matched Layers in plasmas and metamaterials

Participants: Eliane Bécache, Patrick Joly, Maryna Kachanovska, Valentin Vinoles.

This work is a part of the PhD of Valentin Vinoles and is the subject of the post-doc of Maryna Kachanovska. It deals with the stability of Perfectly Matched Layers (PMLs) in dispersive media and is motivated by the fact that classical PMLs are unstable in negative index metamaterials and in some anisotropic plasmas. This led us to derive a new necessary criterion of stability which is valid for a large class of dispersive models and for more general PMLs than the classical ones. This criterion has been used to design new stable PMLs for negative index metamaterials and uniaxial anisotropic plasmas.

6.1.4. Retrieval method for anisotropic metamaterials

Participants: Aurore Castanié, Jean-François Mercier.

This work has been done during the post-doc of Aurore Castanié, in collaboration with Agnès Maurel from Institut Langevin at ESPCI and Simon Felix from the LAUM (Laboratoire d'Acoustique de l'Université du Maine). Electromagnetic or acoustic metamaterials can be described in terms of equivalent effective, in general anisotropic, media and several techniques exist to determine the effective permeability and permittivity (or effective mass density and bulk modulus in the context of acoustics). Among these techniques, retrieval methods use the measured scattering coefficients for waves incident on a metamaterial slab containing few unit cells. Until now, anisotropic effective slabs have been considered in the literature but they are limited to the case where one of the axes of anisotropy is aligned with the slab interface. We propose an extension to arbitrary orientations of the principal axes of anisotropy and oblique incidence. The retrieval method is illustrated in the electromagnetic case for layered media, and in the acoustic case for array of tilted elliptical particles.

6.2. Wave propagation in heterogeneous media

6.2.1. High order transmission conditions between homogeneous and homogenized periodic half-spaces

Participants: Sonia Fliss, Valentin Vinoles.

This work is a part of the PhD of Valentin Vinoles, and is done in collaboration with Xavier Claeys (LJLL, Paris VI). It is motivated by the fact that classical homogenization theory poorly takes into account interfaces, which is particularly unfortunate when considering negative materials, because important phenomena arise precisely at their surface (plasmonic waves for instance). To overcome this limitation, we want to construct high order transmission conditions. Using matched asymptotics, we have treated the case of a plane interface between a homogeneous and a homogenized periodic half space. The analysis is based on an original combination of Floquet-Bloch transform and a periodic version of Kondratiev techniques. The obtained conditions involve Laplace- Beltrami operators at the interface and requires to solve cell problems in infinite strips.

6.2.2. Multiple scattering by small homogeneities

Participants: Patrick Joly, Simon Marmorat.

This is the topic of the PhD of Simon Marmorat, done in collaboration with the CEA-LIST and with Xavier Claeys (LJLL, Paris VI). We aim at developing an efficient numerical approach to simulate the propagation of waves in concrete, which is modelled as a smooth background medium, with many small embedded heterogeneities. This kind of problem is very costly to handle with classical numerical methods, due the refined meshes needed around the inclusions. To overcome these issues, two models have been developed, which rely on the asymptotic analysis of the problem: each of them can be interpreted as a full space wave equation, which can be discretized using a defects-free mesh, coupled to some auxiliary unknowns accounting for the presence of the inclusions. While the first model is established by using a special Galerkin approximation in the vicinity of the inclusions, the second model only focuses on the far field. The challenge is then to simulate source points coupled to the incident field and this is achieved thanks to the introduction of a special relaxed version of the Dirac mass. Rigorous error estimates as well as some numerical tests have been established, highlighting the efficiency of the two methods.

6.2.3. Finite Element Heterogeneous Multiscale Method for Maxwell's Equations

Participants: Patrick Ciarlet, Sonia Fliss, Christian Stohrer.

This work is the subject of the post-doc of Christian Stohrer. The standard Finite Element Heterogeneous Multiscale Method (FE-HMM) can be used to approximate the effective behavior of solutions to the classical Helmholtz equation in highly oscillatory media. Using a novel combination of well-known results about FE-HMM and the notion of T-coercivity, we derive an a priori error bound. Numerical experiments corroborate the analytical findings. We work now on the application of HMM in presence of interfaces, for Maxwell's equations and finally in presence of high contrast materials.

6.2.4. Effective boundary conditions for strongly heterogeneous thin layers

Participants: Matthieu Chamaillard, Patrick Joly.

This topic is the object of the PhD of Matthieu Chamaillard, done in collaboration with Houssem Haddar (CMAP École Polytechnique). We are interested in the construction of effective boundary conditions for the diffraction of waves by an obstacle covered with a thin coating whose physical characteristics vary "periodically". The width of the coating and the period are both proportional to the same small parameter δ. In the scalar case, we proved that the error between the exact model (with the thin coat) and the one with the effective boundary condition of order n for $n \in \{1, 2\}$ is of the order $\mathcal{O}(\delta^{n+1})$. This has been checked numerically for some two dimensional configurations. Recently, we also succeeded to extend our theoretical work to Maxwell equations. We found a first order boundary condition of the form $E \times n = \delta ik \mathcal{Z}_\Gamma (n \times (H \times n))$ where n is the unit outward normal to the boundary Γ and $\mathcal{Z}_\Gamma$ is a second order tangential differential operator along Γ. The coefficients of this operator depend only on the deformation mapping ψ_Γ and the material properties of the coating, through the resolution of particular unbounded cell problems in the flat reference configuration. When the coating is homogeneous, one recovers the well known first order thin layer condition. We have moreover proven that this effective condition provides an error of the order $\mathcal{O}(\delta^2)$.

6.3. Spectral theory and modal approaches for waveguides

6.3.1. Guided modes in ladder-like open periodic waveguides

Participants: Sonia Fliss, Patrick Joly, Elizaveta Vasilevskaya.

This work is done in the context of the PhD of Elizaveta Vasilevskaya, in collaboration with Bérangère Delourme, from Paris 13 University. We consider the theoretical and numerical aspects of the wave propagation in ladder-like periodic structures. We exhibit situations where the introduction of a lineic defect into the geometry of the domain leads to the appearance of guided modes and we provide numerical simulations to illustrate the results. From the theoretical point of view, the problem is studied by asymptotic analysis methods, the small parameter being the thickness of the domain, so that when the thickness of the structure is small enough, the domain approaches a graph. Numerical computations are based on specific transparent conditions for periodic media.

6.3.2. Absence of trapped modes for a class of unbounded propagative media

Participants: Anne-Sophie Bonnet-Ben Dhia, Christophe Hazard, Sonia Fliss, Antoine Tonnoir.

We have proposed a new approach to prove that there does not exist square-integrable solutions to the two-dimensional Helmholtz equation in a homogeneous conical domain with a vertex angle greater than π. This shows that for a medium filling the whole plane, there can be no trapped modes if all the inhomogeneities (penetrable or not) are concentrated in a conical domain with a vertex angle less than π. The proof uses the compatibility of Fourier representations of the field in different half-spaces. One interesting consequence of our result concerns the case of curved open waveguides (e.g., bended optical fibers). Unlike closed waveguides for which trapped modes confined near the bend may occur, our result implies that trapped modes cannot exist if the core of the waveguide is located in a cone with vertex angle less than π. Our results can be extended to higher space dimensions, and to some Y-junctions of open waveguides (using a generalized Fourier transform instead of the usual one).

6.3.3. Reduced graph models for networks of thin co-axial electromagnetic cables

Participants: Geoffrey Beck, Patrick Joly.

This work is the object of the PhD of Geoffrey Beck and is done collaboration with Sébastien Imperiale (Inria, MEDISIM). The general context is the non destructive testing by reflectometry of electric networks of co-axial cables with heterogeneous cross section and lossy materials, which is the subject of the ANR project SODDA. We consider electromagnetic wave propagation in a network of thin coaxial cables (made of a dielectric material which surrounds a metallic inner-wire). The goal is to reduce $3D$ Maxwell's equations to a quantum graph in which, along each edge, one is reduced to compute the electrical potential and current by solving 1D wave equations (the telegrapher's model) coupled by vertex conditions. Using the method of matched asymptotics, we have derived and justified improved Kirchhoff conditions.

6.3.4. Geometrical transformations for waveguides of complex shapes

Participant: Jean-François Mercier.

In collaboration with Agnès Maurel from the Langevin Institut and Simon Felix from the LAUM, we have developed multimodal methods to describe the acoustic propagation in rigid waveguides of general shapes, with varying curvature and cross section. A key feature is the use of a flexible geometrical transformation to a virtual space in which the waveguide is straight but associated to Robin boundary conditions. We have revisited an efficient method developed earlier which consists in adding two extra non-physical modes to the usual modal expansion of the field on the Neumann guided modes, in order to obtain a better convergence of the modal series.

This method has been extended to a half guide with an end wall of general shape, transformed into a flat surface by a geometrical transformation, thus avoiding to question the Rayleigh hypothesis. The transformation only affects a bounded inner region that naturally matches the outer region, which allows to easily select the ingoing and outgoing waves.

6.4. Inverse problems

6.4.1. Quasi-Reversibility method and exterior approach for evolution problems

Participants: Eliane Bécache, Laurent Bourgeois.

This work is a collaboration with Jérémi Dardé from Toulouse University and has been the object of the internship of Lucas Franceschini, student at ENSTA. We address some linear ill-posed problems involving the heat or the wave equation, in particular the backward heat equation and the heat/wave equation with lateral Cauchy data. The main objective is to introduce some variational mixed formulations of quasi-reversibility which enable us to solve these ill-posed problems by using classical Lagrange finite elements. We have also designed a new approach called the "exterior approach" to solve inverse obstacle problems with initial condition and lateral Cauchy data for heat/wave equation. It is based on a combination of an elementary level set method and the quasi-reversibility methods we have just mentioned. Some numerical experiments have proved the feasibility of our strategy in all those situations.

6.4.2. Uniqueness and non-uniqueness results for the inverse Robin problem

Participant: Laurent Bourgeois.

This work is a collaboration with Laurent Baratchart and Juliette Leblond (Inria, APICS). We consider the classical Robin inverse problem, which consists in finding the ratio between the normal derivative and the trace of the solution (the Robin coefficient) on a subset of the boundary, given the Cauchy data (both the normal derivative and the trace of the solution) on the complementary subset. More specifically, we consider a Robin coefficient which is merely in L^∞ and a Neumann data in L^2. In the $2D$ case we prove uniqueness of the Robin coefficient for a problem governed in a Lipschitz domain by a conductivity equation with a conductivity chosen in $W^{1,r}$, where $r > 2$. We also prove a non-uniqueness result in the $3D$ case. In two dimensions, the proof relies on complex analysis, while in higher dimension, the proof relies on a famous counterexample to unique continuation by Bourgain and Wolff.

6.4.3. Higher-order expansion of misfit functional for defect identification in elastic solids

Participants: Marc Bonnet, Rémi Cornaggia.

This work, done in the context of the PhD of Rémi Cornaggia, concerns the defect identification by time-harmonic elastodynamic measurements. We propose a generalization to higher orders of the concept of topological derivative, by expanding the least-squares functional in powers of the small radius of a trial inclusion. This expansion is facilitated by resorting to an adjoint state. With this approach, a region of interest may be exhaustively probed at reasonable computational cost.

6.4.4. Inverse scattering and invisibility with a finite set of emitted-received waves

Participant: Anne-Sophie Bonnet-Ben Dhia.

In collaboration with Lucas Chesnel from CMAP at Ecole Polytechnique and Sergei Nazarov from Saint-Petersburg University, we investigate a time harmonic acoustic scattering problem by a compactly supported penetrable inclusion in the free space. We consider cases where an observer can produce incident plane waves and measure the far field pattern of the resulting scattered field only in a finite set of directions. In this context, we say that a wavenumber is a non-scattering wavenumber if the associated relative scattering matrix has a non trivial kernel. Under certain assumptions on the physical coefficients of the inclusion, we have shown that the non-scattering wavenumbers form a (possibly empty) discrete set. Then, for a given real wavenumber, we built a constructive technique (which provides a numerical algorithm) to prove that there exist inclusions for which the corresponding relative scattering matrix is null. These inclusions have the important property to be impossible to detect from far field measurements.

6.4.5. Energy-based cost functional for three-dimensional transient elastodynamic imaging

Participant: Marc Bonnet.

This work is a collaboration with Wilkins Aquino (Duke University, USA). It is concerned with large-scale three-dimensional inversion under transient elastodynamic conditions by means of the modified error in constitutive relation (MECR), an energy-based, cost functional. Each evaluation of a time-domain MECR cost functional involves the solution of two elastodynamic problems (one forward, one backward), which moreover are coupled (unlike the case of L^2 misfit functionals). This coupling creates a major computational bottleneck, making MECR-based inversion difficult for spatially 2D or 3D configurations. To overcome this obstacle, we propose an approach whose main ingredients are (a) setting the entire computational procedure in a consistent time-discrete framework that incorporates the chosen time-stepping algorithm, and (b) using an iterative successive over-relaxation-like method for the resulting stationarity equations. The resulting MECR-based inversion algorithm is formulated under quite general conditions, allowing for 3D transient elastodynamics, straightforward use of available parallel solvers, a wide array of time-stepping algorithms commonly used for transient structural dynamics, and flexible boundary conditions and measurement settings. The proposed MECR algorithm is then demonstrated on computational experiments involving 2D and 3D transient elastodynamics and up to over 500 000 unknown elastic moduli.

6.5. Integral equations

6.5.1. Fast solution of the BEM system in 3-D frequency-domain elastodynamics

Participants: Stéphanie Chaillat, Patrick Ciarlet, Luca Desiderio.

The main advantage of the Boundary Element Method (BEM) is that only the domain boundaries are discretized leading to a drastic reduction of the total number of degrees of freedom. In traditional BE implementation the dimensional advantage with respect to domain discretization methods is offset by the fully-populated nature of the BEM coefficient matrix. Using the $\mathcal{H}$-matrix arithmetic and low-rank approximations (performed with Adaptive Cross Approximation), we derive a fast direct solver for the BEM system in 3-D frequency-domain elastodynamics. We assess the numerical efficiency and accuracy on the basis of numerical results obtained for problems having known solutions. In particular, we study the efficiency of low-rank approximations when the frequency is increased. The efficiency of the method is also illustrated to study seismic wave propagation in 3-D domains. This is done in partnership with SHELL company in the framework of the PhD of Luca Desiderio.

6.5.2. OSRC preconditioner for 3D elastodynamics

Participant: Stéphanie Chaillat.

This work is done in collaboration with Marion Darbas from University of Picardie and Frédérique Le Louer from Technological University of Compiègne. The fast multipole accelerated boundary element method (FM-BEM) is a possible approach to deal with scattering problems of time-harmonic elastic waves by a three-dimensional rigid obstacle. In 3D elastodynamics, the FM-BEM has been shown to be efficient with solution times of order $O(N \log N)$ per iteration (where N is the number of BE degrees of freedom). However, the number of iterations in GMRES can significantly hinder the overall efficiency of the FM-BEM. To reduce the number of iterations, we propose a clever integral representation of the scattered field which naturally incorporates a regularizing operator. When considering Dirichlet boundary value problems, the regularizing operator is a high-frequency approximation to the Dirichlet-to-Neumann operator, and is constructed in the framework of the On-Surface Radiation Condition (OSRC) method. This OSRC-like preconditioner is successfully applied to Dirichlet exterior problems in 3D elastodynamics.

6.5.3. Boundary Integral Formulations for Modeling Eddy Current Testing

Participants: Marc Bonnet, Audrey Vigneron.

This work was a part of the PhD thesis of Audrey Vigneron, and has been done in collaboration with Edouard Demaldent from CEA-List. It concerns the simulation of eddy current non-destructive testing, which aims to assess the presence of defects (cut, corrosion ...) in a conductive, and possibly magnetic, medium. We propose a simple block-SOR solution method for the PMCHWT-type Maxwell integral formulation, that is well suited for the low-frequency, high-conductivity limit typical of eddy current testing methods. We also derive an asymptotic expansion of the Maxwell integral formulation in powers of some relevant (small) non-dimensional number and show its relation to Hiptmair's eddy current integral formulation. Both aspects are validated on 3D numerical experiments.

6.6. Domain decomposition methods

6.6.1. Transparent boundary conditions with overlap in elastic waveguides

Participants: Anne-Sophie Bonnet-Ben Dhia, Sonia Fliss, Antoine Tonnoir.

This work is a part of the PhD of Antoine Tonnoir and is done in partnership with Vahan Baronian form CEA-LIST. We have conceived new transparent boundary conditions for the time-harmonic diffraction problem in an acoustic or elastic waveguide. These new conditions use the natural modal decomposition in the waveguide and are said "with overlap" by analogy with the domain decomposition methods. Among their main advantages, they can be implemented in general elastic anisotropic waveguides, for which usual Dirichlet to Neumann maps are not available. Moreover, the traditional benefit of the overlap for iterative resolution is obtained, independently of the size of the overlap.

6.6.2. Electromagnetic scattering by objects with multi-layered dielectric coatings

Participants: Patrick Joly, Matthieu Lecouvez.

This is the object of the PhD thesis of Matthieu Lecouvez in collaboration with the CEA-CESTA and Francis Collino. We are interested in the diffraction of time harmonic electromagnetic waves by perfectly conducting objects covered by multi-layered (possibly thin) dielectric coatings. This problem is computationally hard when the size of the object is large (typically 100 times larger) with respect to the incident wavelength. In such a situation, the idea is to use a domain decomposition method in which each layer would constitute a subdomain. The transmission conditions between the subdomains involve some specific impedance operators in order to achieve a geometric convergence of the method (compared to the slow algebraic convergence obtained with standard Robin conditions). We propose a practical solution that uses approximations of nonlocal integral operators with appropriate Riesz potentials.

6.6.3. Domain Decomposition Methods for the neutron diffusion equation

Participants: Patrick Ciarlet, Léandre Giret.

Studying numerically the steady state of a nuclear core reactor is expensive, in terms of memory storage and computational time. In particular, one must solve the neutron diffusion equation discretized by finite element techniques, totaling millions of unknowns or more, within a loop. Iterating in this loop allows to compute the smallest eigenvalue of the system, which determines the critical, or non-critical, state of the 3D core configuration. This problem fits within the framework of high performance computing so, in order both to optimize the memory storage and to reduce the computational time, one can use a domain decomposition method, which is then implemented on a parallel computer. The definition of an efficent DDM has been recently addressed for conforming meshes. The development of non-conforming, hence more flexible, methods is under way. Since one is dealing with highly heterogeneous configurations, the regularity of the exact solution can be very low, which then deteriorates the convergence rate of the discretized solution to the exact one. Next, the optimization of the eigenvalue loop will be studied.
This topic is developed in partnership with CEA-DEN (Erell Jamelot). Realistic computations are carried out with the APOLLO3 neutronics code.

6.7. Aeroacoustics

6.7.1. Time-harmonic acoustic scattering in a rotationnal flow

Participants: Antoine Bensalah, Patrick Joly, Jean-François Mercier.

This activity is done in the framework of the PhD of Antoine Bensalah, in partnership with EADS. We study the time-harmonic acoustic radiation in a fluid in a general flow which is not curl free, but has restricted vortical areas. The objective is to take into account the complicated coupling between acoustics and hydrodynamics. The Galbrun approach developed previously in 2D is too expensive in terms of degrees of freedom for 3D simulations. As an alternative, we propose to consider instead the Goldstein equations, which are vectorial only in the vortical areas and remain scalar elsewhere. Extending the proof done for the Galbrun equation, it is possible to prove that the Goldstein equations are well-posed in a domain Ω if the flow is Ω-filling (each point of Ω is reached by a streamline coming from the inflow boundary in a finite time). Then we focused on the case of a rotating flow in an annular geometry, which is not Ω-filling and we proved the well-posedness of the problem .

REGULARITY Project-Team

6. New Results

6.1. Highlights of the Year

The article "Christiane's Hair" by Jacques Lévy-Véhel and Franklin Mendivil has received the Paul R. Halmos - Lester R. Ford award of the Mathematical Association of America.

6.2. Modelling the exchange of cultural goods on the Internet

Participant: Jacques Lévy Véhel.

In collaboration with Pierre Emmanuel Lévy Véhel and Victor Lévy Véhel.

Illegal sharing of cultural goods on the Internet has become a massive reality in today's connected society. Numerous studies have been performed to try and evaluate the impact of these practices on the industry of cultural goods, and how much harm, if any, they have entailed. The effect of legal and technical responses to limit pirating has also been investigated, showing in general inconclusive effect. Instead of penalizing illegal actors - providers and/or consumers -, a totally different approach has been proposed recently by the french government agency Hadopi. The idea is to offer the possibility to sites that illegally share cultural goods to become legal in exchange of a retribution proportional to their activity. In the frame of a contract with the Hadopi, we have built a model that studies the economic feasibility of such a scheme under various assumptions on the behaviour of the different actors involved. Our main finding is that, supposing that more popular goods are more prone to pirating, a retribution of the order of the increase in benefit per user gained by legalized sites does indeed lead to a win-win situation for both producers/sellers of cultural goods and willing-to-be-legalized sites. This will be the case under two conditions: the proportion of pirates is large enough (which seems largely true) and the increase in the amount of money that forums will make from advertisement when becoming legal is sufficient [43].

An extension of our work is under way, that will consider further actors and refined modelling of the way illegal sharing takes place. Calibration issues will also be investigated more closely.

6.3. Financial risk analysis

Participant: Jacques Lévy Véhel.

Financial regulations have fundamentally changed since the Basel II Accords. Among other evolutions, Basel II and III explicitly impose that computations of capital requirements be model-based. This paradigm shift in risk management has been the source of strong debates among both practitioners and academics, who question whether such model-based regulations are indeed more efficient.

A common feeling in the industry is that regulations will sometimes give a false impression of security: risk manager tend to think that a financial company that would fulfil all the criteria of, say, the Basel III Accords on capital adequacy, is not necessarily on the safe side. This is so mainly because many risks, and most significantly systemic or system-wide risks, are not properly modelled, and also because it is easy to manipulate to some extent various risk measures, such as VaR.

In parallel, a fast growing body of academic research provides various arguments explaining why current regulations are not well fitted to address risk management in an adequate way, and may even, in certain cases, worsen the situation.

We use the term *regulation risk* to describe the fact that, in some situations, prudential rules are themselves the source of a systemic risk. We have shown how a combination of model risk and regulation risk leads to an effect which is exactly the opposite of what the regulator tries to enforce. More precisely, we explain how wrongly assuming a Gaussian dynamics (or, more generally, a left-light-tailed one) when the "true" one is pure jump (or, more generally, left-heavy-tailed), and imposing as a constraint *minimizing* VaR at constant volume results in effect in movements that will *maximize* VaR. This effect is related to the fact that regulations fail to consider that risk is endogenous. In a nutshell, the idea is simply that, by treating jumps in the evolution of prices as exceptional events and essentially ignoring them in model-based VaR computations, one misses an essential dimension of risk, and acts in a way that will in effect favour sudden large movements in the markets and ultimately increase VaR. Our simple setting predicts that VaR constraints result in an *increased* intensity of jumps and a *decrease* in volatility - a fact confirmed experimentally on certain datasets. This is a mathematical translation of the common feeling of practitioners that regulations give a false impression of security characterized by low volatility but increased risk of sudden large movements.

6.4. Functional central limit theorem for multistable Lévy motions

Participants: Xiequan Fan, Jacques Lévy Véhel.

We prove a functional central limit theorem (FCLT) for the independent-increments multistable Lévy motions (MsLM) $L_I(t), t \in [0,1]$, as well as of integrals with respect to these processes, using weighted sums of independent random variables. In particular, we prove that multistable Lévy motions are stochastic Hölder continuous and strongly localisable.

Theorem 0.1 Let $(\alpha_n(u))_n, \alpha(u), u \in [0,1]$, be a class of càdlàg functions ranging in $[a,b] \subset (0,2]$ such that the sequence $(\alpha)_n$ tends to α in the uniform metric. Let $(X(k,n))_{n\in\mathbb{N},\ k=1,\dots,2^n}$ be a family of independent and symmetric $\alpha_n(\frac{k}{2^n})-$stable random variables with unit scale parameter, i.e., $X(k,n) \sim S_{\alpha_n(\frac{k}{2^n})}(1,0,0)$. Then the sequence of processes

$$L_I^{(n)}(u) = \sum_{k=1}^{\lfloor 2^n u \rfloor} \left(\frac{1}{2^n}\right)^{1/\alpha_n(\frac{k}{2^n})} X(k,n), \qquad u \in [0,1], \tag{8}$$

tends in distribution to $L_I(u)$ in $(D[0,1], d_S)$, where $\lfloor x \rfloor$ is the largest integer smaller than or equal to x. In particular, if α satisfies

$$(\alpha(x) - \alpha(x+t)) \ln t \to 0 \tag{9}$$

uniformly for all x as $t \searrow 0$, then $L_I(u)$ is localisable at all times.

We have defined integrals of MsLM, and given criteria for convergence,independence, stochastic Hölder continuity and strong localisability of such integrals.

6.5. Deviation inequalities for martingales with applications

Participant: Xiequan Fan.

In the papers [36], [37] we study some general exponential inequalities for supermartingales. The inequalities improve or generalize many exponential inequalities of Bennett (1962), Freedman (1975), van de Geer (1995), de la Peña (1999) and Pinelis (2006). Moreover, our concentration inequalities also improve some known inequalities for sums of independent random variables. Applications associated with linear regressions, autoregressive processes and branching processes are provided. In particular, an interesting application of de la Peña's inequality to self-normalized deviations is also provided.

We also considered an $\mathcal{X}$-valued Markov chain $X_1, X_2, ..., X_n$ belonging to a class of iterated random functions, which is "one-step contracting" with respect to some distance d on $\mathcal{X}$. If f is any separately Lipschitz function with respect to d, we use a well known decomposition of $S_n = f(X_1, ..., X_n) - \mathbb{E}[f(X_1, ..., X_n)]$ into a sum of martingale differences d_k with respect to the natural filtration $\mathcal{F}_k$. We show that each difference d_k is bounded by a random variable η_k independent of $\mathcal{F}_{k-1}$. Using this very strong property, we obtain a large variety of deviation inequalities for S_n, which are governed by the distribution of the η_k's. Finally, we give an application of these inequalities to the Wasserstein distance between the empirical measure and the invariant distribution of the chain.

6.6. Self-stabilizing Lévy motions

Participants: Xiequan Fan, Jacques Lévy Véhel.

Self-stabilizing processes have the property that the "local intensities of jumps" varies with amplitude. They are good models for, e.g., financial and temperature records.

The main aim of our work is to establish the existence of such processes and to give a simple construction. Formally, one says that a stochastic process $S(t), t \in [0,1]$, is a self-stabilizing process if, for almost surely all $t \in [0,1)$, S is localisable at t with tangent process S'_t an $g(S(t))-$stable process, with respect to the conditional probability measure $\mathbb{P}_{S(t)}$. In other words,

$$\lim_{r \searrow 0} \frac{S(t+ru) - S(t)}{r^{1/g(S(t))}} = S'_t(u), \tag{10}$$

where convergence is in finite dimensional distributions with respect to $\mathbb{P}_{S(t)}$. Heuristically, if $S'_t(u) = L_{g(S(t))}(u)$, equality (8) implies that

$$S(t+ru) - S(t) \approx r^{1/g(S(t))} L_{g(S(t))}(u) = (ru)^{1/g(S(t))} L_{g(S(t))}(1),$$

when r is small. Thus it is natural to define $S(t) = \lim_{n\to\infty} S_n(\frac{\lfloor nt \rfloor}{n})$, where

$$S_n\left(\frac{k+1}{n}\right) - S_n\left(\frac{k}{n}\right) = n^{-1/g(S_n(k/n))} L_{g(S_n(k/n))}(1).$$

This inspiration allows us to build Markov processes that converge to a self-stabilizing process. Note that, when $\alpha(x) \equiv 2$, this is simply Donsker's construction. The main difficult is to prove the weak convergence of S_n. To this aim, we make use of a generalization of the Arzelà-Ascoli theorem.

Definition 0.1 We call the sequence $(f_n(\theta))_{n\geq 1}$ is sub-equicontinuous on $I \subset \mathbb{R}^d$, if for any $\varepsilon > 0$, there exist $\delta > 0$ and a sequence of nonnegative numbers $(\varepsilon_n)_{n\geq 1}$,$\varepsilon_n \to 0$ as $n \to \infty$, such that, for all functions f_n in the sequence,

$$|f_n(\theta_1) - f_n(\theta_2)| \ \leq \ \varepsilon + \varepsilon_n, \qquad \theta_1, \theta_2 \in I, \tag{11}$$

whenever $||\theta_1 - \theta_2|| < \delta$ (if $\varepsilon_n = 0$ for all n, then $(f_n(\theta))_{n\geq 1}$ is just equicontinuous).

The slightly generalized version of the Arzelà-Ascoli theorem reads:

Lemma 0.1 Assume that $(f_n)_{n\geq 1}$ be a sequence of real-valued continuous functions defined on a closed and bounded set $\Pi_{i=1}^d [a_i, b_i] \subset \mathbb{R}^d$. If this sequence is uniformly bounded and sub-equicontinuous, then there exists a subsequence $(f_{n_k})_{k\geq 1}$ that converges uniformly.

The following theorem states that self-stabilizing processes do exist.

Theorem 0.2 Let g be a Hölder function defined on $\mathbb{R}$ and ranging in $[a, b] \subset (0, 2]$. There exists a self-stabilizing process $S(t), t \in [0,\ 1]$, that it is tangent at all u to a $g(S(u))-$stable Lévy process under the conditional expectation with respect to $S(u)$. Moreover, the process $S(t), t \in [0,\ 1]$, satisfies, for all $(\theta_j, t_j) \in \mathbb{R} \times [0, 1],\ j = 1, 2, ..., d$,

$$\mathbb{E}_{S(t_1)}\left[\exp\left\{i\sum_{j=2}^{d}\theta_j\left(S(t_j)-S(t_1)\right)+\int|\sum_{j=2}^{d}\theta_j\mathbf{1}_{[t_1,t_j]}(z)|^{g(S(z))}dz\right\}\right]=1. \tag{12}$$

We are currently studying the main properties of self-stabilizing processes.

SELECT Project-Team

6. New Results

6.1. Model selection in Regression and Classification

Participants: Gilles Celeux, Serge Cohen, Clément Levrard, Erwan Le Pennec, Pascal Massart, Nelo Molter Magalhaes, Lucie Montuelle.

Unsupervised segmentation is an issue similar to unsupervised classification with an added spatial aspect. Functional data is acquired on points in a spatial domain and the goal is to segment the domain in homogeneous domain. The range of applications includes hyperspectral images in conservation sciences, fMRi data and all spatialized functional data. Erwan Le Pennec and Lucie Montuelle are focusing on the questions of the way to handle the spatial component from both the theoretical and the practical point of views. They study in particular the choice of the number of clusters. Furthermore, as functional data require heavy computation, they are required to propose numerically efficient algorithms. With Serge Cohen and an X intern some progress have been made on the use of logistic weights in the hyperspectral setting.

Lucie Montuelle has studied a model of mixture of Gaussian regressions in which the proportions are modeled using logistic weights. Using maximum likelihood estimators, a model selection procedure has been applied, supported by a theoretical guarantee. Numerical experiments have been conducted for regression mixtures with parametric logistic weights, using EM and Newton algorithms. This work is published in Electronic Journal of Statistics.

Another subject considered by Erwan Le Pennec and Lucie Montuelle was the obtention of oracle inequalities in deviation for model selection aggregation in the fixed design regression framework. Exponential weights are widely used but sub-optimal. They aggregate linear estimators and penalize Stein's unbiased risk estimate used in exponential weights to derive such inequalities. Furthermore if the infinity norm of the regression function is known and taken into account in the penalty, then a sharp oracle inequality is available. Pac-Bayesian tools and concentration inequalities play a key role in this work. These results may be found in a prepublication on arxiv or in Lucie Montuelle's PhD thesis.

In collaboration with Sylvain Arlot, Matthieu Lerasle an Patricia Reynaud-Bourret (CNRS) Nelo Molter Magalhaes considers estimator selection problem with the L^2 loss. They provide a theoretical minimal and optimal penalty. They define practical cross-validation procedures and provide non-asymptotic and first order optimal results for these procedures.

Emilie Devijver and Pascal Massart focused on the Lasso for high dimension finite mixture regression models. An ℓ_1 oracle inequality have been get for this estimator for this model, for a specific regularization parameter. Moreover, for maximum likelihood estimators, restricted to relevant variables and to low rank, theoretical results have been proved to support methodology.

Pascal Massart and Clément Levrard continue their work on the properties of the k-means algorithm in collaboration with Gérard Biau (Université Paris 6). Most of the work achieved this year was devoted to the obtention of fast convergence rates for the k-means quantizer of a source distribution in the high-dimensional case. It has been proved that the margin condition for vector quantization introduced last year can be extended to the infinite dimensional Hilbert case, and that this condition is sufficient for the source distribution to satisfy some natural properties, such as the finiteness of the set of optimal quantizers. When this condition is satisfied, a dimension-free fast convergence rate can be derived. In addition, this margin condition provides theoretical guarantees for methods combining k-means and variable selection through a Lasso-type procedure. Its implementation is still in process, however early experiments shows that this procedure can retrieve active variables in the Gaussian mixture case.

Among selection methods for nonparametric estimators, a recent one is the procedure of Goldenshluger-Lespki. This method proposes a data-driven choice of m to select an estimator among a collection $(\widehat{s}_m)_{m\in M}$. The selected $\widehat{m}$ is chosen as a minimiser of $B(m)+V(m)$ where $B(m)=\sup\{[\|\widehat{s}_m-\widehat{s}_{m'}\|-V(m')]_+,\ m'\in M\}$ and $V(m)$ is a penalty term to be suitably chosen. Previous results have established oracle inequalities to ensure that if $V(m)$ is large enough the final estimator $\widehat{s}_{\widehat{m}}$ is almost as efficient as the best one in the collection. The aim of the work of Claire Lacour and Pascal Massart was to give a practical way to calibrate $V(m)$. To do this they have evidenced an explosion phenomenon: if V is chosen smaller than some critical V_0, the risk $\|s-\widehat{s}_{\widehat{m}}\|$ is proven to dramatically increase, though for $V>V_0$ this risk is quasi-optimal. Simulations have corroborated this behavior.

The well-documented and consistent variable selection procedure in model-based cluster analysis and classification, that Cathy Maugis (INSA Toulouse) has designed during her PhD. thesis in SELECT, makes use of stepwise algorithms which are painfully slow in high dimensions. In order to circumvent this drawback, Gilles Celeux in collaboration with Mohammed Sedki (Université Paris XI) and Cathy Maugis), proposed to sort the variables using a lasso-like penalization adapted to the Gaussian mixture model context. Using this rank to select the variables they avoid the combinatory problem of stepwise procedures. After tests on challenging simulated and real data sets, their algorithm finalised and show good performances.

In collaboration with Jean-Michel Marin (Université de Montpellier) and Olivier Gascuel (LIRMM), Gilles Celeux has continued a research aiming to select a short list of models rather a single model. This short list of models is declared to be compatible with the data using a p-value derived from the Kullback-Leibler distance between the model and the empirical distribution. And, the Kullback-Leibler distances at hand are estimated through non parametric and parametric bootstrap procedures.

6.2. Statistical learning methodology and theory

Participants: Vincent Brault, Gilles Celeux, Christine Keribin, Erwan Le Pennec, Lucie Montuelle, Michel Prenat, Solenne Thivin.

Vincent Brault, Ph D. student of Gilles Celeux and Christine Keribin defended his thesis on the Latent Block Model (LBM) for categorical data. Their work investigated a Gibbs algorithm to avoid solutions with empty clusters on synthetic as well as real data (Congressional Voting Records and genomic data. They detailed the link between the information criteria ICL and BIC, compared them on synthetic and real data, and conjectured that these criteria are both consistent for LBM, which is not a standard behavior. Hence, ICL has to be preferred for LBM. This work is now published in *Statistics and Computing*.

Vincent Brault has achieved a detailed bibliographical review on coclustering with Aurore Lomet (UTC) which is currently under revision. He has also worked in collaboration with Mahindra Mariadassou (INRA) to overview the state of the art on theoretical results for latent or stochastic block model.

Vincent Brault, Christine Keribin and Mahindra Mariadassou have started a collaboration to tackle the consistency and asymptotic normality for the maximum likelihood and variational estimators in a stochastic or latent block model.

Gilles Celeux has started a collaboration with Jean-Patrick Baudry on strategies to avoid the traps of the EM algorithm in mixture analysis. They anayse the effect of the spurious local maximizers and the regulariszed algorithms to avoid these spurious solutions. They explore the link of the degree of regularization and the slope heuristics. Moreover, they propose and study strategies to initiate the EM algorithm embedding the solution with K components and the starting position with $K+1$ component to avoid suboptimal solutions.

Erwan Le Pennec is supervising Solenne Thivin in her CIFRE with Michel Prenat and Thales Optronique. The aim is target detection on complex background such as clouds or sea. Their approach is a local approach based on test decision theory. They have obtained theoretical and numerical results on a segmentation based approach in which a simple Markov field testing procedure is used in each cell of a data driven partition. They also have obtained experimental results on images (or patches) unsupervised classification, with the aim of better calibrate the detection procedure. The classification is based on features which are defined in cloud texture modeling activity.

Erwan Le Pennec and Michel Prenat have also collaborated on a cloud texture modeling using a non-parametric approach. Such a modeling coud be used to better calibrate the detection procedure: it can lead to more examples than the one acquired and it could be the basis of an ensemble method.

6.3. Reliability

Participants: Yves Auffray, Gilles Celeux, Rémy Fouchereau, Patrick Pamphile, Jana Kalawoun.

In 2014, in the framework of a CIFRE convention with Snecma-SAFRAN Rémy Fouchereau has defended a thesis on the modeling of fatigue lifetime supervised by Gilles Celeux and Patrick Pamphile. In aircraft, space and nuclear industry, fatigue test is the main basic tool for analyzing fatigue lifetime of a given material, component, or structure. A sample of the material is subjected to cyclic loading S (stress, force, strain, etc.), by a testing machine which counts N, the number of cycles to failure. Fatigue test results are plotted on a SNcurve. A probabilistic model for the construction of SN-curve is proposed. In general, fatigue test results are widely scattered for High Cycle Fatigue region and "duplex" SN-curves appears for Very High Cycle region. That is why classic models from mechanic of rupture theory on one hand, probability theory on the other hand, do not fit SN-curve on the whole range of cycles. We have proposed a probabilistic model, based on a fracture mechanic approach: few parameters are required and they are easily interpreted by mechanic or material engineers.This model has been applied to both simulated and real fatigue test data sets. The SN-curves have been well fitted on the whole range of cycles. The parameters have been estimated using the EM algorithm, combining Newton-Raphson optimisation method and Monte Carlo integral estimations. The model has been then improved taking into account production process information, thanks to a clustering approach. Thus, we have provided engineers with a probabilistic tool for reliability design of mechanical parts, but also with a diagnostic tool for material elaboration.

Since two years SELECT collaborates with CEA for the estimation of the battery State of Charge (SoC). For vehicles powered by an electric motor, SoC estimation is essential to guarantee vehicle autonomy, as well as safe utilization. The aim is to create a reliable SoC model to closely fit the battery dynamic, in embedded applications (e.g. Electric Vehicle). Jana Kalawoun started a thesis supervised by Gilles Celeux, Patrick Pamphile and Maxime Montaru (CEA) on this topic. The SoC is modeled by a Switching Markov State-Space Model. The parameters are estimated by combining the EM algorithm and Particle Filter methods. The model is validated using real-life electric vehicle data. It has been proved to be highly superior to a simple state space model. The optimal number of battery modes is then identified, using different model selection criteria as BIC or the slope heuristics.

Yves Auffray and Gilles Celeux proposed a solution to a reliability problem on Dassault's F7X business jet brakes. As the origin brake version showed poor reliability performance, an increased frequency inspection of the brakes had been decided and, after a while, a new brake version adopted. The new version has not shown any failure since its adoption. Then the question was : is it possible to relax the brakes inspection frequency ?
On the basis of first brake version failure data, the parameters of a Weibull law was estimated : $\eta = 3169, \beta = 1.38$. Under the hypothesis that the new brake version would follow the same Weibull law, the probability that none of them broke was $1.67\ 10^{-6}$. This led to reject that hypothesis.
A Weibull model for the new brakes was then estimated. The shape parameter beeing leaved conservatively unchanged, the scale parameter was estimated so that the no failure event probability amounts to 0.05. This led to $\eta = 9326$.
From the resulting Weibull model, dates $D_0, D_1, \cdots, D_k, \cdots$ of inspection for the new brakes was established so that : $\mathbb{P}(T \leq D_0 + D_1 + \cdots + D_k | T > D_0 + \cdots + D_{k-1}) = 0.01$.
Dassault has adopted this far less constraining inspection calendar.

6.4. Statistical analysis of genomic data

Participants: Vincent Brault, Gilles Celeux, Mélina Gallopin, Christine Keribin, Yann Vasseur.

In collaboration with Florence Jaffrezic and Andrea Rau (INRA, animal genetic department), Mélina Gallopin is a third year PhD student under the supervision of Gilles Celeux. This thesis is concerned with the modelization and model selection in the analysis of RNA-seq data. This year, they proposed a model selection criterion for model-based clustering of annotated gene expression data. This criterion is a ICL-like criterion taking into account the annotations. They are also working on a objective comparison of discrete and continuous modelling after a transformations for RNA-seq data based on a comparison of the likelihoods (eventually penalized) of the models in competition.

The subject of Yann Vasseur PhD Thesis, supervised by Gilles Celeux and Marie-Laure Martin-Magniette (INRA URGV), is the inference of a regulatory network on Transcriptions Factors (TFs), which are specific genes, of *Arabidopsis thaliana*. In that purpose, a transciptome dataset with a sensibly equal size of TFS and statistical units is available. The first aim consists of reducing the dimension of the network to avoid high dimension difficulties. Representing this network with a Gaussian Graphical Model, the following procedure has been defined:

1. *Selection step*: choosing the set of TFs regulators (supports) of each TF.
2. *Classification step*: deducing co-factors groups (TFs with similary expression levels) from these supports.

Thus, the reduced network would be built on the co-factors groups. Currently, several selection methods based on Gauss-LASSO and resampling procedures have been applied on the dataset. The study of the stability and the parameters calibration of these methods are in progress. The TFs are clustered with the Latent Block Model in a number of co-factors groups selected with the BIC or the exact ICL criterion.

In collaboration with Marie-Laure Martin-Magniette, Cathy Maugis and Andrea Rau, Gilles Celeux studied gene expression gotten from high-throughput sequencing technology. They focus on the question of clustering digital gene expression profiles as a means to discover groups of co-expressed genes. They propose a Poisson mixture model using a rigorous framework for parameter estimation as well as the choice of the appropriate number of clusters. They illustrate co-expression analyses using this approach on two real RNA-seq datasets. A set of simulation studies also compares the performance of the proposed model with that of several related approaches developed to cluster RNA-seq or serial analysis of gene expression data. The proposed method is implemented in the open-source R package `HTSCluster`, available on CRAN.

6.5. Model based-clustering for pharmacovigilance data

Participants: Gilles Celeux, Christine Keribin, Valérie Robert.

In collaboration with Pascale Tubert-Bitter, Ismael Ahmed and Mohamed Sedki, Gilles Celeux and Christine Keribin has started a research concerning the detection of associations between drugs and adverse events in the framework of the PhD of Valerie Robert. At first, this team has developed a model-based clustering inspired of the latent black model which consists in co-clustering rows and columns of two binary tables imposing the same row ranking. Then it enables to highlight subgroups of individuals sharing the same drug profile and subgroups of adverse effects and drugs with strong interaction. Besides, some sufficient conditions are provided to obtain the identifiability of the model and some studies are experimented on simulated data.

6.6. Curves classification, denoising and forecasting

Participants: Émilie Devijver, Pascal Massart, Jean-Michel Poggi, Vincent Thouvenot.

In collaboration with Farouk Mhamdi and Meriem Jaidane (ENIT, Tunis, Tunisia), Jean-Michel Poggi proposed a method for trend extraction from seasonal time series through the Empirical Mode Decomposition (EMD). Experimental comparison of trend extraction based on EMD, X11, X12 and Hodrick Prescott filter are conducted. First results show the eligibility of the blind EMD trend extraction method. Tunisian real peak load is also used to illustrate the extraction of the intrinsic trend.

Jean-Michel Poggi, co-supervising with Anestis Antoniadis (Université Joseph Fourier Grenoble) the PhD thesis of Vincent Thouvenot, funded by a CIFRE with EDF. The industrial motivation of this work is the recent development of new technologies for measuring power consumption by EDF to acquire consumption data for different mesh network. The thesis will focus on the development of new statistical methods for predicting power consumption by exploiting the different levels of aggregation of network data collection. From the mathematical point of view, the work is to develop generalized additive models for this type of kind of aggregated data for the modeling of functional data, associating closely nonparametric estimation and variable selection using various penalization methods.

Jean-Michel Poggi and Pascal Massart are the co-advisors of the PhD thesis of Émilie Devijver, strongly motivated by the same kind of industrial forecasting problems in electricity, which is dedicated to curves clustering for the prediction. A natural framework to explore this question is mixture of regression models for functional data. They extend to functional data the recent work by Bühlmann and coauthors dealing with the simultaneous estimation of mixture regression models in the scalar case using Lasso type methods. It is based on the technical tools of the work of Caroline Meynet (which completes her thesis Orsay under the direction of P. Massart), which deals with the clustering of functional data using Lasso methods choosing simultaneously number of clusters and selecting significant wavelet coefficients. Nevertheless, they also propose a procedure dealing with low rank estimator. Simulations and benchmark data have been conducted for high-dimensional finite mixture regression models.

Jean-Michel Poggi, co-supervising with Meriem Jaëdane, Raja Ghozi (ENIT Tunisie) and from the industrial side, Sylvie Sevestre-Ghalila (CEA LinkLab), the PhD thesis of Neska El Haouij, funded by a kind of CIFRE with CEA LinkLab. The industrial motivation of this work is the recent development of new technologies for sensory measurements, environmental and physiological to explain and improve the driving tasks . The thesis aims to explain sensory aspects involved in automated decision to the car interior, by objectivization. The thesis will focus on the use and development of experimental designs and statistical methods to quantify and explain driving ability in to the modeling using functional explanatory factors. Statistical contributions of this work will involve nonparametric estimation and variable selection and/or models.

6.7. Statistical analysis of medical images

Participants: Christine Keribin, Yves Rozenholc.

Yves Rozenholc and C. Keribin work the genomic tumoral alterations and supervised a Master student Yi LIU. The study of genomic DNA alterations (recurrent regions of alteration, patterns of instability) contributes to tumor classification, and becomes of great importance for the personalization of cancer treatments. The use of Single-Nucleotide Polymorphism (SNP) arrays or of New Generation Sequences (NGS) techniques allows the simultaneous estimation of segmented copy number (CN) and B-allele frequency (BAF) profiles along the whole genome. In this context, Popova (2009) proposed the GAP method, based on pattern recognition with (BAF, CN) maps to detect genotype status of each segment in complex tumoral genome profiles. It takes into account the fact that the observations on these maps are necessarily placed on centers that depend –up to a proper scaling of the CN– only on the unknown proportion of non tumoral tissue in the sample. Being deterministic and manually tuned, this method appears sensitive to noise. To overcome this drawback, they set a mixture model, allowing the automatic estimation of the proportion of non tumoral tissue and the test of genotype for each segment along the whole genome. They develop the estimation with an adapted EM algorithm that has been tested on simulated data. This work has already been presented (ERCIM 14, SEQBIO14) and provides many potential developments.

TAO Project-Team

6. New Results

6.1. Highlights of the Year

- The European commission has chosen Crystal-Supergrids (http://www.artelys.com/news/120/90/Energy-The-European-Commission-Chooses-Artelys-Crystal) for energy modeling and planning in Europe. Crystal-Supergrids is based on the Post project, an ADEME project between Artelys and Inria-TAO.
- The HiggsML challenge was the all-time most popular challenge organized by Kaggle. Cécile Germain-Renaud, Balázs Kégl and Marc Schoenauer were part of the organizing committee.
- Creation of the Center for Data Science, an interdisciplinary institute of the Université Paris-Saclay. Co-chaired by Balázs Kégl, with more than 250 permanent researchers in 35 laboratories, the CDS organizes continued cross-fertilization of machine learning and domain sciences.
- Best Paper Award at PPSN .

BEST PAPERS AWARDS :
[36] **13th International Conference on Parallel Problem Solving from Nature**. I. LOSHCHILOV, M. SCHOENAUER, M. SEBAG, N. HANSEN.

6.2. Optimal Decision Making under Uncertainty

Participants: Olivier Teytaud [correspondent], Jean-Joseph Christophe, Jérémie Decock, Nicolas Galichet, Marc Schoenauer, Michèle Sebag, Weijia Wang.

The UCT-SIG works on sequential optimization problems, where a decision has to be made at each time step along a finite time horizon, and the underlying problem involves uncertainties along an either adversarial or stochastic setting.

After several years of success in the domain of GO, the most prominent application domain here is now energy management, at various time scales, and more generally planning. Furthermore, the work in this SIG has also lead to advances in continuous optimization at large, that somehow overlap with the work in the OPT-SIG (see 6.3).

The main advances done this year include:

Bandit-based Algorithms Active learning for the identification of biological dynamical systems has been tackled using Multi-Armed Bandit algorithms [35]. Weijia Wang's PhD [5] somehow summarizes the work done in TAO regarding Multi-objective Reinforcement Learning with MCTS algorithm. Differential Evolution was applied as an alternative to solve non-stationary Bandit problems [45].

Continuous optimization: parallelism, real-world, high-dimension and cutting-plane methods Our work in continuous optimization extends testbeds as follows: (i) including higher dimension (many testbeds in evolutionary algorithms consider dimension ≤ 40 or ≤ 100) (ii) taking into account computation time and not only the number of function evaluations (this makes a big difference in high dimension) (iii) including real world objective functions (iv) including parallelism, in particular, parallel convergence rates for differential evolution and particle swarm optimization [21]. We have a parallel version of cutting plane methods, which use more than black-box evaluations of the objective functions - we keep in mind that some of our black-box methods, on the other hand, also do not need convexity or the existence of a gradient.

Noisy optimization We have been working on noisy optimization in discrete and continuous domains. In the discrete case, we have shown the impact of heavy tails, and we have shown that resampling can solve some published open problems in an anytime manner. In the continuous case, we have shown [16] that a classical evolutionary principle (namely the step-size proportional to the distance to the optimum) implies that the optimal rates can not be reached - more precisely, we can have simple regret at best $O(1/\sqrt{number\ of\ fitness\ evaluations})$ in the simple case of an additive noise, whereas some published algorithms reached $O(1/number\ of\ fitness\ evaluations)$. One of the most directly applicable of our works is bias correction when the objective function $f(x)$ has the form $f(x) = \mathbb{E}_\omega f(x, \omega)$ and is approximated by $f(x) = \frac{1}{N}\sum_{i=1}^{N} f(x, \omega_i)$ for a given finite sample $\omega_1, \cdots, \omega_N$. We have also worked on portfolios of noisy optimizers [20], [34].

Discrete-time control with constrained action spaces. While Direct Policy Search is a reliable approach for discrete time control, it is not easily applicable in the case of a constrained high-dimensional action space. In the past, we have proposed DVS (Direct Value Search) for such cases [54]. The method is satisfactory, and we have additional mathematical results; in particular we prove positive results for non-Markovian, non-convex problems, and we prove a polynomial-time decision making and, simultaneously, exact asymptotic consistency for a non-linear transition [24]. Related work [60] also proposes to directly learn the value function, in a RL context, using some trajectories known to be bad.

Games. While still lightly contributing to the game of GO with our taiwanese partners [8], we obtained significant improvements in randomized artificial intelligence algorithms by decomposing the variance of the result into (i) the random seed (ii) the other random contributions such as the random seed of the opponent and/or the random part in the game. By optimizing our probability distribution on random seeds, we get significant improvements in e.g. phantom Go. This is basically a simple tool for learning opening books [44].

Adversarial bandits. High-dimensional adversarial bandits lead to two main drawbacks: (i) computation time (ii) highly mixed nature of the obtained solution. We developped methods which focus on sparse solution. Provably consistent, these methods are faster when the Nash equilibrium is sparse, and provides highly sparse solutions[17].

6.3. Continuous Optimization

Participants: Ouassim Ait Elhara, Asma Atamna, Anne Auger, Alexandre Chotard, Nikolaus Hansen, Yann Ollivier, Marc Schoenauer, Michèle Sebag, Olivier Teytaud, Luigi Malago, Emmanuel Benazera.

Our main expertise in continuous optimization is on stochastic search algorithms. We address theory, algorithm design, and applications. The methods we investigate are adaptive techniques able to learn iteratively parameters of the distribution used to sample solutions. The Covariance Matrix Adaptation Evolution Strategy (CMA-ES) is nowadays one of the most powerful methods for derivative-free continuous optimization. We work on different variants of the CMA-ES to improve it in various contexts as described below. We are well recognized in the field and were invited to write a book chapter on the design of continuous stochastic search[50].

Online adaptation of CMA-ES hyperparameters CMA-ES uses clever mechanisms to adapt the covariance matrix and the step-size, based on the evolution path. However, these mechanisms in turn use learning parameters, that were adjusted by trial-and-error in the seminal algorithm. However, thanks to the invariance properties of the algorithm, these values have been demonstrated to be very robust. An original mechanism has been proposed to adapt these hyper-parameters online, maximizing the likelihood of the selected sample at time to adapt the hyperparameters at time t-1. The corresponding paper published at PPSN received the Best Paper Award [36].

Linear Time and Space Complexity CMA-ES for Large-Scale Optimization We have been proposing a large-scale version of CMA-ES where the covariance matrix is restriced to a linear numbers of parameters. The update for the covariance matrix has been derived using the Information

Geometric Optimization (IGO) framework and cumulation concepts borrowed from the original CMA have been additionally included [14]. This work is part of a joint project between the TAO team and Shinshu university in Japan funded by the Japanese governement. In this context, Luigi Malago is visiting the team working on extending the proposed algorithm to a richer model.

Evaluation of Black-Box Optimizers We have been focusing on appraising the performance of step-size adaptation mechanisms for stochastic adaptive algorithms. We have shown that a too restrictive choice of test functions for the design of a method leads to misleading conclusions and proposed a thorough framework for evaluating step-size mechanism [29]. We have been pursuing our effort for *thorough and rigorous* benchmarking of black-box algorithms by organizing two more Black-Box-Optimization Benchmarking workshops that will take place at CEC 2015 and GECCO 2015. Those workshops are based on the platform COCO that we develop in the context of the ANR NumBBO project.

Theoretical Analysis of Stochastic Adaptive Algorithms We have analyzed the CSA-ES algorithm using resampling for **constrained optimization** on a linear function with a linear contraint. We have studied the behavior of the algorithm and proven success of failure of the algorithm depending on internal parameters of the algorithm [22]. We have extended a previous work on a linear function from the use of standard normal distribution to more general ones [23]. The published paper has been invited for an extension in an ECJ special issue. The extended paper had been submitted in december 2014. We have been providing a general methodology to prove the linear convergence of Comparison-based Step-size Adaptive Randomized Search on scaling-invariant functions by analyzing the stability of underlying Markov chains [57].

CMA-ES Library Besides our continuous work on implementations of CMA-ES (see e.g. github, PyPI), we have created a new library in C++11 (libcmaes). As part of the ANR SIMINOLE project, the library has been coupled with ROOT, the data analysis framework used at CERN, and generally in physics.

6.4. Applications to E-science

Participants: Cécile Germain-Renaud [correspondent], Marco Bressan, Philippe Caillou, Dawei Feng, Cyril Furtlehner, Blaise Hanczar, Karima Rafes, Balázs Kégl, Michèle Sebag.

The E-S-SIG explores the issues related to applications to E-Science, starting with modeling and optimizing very large scale computational grids, in particular in the context of Physics, to social sciences modelling with multi-agent systems.

The Higgs boson Machine Learning challenge The HiggsML challenge [0] has been set up to promote collaboration between high-energy physicists and computer scientists. The challenge, hosted by Kaggle, has drawn a remarkably large audience (with 1700+ teams it is one of the all-time most popular Kaggle challenges) and large coverage both in the social networks and in the media.

The goal of the challenge is to improve the procedure that classifies events produced by the decay of the Higgs boson versus events produced by other (background) processes, based on a training set of 250,000 examples. The challenge is a premier: it is the first time that a CERN experiment (ATLAS) made public such a large set of the official event and detector simulations. It also features a unique formal objective representing an approximation of the median significance (AMS) of a discovery (counting) test, which generates interesting algorithmic/theoretical questions beyond the usual challenges of finding and tuning the best classification algorithm [55].

A follow-up, the HEPML workshop was organized at NIPS14 [0], reporting on the results and the winning algorithms. The dataset and a software toolkit are available from the CERN Data Portal [0]

[0] https://www.kaggle.com/c/higgs-boson
[0] http://nips.cc/Conferences/2014/Program/event.php?ID=4292
[0] http://opendata.cern.ch

The Center for Data Science is a Lidex of the Université Paris-Saclay (UPSay), headed by Balazs Kégl and Arnak Dalalyan, gathering over 52 research teams and 34 labs with the goal of designing and applying automated methods to analyze massive and complex scientific datasets in order to extract useful information. Data science projects require expertise from a vast spectrum of disciplines (statistics, signal processing, machine learning, data mining, data visualization, high performance computing), besides the mastery of the scientific domain where the data originate from.

The goal of CDS is to establish an institutionalized agora in which scientists can find each other, exchange ideas, initiate and nurture interdisciplinary projects, and share their experience on past data science projects. To foster synergy between data analysts and data producers CDS organizes actions to provide initial resources for helping collaborations to get off the ground, to mitigate the non-negligible risk taken by researchers venturing into interdisciplinary data science projects, and to encourage the use of unconventional forms of information transmission and dissemination essential in this communication-intensive research area. The CDS fits perfectly in the recent surge of similar initiatives, both at the international and at the national level, and it has the potential to make the University Paris-Saclay one of the international fore-runners of data science [0].

Fault management As Lamport formulated decades ago, fault management in distributed systems exemplifies the unreachability of exact prior knowledge. Real-world large scale system add a supplementary complexity, which is non-stationarity.

- [12] models the system state and its ruptures (non-stationarity) through the flow of jobs as a stream (scalability), with a traceability goal (interpretability). These new streaming approaches involve self-calibration of the model based on scale invariance.
- D. Feng's PhD thesis [3] formulates the problem of probe selection for fault prediction based on end-to-end probing as a Collaborative Prediction (CP) problem, based on the reasonable assumption of an underlying factorial model. [26] extends the matrix completion/compressd sensing setup to a sequential (tensor) context. We propose and evaluate a new algorithme, *Sequential Matrix Factorization* (SMF) that combines matrix completion with a self-calibrating exploration/exploitation balancing heuristic. Its active learning version (SMFA) exhibits superior performance over state-of-the-art methods.

Distributed system observation The work on distributed system automated analaysis and description[7] has been persued thru the continued development of the GAMA multi-agent framework https://code.google.com/p/gama-platform/wiki/GAMA. Philipps Caillou is associated to the new young researcher ANR ACTEUR, coordinated by Patrick Taillandier (IDEES, Rouen university), which will give an additional strucuture for further collaborations.

Identifying leaders in Social Networks The Modyrum contract with the SME Augure (funding Marco Bressan's Post-doc) aims at providing criteria to identify the trend leaders from blogs, tweets and other web-site posts. The same methods is being applied to fashion leaders in business as well as to opinion leaders in politics.

6.5. Designing criteria

Participants: Jamal Atif, Aurélien Decelle, Cyril Furtlehner, Yoann Isaac, Alexandre Quemy, Yann Ollivier, Marc Schoenauer, Michèle Sebag.

This SIG, rooted on the claim that *What matters is the criterion*, aims at defining new learning or optimization objectives reflecting fundamental properties of the model, the problem or the expert prior knowledge.

A statistical physics perspective With motivating applications in large scale inference problems like traffic congestions we are pursuing our quest of practical solutions to inverse problems like in [39] where a method is proposed to invert a Gaussian Markov random field with topological and spectral constraints well suited to subsequent use of belief propagation as inference algorithm

[0] http://www.datascience-paris-saclay.fr/en

(see https://who.rocq.inria.fr/Jean-Marc.Lasgouttes/star-ips for the implementation). A more specific model for traffic inference has also been developped in [11]. A method adapted to the generalized belief propagation framework, aiming at adressing directly and systematically the loop corrections without loss of scalability is about to be completed.

Multi-objective ATC The new Bayesian approach of Air Traffic Control belongs to this SIG, but was described in the Section 4.2 . Main publications are Gaétan Marceau's PhD [4] and the corresponding PPSN paper [38], [59].

Programming by Feedback Riad Akrour's PhD work on Preference Based Learning [1] culminated with the addition of a model for the user's competence in the interactive learning loop. In the resulting original paradigm, the user is sequentially proposed a series of behaviors and is only asked "Hot-or-cold" questions. The *Programming by Feedback* paradigm [15] will hopefully initiate a general way to allow non-digitally-proficient users to nevertheless control the behavior of software-based agents in their environment.

Multi-objective AI Planning This activity had almost stopped since the end of the DESCARWIN ANR project. However, a productive intership resulted in some new benchmarks in the ZenoTravel domain together with an exact solver ensuring the knowledge of the true Pareto front [41], [40].

Algorithm Selection Algorithm Selection can be viewed as a Collaborative Filtering problem, in which a problem "likes" an algorithm that is able to solve it. Initiated during Mustafa Misir's ERCIM postdoc in 2013, this idea has also been applied for Process Management [43], and is the basis of François Gonards's PhD funded by IRT SystemX in the context of aeronautics and car industry.

Outlier rejection in classification An original approach based on One-Class SVM has been proposed during Blaise Hanczar's on year *delegation* at TAO [28].

Learning sparse representations by auto-encoders Auto-encoders (AE) are a widely used tool for unsupervised learning, which consists of a neural network trained to reconstruct its own input via smaller-dimensional layers. The usual training criterion is the reconstruction error, however, the usual justification for AE is to learn a more compact data representation. In [62] we formalize this latter criterion using Minimum Description Length (MDL) and establish a comparison with the traditional reconstruction criterion. The MDL criterion has an interpretation as a denoising reconstruction and fully determines an optimal noise level, contrary to the literature on denoising AEs. More surprisingly, AE (aka Auto-associators) can also be used to learn sparse representations in the context of supervised learning [51].

AMIB Project-Team

5. New Results

5.1. RNA

To mitigate the current absence of a selective scientific event dedicated to RNA computational biology, impeding the dissemination of recent methodological results, AMIB members have participated in the creation of the *Computational Methods for Structural RNAs* workshops (CMSR'14). This first installment of the event was hosted in Strasbourg as a workshop of the 2014 edition of European Conference on Computational Biology. Its proceedings were published by McGill University [33], and extended versions of selected articles were invited to appear in the *Journal of Computational Biology*.

5.1.1. RNA visualization

The field of RNA visualization is now rich with multiple tools that accommodate different needs, arising from a variety of application contexts. In order to help end-users navigate through the jungle of available options, Y. Ponty and F. Leclerc (IGM, Univ. Paris-Sud) have contributed a review of existing tools, and illustrate their usage to address a collection of typical use-cases [35].

5.1.2. RNA design and structures

The past couple of years have seen the multiplication of heuristic or exponential time algorithms for the RNA design problem. This situation motivates a survey, which s currently lacking, that would focus on the relative merits of existing algorithms, and assess their applicability towards the typical goals of synthetic biology. Such an objective evaluation is at the core of the PhD project of Vincent Le Gallic, which was started in September 2014.

With Antoine Soulé, a PhD student of J-M Steyaert and J. Waldispühl (McGill), a comparative study of the various softwares for the inverse RNA folding problem is under revision and a new version of RNAMUTANT in the langage GAP-L with enrichment has been designed.

Besides, we have published a general survey on RNA structure comparison [9].

5.1.3. RNA splicing regulation

RNA splicing is a modification of the nascent pre-messenger RNA (pre-mRNA) transcript in which introns are removed and exons are joined. The U2AF heterodimer protein has been well studied for its role in defining functional 3' splice sites in pre-mRNA splicing, but multiple critical problems are still outstanding, including the functional impact of their cancer-associated mutations. In collaboration with Xiang-Dong Fu's groups in San Diego and Wuhan, , through genome-wide analysis of U2AF-RNA interactions, we reported in [16] that U2AF has the capacity to define 88% of functional 3' splice sites in the human genome. Numerous U2AF binding events also occur in other genomic locations, and metagene and minigene analysis suggests that upstream intronic binding events interfere with the immediate downstream 3' splice site associated with either the alternative exon to cause exon skipping or competing constitutive exon to induce inclusion of the alternative exon.

5.1.4. RNA 3D structure modelling

Conformational diversity for RNA ensemble analyses is often provided by sophisticated molecular dynamics simulations. Long trajectories with specialized force fields on dedicated supercomputers are required to adequately sample conformational space, limiting ensemble analyses to modestly-sized RNA molecules. To avoid these limitations, we developed an efficient conformational sampling procedure, Kino-geometric sampling for RNA (KGSrna), which can report on ensembles of RNA molecular conformations orders of magnitude faster than MD simulations. In the KGSrna model, the RNA molecule is represented with

rotatable, single bonds as degrees-of-freedom and groups of atoms as rigid bodies. In this representation, non-covalent bonds form distance constraints, which create nested, closed cycles in a rooted spanning tree. Torsional degrees-of-freedom in a closed ring demand carefully coordinated changes to avoid breaking the non-covalent bond, which greatly reduces the conformational flexibility. The reduced flexibility from a network of nested, closed rings consequently deforms the biomolecule along preferred directions on the conformational landscape. This new procedures projects degrees-of-freedom onto a lower-dimensional subspace of the conformation space, in which the geometries of the non-covalent bonds are maintained exactly under conformational perturbation. The dimensionality reduction additionally enables efficient exploration of conformational space and reduces the risk of overfitting sparse experimental data. Kinogeometric sampling of 3D RNA models can recover the conformational landscape encoded by proton chemical shifts in solution and is thus of great help to interpret NMR experimental data [11]. The computational efficiency of this approach, combined to its inherent parallel nature could also be adapted to model large assemblies on parallel platforms.

Our expertise was also essential in modelling junction of the RNA structure of a large biomolecule of interest, the tmRNA so as to study its interaction with the SmpB protein. Results obtained in collaboration with experiementalists, mainly P. Vachette at IBBMC and S. Nonin-Lecomte at the LCRB were made available in [15].

5.2. Sequences

5.2.1. Random generation

In collaboration with the Simon Fraser University (Vancouver, Canada), we have explored a random generation strategy, under a Boltzmann distribution, to assess the robustness of predicted adjacencies in ancestral genomes using a parsimony-based approach. The sampling algorithm was used to estimate the Boltzmann probability of ancestral adjacencies, which was then used as a filter to weed out unsupported predictions, leading to the resolution of a large number of syntenic inconsistencies [23].

5.2.2. Combinatorics of motifs

An algorithm for pvalue computation has been proposed in [40] that takes into account a Hiddden Markov Model and an implementation, SUFPREF, has been realized (http://server2.lpm.org.ru/bio).

Combinatorics of clumps have been extensively studied, leading to the definition of the so-called *canonic clumps*. It is shown in [26] that they contain the necessary information needed to calculate, approximate, and study probabilities of occurrences and asymptotics. This motivates the development of a *clump automaton*. It allows for a derivation of pvalues, decreasing the space and time complexity of the generating function approach or previous weighted automata. An extension to degenerate patterns is currently realized and implemented in a collaboration with J. Holub (Praha U.) and E. Furletova (IMPB).

During her master thesis at King's College, A. Héliou and collaborators designed the first linear-time and linear-space algorithm for computing all minimal absent words based on the suffix array [6]. In a typical application, one would be interested in computing minimal absent words to compare and study genomes in linear time by considering this negative information.

In a collaboration with AlFarabi University, where M. Régnier acts as a foreign co-advisor), word statistics were used to identify mRNA targets for miRNAs involved in various cancers [7].

5.2.3. Prediction and functional annotation of ortholog groups of proteins

In comparative genomics, orthologs are used to transfer annotation from genes already characterized to newly sequenced genomes. Many methods have been developed for finding orthologs in sets of genomes. However, the application of different methods on the same proteome set can lead to distinct orthology predictions.

In [38], [14] we developed a method based on a meta-approach that is able to combine the results of several methods for orthologous group prediction. The purpose of this method is to produce better quality results by using the overlapping results obtained from several individual orthologous gene prediction procedures. Our method proceeds in two steps. The first aims to construct seeds for groups of orthologous genes; these seeds correspond to the exact overlaps between the results of all or several methods. In the second step, these seed groups are expanded by using HMM profiles.

We evaluated our method on two standard reference benchmarks, OrthoBench and Orthology Benchmark Service. Our method presents a higher level of accurately predicted groups than the individual input methods of orthologous group prediction. Moreover, our method increases the number of annotated orthologous pairs without decreasing the annotation quality compared to twelve state-of-the-art methods.

5.3. 3D Modelling and Interactions

5.3.1. Transmembrane proteins

Transmembrane beta-barrel proteins (TMB) account for 20 to 30% of identified proteins in a genome but, due to difficulties with standard experimental techniques, they are only 2% of the RCSB Protein Data Bank. As TMB perform many vital functions, the prediction of their structure is a challenge for life sciences, while the small number of known structures prohibits knowledge-based methods for structure prediction. We study and design algorithmic solutions addressing the secondary structure, an abstraction of the 3D conformation of a molecule, that only retains the contacts between its residues. As TMBs are strongly structured objects, model based methodologies [18] are an interesting alternative to conventional methods. The efficiently obtained 3D structures provide a good model for further 3D and interaction analyses.

5.3.2. 3D Interaction prediction

While protein-RNA complexes provide a wide range of essential functions in the cell, their atomic experimental structure solving is even more difficult than for proteins. Protein-RNA complexes provide a wide range of essential functions in the cell. Docking approaches that have been developed for proteins are often challenging to adapt for RNA because of its inherent flexibility and the structural data available being relatively scarce.We adapted the reference RosettaDock protocol for protein-RNA complexes both at the nucleotide and atomic levels. Using a genetic algorithm-based strategy, and a non-redundant protein-RNA dataset, we derived a RosettaDock scoring scheme able not only to discriminate but also score efficiently docking decoys. The approach proved to be both efficient and robust for generating and identifying suitable structures when applied to two protein-RNA docking benchmarks in both bound and unbound settings. It also compares well to existing strategies. This is the first approach that currently offers a multi-level optimized scoring approach integrated in a full docking suite, leading the way to adaptive fully flexible strategies [28], [12]. This work is part of the PhD thesis of Adrien Guilhot-Gaudeffroy. While the previously described approaches perform well in a rigid or semi-flexible docking setting, the generation of putative conformations for flexible molecules (sampling) is still a difficult question that has to be addressed in a multi-scale setting involving new algorithms. Docking these sampled conformations will also certainly require improvement in clustering approaches.

5.4. Data Integration

With the increasing popularity of scientific workflows, public and private repositories are gaining importance as a means to share, find, and reuse such workflows.

As the sizes of these repositories grow, methods to compare the scientific workflows stored in them become a necessity, for instance, to allow duplicate detection or similarity search. Scientific workflows are complex objects, and their comparison entails a number of distinct steps from comparing atomic elements to comparison of the workflows as a whole. Various studies have implemented methods for scientific workflow comparison and came up with often contradicting conclusions upon which algorithms work best. Comparing these results is cumbersome, as the original studies mixed different approaches for different steps and used different evaluation data and metrics.

In collaboration with members of the University of Humboldt (Berlin), we first contribute to the field [17] by (i) comparing in isolation different approaches taken at each step of scientific workflow comparison, reporting on an number of unexpected findings, (ii) investigating how these can best be combined into aggregated measures, and (iii) making available a gold standard of over 2000 similarity ratings contributed by 15 workflow experts on a corpus of 1500 workflows and re-implementations of all methods we evaluated. In this context, we have designed new approaches based on consensus ranking [21] to provide a consensus of the experts' answers.

Then, with members of the University of Pennsylvania, we have presented a novel and intuitive workflow similarity measure that is based on layer decomposition [27] (designed during the month SCB spent at UPenn). Layer decomposition accounts for the directed dataflow underlying scientific workflows, a property which has not been adequately considered in previous methods. We comparatively evaluate our algorithm using our gold standard and show that it a) delivers the best results for similarity search, b) has a much lower runtime than other, often highly complex competitors in structure-aware workflow comparison, and c) can be stacked easily with even faster, structure-agnostic approaches to further reduce runtime while retaining result quality.

Another way to make scientific workflows easier to reuse is to reduce their structural complexity to make them easier to apprehend. In particular, we have continued to work in collaboration with the University of Manchester on DistillFlow, an approach to remove the structural redundancy in workflows. Our contribution is four fold. Firstly, we identify a set of anti-patterns that contribute to the structural workflow complexity. Secondly, we design a series of refactoring transformations to replace each anti-pattern by a new semantically-equivalent pattern with less redundancy and simplified structure. Thirdly, we introduce a distilling algorithm that takes in a workflow and produces a distilled semantically-equivalent workflow [8]. Lastly, we provide an implementation of our refactoring approach (dedicated demo published [24]) that we evaluate on both the public Taverna workflows and on a private collection of workflows from the BioVel project. On going work includes extending the list of anti-patterns to be considered and identifying *good patterns*, that is, patterns which are easy to maintain and have systematically been able to be executed. This has been done in the context of the master internship of Stéphanie Kamgnia Wonkap [37]. First results obtained are promising.

5.5. Systems Biology

5.5.1. Analyzing SBGN-AF Networks Using Normal Logic Programs

A wide variety of signaling networks are available in the literature or in databases under the form of influence graphs. In order to understand the systems underlying these networks and to modify them for a medical purpose, it is necessary to understand their dynamics. Consequently, a variety of modelling techniques for these networks have been developped. In particular, it is possible to model their dynamical behavior with Boolean networks. The construction of these Boolean networks starting from influence graphs requires a parametrization of some Boolean functions. This task is most often realized by interpreting experimental results, that can be hard to obtain.

We introduced a method that allows to model any influence graph expressed in the Systems Biology Graphical Notation Activity Flow language (SBGN-AF) under the form of a Boolean network [32], [29]. The parametrization does not rely on any experimental results but on general principles that govern the dynamics of signaling networks. Together with the translation of a SBGN-AF influence graph into predicates, these general principles expressed under the form of logic rules form a first-order normal logic program (NLP) equivalent to a Boolean network. We show that the trajectories as well as the steady-state of any SBGN-AF network can be obtained by computing the orbits and the supported models of its corresponding NLP, respectively.

5.5.2. Scalable methods for analysing dynamics of automata networks

In collaboration with T. Chatain, S. Haar, S. Schwoon, and L. Jezeguel (INRIA MEXICO), we explored new techniques for computing the reachable attractors in automata networks using Petri net unfoldings [22]. Attractors of network dynamics represent the long-term behaviours of the modelled system. Their characterization is therefore crucial for understanding the response and differentiation capabilities of a dynamical system. In the scope of qualitative models of interaction networks, the computation of attractors

reachable from a given state of the network faces combinatorial issues due to the state space explosion. Our new algorithm relies on Petri net unfoldings that can be used to compute a compact representation of the dynamics, in particular by exploiting the concurrency of the transitions in order to remove redundant sequences of transitions. We illustrate the applicability of the algorithm with Petri net models of cell signalling and regulation networks, Boolean and multi-valued. The proposed approach aims at being complementary to existing methods for deriving the attractors of Boolean models, while being generic since it actually applies to any safe Petri net.

In collaboration with M. Folschette, M. Magnin, O. Roux (IRCCyN, Nantes), and K. Inoue (Nii, Tokyo), we developed a framework for identifying classical Boolean or discrete networks models from Proces Hitting (PH) models [10]. The PH allows to model non-deterministic cooperations between interacting components, and we have shown that the dynamics of a single PH can embed (include) the dynamics of multiple discrete networks, where transitions functions are deterministic. Hence, if a behaviour is shown impossible at the PH model, it is necessary impossible in any included discrete models. Such kind of analysis is relevant in systems biology, where the cooperations between components are often under-determined and the enumeration of all compatible discrete models is intractable: our framework allows to reason on the dynamics of a single abstract model.

Finally, a chapter summarizing the recent advances on static analysis for dynamics of large biological networks has been published as part of the *Logical Modeling of Biological Systems* handbook [30].

GALEN Project-Team

6. New Results

6.1. Highlights of the Year

- Handbook of Biomedical Imaging: Methodologies and Clinical Research [38] - co-edited from Nikos Paragios, James Duncan and Nicholas Ayache - has been published from Springer Publishing house.
- Nikos Paragios was admitted as a senior fellow at the Insitut Universitaire de France and has been awarded an IBM Faculty award. He has also been one of the plenary invited lecturers at the IARP International Conference in Pattern Recognition (ICPR'2015, Stockholm).

BEST PAPER AWARD :
[26] **Sparsity Techniques in Medical Imaging (STMI)**. M. MISYRLIS, A. KONOVA, M. BLASCHKO, J. HONORIO, N. ALIA-KLEIN, R. GOLDSTEIN, D. SAMARAS.

6.2. Rounding-based Moves for Metric Labeling

Paticipants: M. Pawan Kumar

Metric labeling is an important special case of energy minimizaton in Markov random fields. While the best known polynomial-time algorithm for the problem is the linear programming (LP) relaxation, in practice it is slow to solve it. In [25], we introduced a new family of efficient move-making algorithms for metric labeling. These algorithms mimic the rounding procedues used for converting a fractional LP solution to a feasible integral solution. Our algorithms provide a matching theoretical guarantee to the LP relaxation, while requiring significantly less computational time.

6.3. Optimizing Average Precision

Paticipants: Puneet Kumar Dokania, Aseem Behl, Pritish Mohapatra, C.V. Jawahar, M. Pawan Kumar

Average precision (AP) is one of the most commonly used measures for ranking. However, due to the inefficiency of optimizing it during learning, a common approach is to use surrogate loss functions such as 0-1 loss. In [27], we proposed a new optimization algorithm for AP-SVM that allows training in a similar time to binary SVM. In [23], we extended the AP-SVM framework to score the samples according to high-order information, as opposed to simple first-order information used in prior work. Finally, in [19], we proposed a novel latent AP-SVM formulation that allows learning from weakly supervised datasets. The advantage of learning with high order and missing information is demonstrated on challenging computer vision problems such as action classification and object detection using standard benchmark datasets.

6.4. Discriminative Training of Deformable Contour Models

Paticipants: Haithem Boussaid, Iasonas Kokkinos and Nikos Paragios

Deformable Contour Models (DCMs) are a main workhorse for medical image analysis - but are not commonly studied from a machine learning perspective. In [21], [20] we haved proposed an integrated machine learning and optimization framework to deploy DCMs in medical image analysis.

Our technical contributions are two-fold: firstly, we use an efficient decomposition-coordination algorithm to solve the optimization problems resulting from Loopy DCMs, by means of the Alternating Direction Method of Multipliers (ADMM); this yields substantially faster convergence than plain Dual Decomposition-based methods.

Secondly, we use structured prediction to exploit loss functions that better reflect the performance criteria used in medical image segmentation. By using the mean contour distance (MCD) as a structured loss during training, we obtain clear test-time performance gains.

We demonstrate the merits of exact and efficient inference with rich, structured models in a large X-Ray image segmentation benchmark, where we obtain systematic improvements over the current state-of-the-art.

6.5. Improved Deformable Part Models for Object Detection

Paticipants: Iasonas Kokkinos, Stavros Tsogkas, Eduard Trulls, Pierre-Andre Savalle, George Papandreou.

In [30] and [36] we have worked on improving the classification accuracy of Deformable Part Models (DPMs) for object detection in two distinct manners. Firstly, in [30] we propose a technique to combine bottom-up segmentation, coming in the form of SLIC superpixels, with sliding window DPM detectors. The merit of our approach lies in 'cleaning up' the low- level features by exploiting the spatial support indicated by segmentation. - tion, for both the root and part filters of DPMs. We use these masks to construct enhanced, background- invariant features to train DPMs. We test our approach on the PASCAL VOC 2007, outperforming the standard DPM in 17 out of 20 classes, yielding an average increase of 1.7AP. Additionally, we demonstrate the robustness of this ap- proach, extending it to dense SIFT descriptors for large dis- placement optical flow.

Secondly, in [36] we have explored the potential of convolutional neural networks as feature extractors for detection with DPMs. In particular, we substitute the Histogram-of-Gradient features of DPMs with Convolutional Neural Network (CNN) features, and demonstrate that we thereby obtain a substantial boost in performance (+14.5 mAP) when compared to the baseline HOG-based models. Some more recent extensions to this work are included in [41] where we explore the potential of explicit scale and aspect ratio search in the context of sliding window detection with CNNs.

6.6. Fine-Grained models of objects and texture

Paticipants: Iasonas Kokkinos, Matthew Blaschko, Stavros Tsogkas, Andrea Vedaldi, Mircea Cimpol, Subhransu Maji, Ross Girshick, Juho Kannala, Esa Rahtu, David Weiss, Ben Taskar, Karen Simonyan.

In [31] and [22] we explore methods for the fine-grained understanding of objects and textures, respectively.

In [22] we introduce a texture dataset that is accompanied by descriptions that capture the essence of the textures in terms of attributes. We explore a broad range of classification techniques for these texture attributes and demonstrate that the learned classifiers help improve generic texture recognition methods.

In [31] we introduce a large-scale dataset of airplanes that is accompanied by thorough human annotations at different levels: airplane types, segment lineouts, attributes, and part descriptions are provided for more than 7000 airplane images. We explore the potential of these rich annotations for the task of constructed fine-grained image descriptions using discriminative training techniques on top of standard image representations (Histogram-of-gradient features).

6.7. Large Scale Video Segmentation

Paticipants: Matthew Blaschko

Spatio-temporal cues are powerful sources of information for segmentation in videos. In [24] we present an efficient and simple technique for spatio-temporal segmentation that is based on a low-rank spectral clustering algorithm. The complexity of graph-based spatio-temporal segmentation is dominated by the size of the graph, which is proportional to the number of pixels in a video sequence. In contrast to other works, we avoid oversegmenting the images into super-pixels and instead generalize a simple graph based image segmentation. Our graph construction encodes appearance and motion information with temporal links based on optical flow. For large scale data sets naïve graph construction is computationally and memory intensive, and has only been achieved previously using a high power compute cluster. We make feasible for the first time large scale graph-based spatio-temporal segmentation on a single core by exploiting the sparsity structure of the problem and a low rank factorization that has strong approximation guarantees.

6.8. Higher Order Graph Matching

Paticipants: Chaohui Wang, Dimitris Samaras, Nikos Paragios

In [42] a generic framework for sparse and dense graph/3D surface matching has been introduced. The framework is endowed with a novel mathematical formulation regarding the matching process along with a novel deformation model. It exploits the power of invariance of higher order clique potentials and through a low to high resolution approach determines optimal correspondences between two sets of 3D points while taking advantage of Mobius tranformation to measure local similarity of shapes/graphs/surfaces. Graph matching of objects undergoing non-rigid deformations along with temporal 3D surface tracking demonstrated the potentials of our method. Inference is solved through an efficient dual decomposition scema.

6.9. Inference of Procedural Grammars from Images

Paticipants: Nikos Paragios

Grammar-like representations are powerful modeling and inference tools in computational vision. In [39] a novel approach towards automatic inference of typology specific building grammars has been introduced. The central idea was to consider that such grammars could be derived through a bottom up approach of common sub-tree reasoning of derivation trees determined through parsing using elementary shape (binary split) grammars. Such an approach performs common subtree reduction within the entire training set and identifies meta-rules (corresponding to the same subtrees) which are then clustered together towards producing a compact, typology specific grammar. Promising results both in terms of grammar compactness as well as in terms of inference demonstrated the potentials of the method that could be used beyond the considered scoped.

6.10. Fully connected CRFs for blood vessel segmentation in retinal images

Paticipants: Matthew Blaschko, José Ignacio Orlando

In [28], we present a novel method for blood vessel segmentation in fundus images based on a discriminatively trained, fully connected conditional random field model. Retinal image analysis is greatly aided by blood vessel segmentation as the vessel structure may be considered both a key source of signal, e.g. in the diagnosis of diabetic retinopathy, or a nuisance, e.g. in the analysis of pigment epithelium or choroid related abnormalities. Blood vessel segmentation in fundus images has been considered extensively in the literature, but remains a challenge largely due to the desired structures being thin and elongated, a setting that performs particularly poorly using standard segmentation priors such as a Potts model or total variation.

6.11. Graph-based Segmentation

Paticipants: Sarah Parisot, Deepak Chittajallu, Ioannis Kakadiaris, Nikos Paragios

In [17] we revisited explicit contour-evolution segmentation methods driven from a graph-based shape prior. Prior knowledge through geometric constraints has been encoded to the model within pair-wise interactions between control points. The segmentation process was driven through an objective function seeking to move the control points towards image locations optimizing the expected visual properties of the organ while satisfying the prior geometric constraints being learned at training. In [18] we have proposed a mathematical formalism for automatic tumor segmentation which was taking advantage of conventional segmentation likelihoods and atlas-based segmentation methods. The central idea was to jointly deform and segment an atlas such that the tumor likelihoods are maximized once projected to the targeted image while relaxing the registration constraints in this area. Furthermore we have endowed to this framework explicit estimation of uncertainties allowing the dynamic sampling of the graph structure resulting on significant speed up of the process while producing quantitative means for the interpretation of the final result.

6.12. Multi-atlas Segmentation

Paticipants: Stavros Alchatzidis, Aristeidis Sotiras, Nikos Paragios

In [33] a novel approach that couples pair-wise deformable registration with multi-atlas segmentation using graphical models was proposed. The method exploits both spaces and seeks to determine the optimal solution which will create the best possible visual agreement between atlases and target image along with their label consistency. The approach optimizes the deformation models and the segmentation labels jointly through an interconnected graph allowing either to relax registration constraints when segmentation labels do indicate or the opposite. The joint optimization of both spaces allowing the "implicit" automatic selection of atlases and therefore improves significantly segmentation performance.

6.13. Higher Order Graph Training throuh Dual Decomposition and Max Margin Principles

Paticipants: Nikos Komodakis, Bo Xiang, Nikos Paragios

In [40] a novel framework based on the structure margin principle was introduced for training higher order graphical models. The idea was to reduce the training of a complex high-order MRF in the parallel training of a series of simple slave MRFs through a principled dual decomposition approach. The theoretical properties of the framework have been studied while the method has been experimentally tested using 2d/3d segmentation problems involving higher order geometric priors that are linear-invariant. The proposed formulation benefits from theoretical guarantees as it concerns performance, computational simplicity while being modular and scalable.

M3DISIM Team

6. New Results

6.1. Highlights of the Year

- Radomir Chabiniok recruited in starting research position (start Febr 2015);
- PhD Defense of Annabelle Collin;
- "Usine Nouvelle" article.

6.2. Modeling

6.2.1. Mechanics of collective unfolding

Participants: Matthieu Caruel [correspondant], Jean-Marc Allain [LMS], Lev Truskinovsky [LMS].

Mechanically induced unfolding of passive crosslinkers is a fundamental biological phenomenon encountered across the scales from individual macro-molecules to cytoskeletal actin networks. In this work we study a conceptual model of athermal load-induced unfolding and use a minimalistic setting allowing one to emphasize the role of long-range interactions while maintaining full analytical transparency. Our model can be viewed as a description of a parallel bundle of N bistable units confined between two shared rigid backbones that are loaded through a series spring. We show that the ground states in this model correspond to synchronized, single phase configurations where all individual units are either folded or unfolded. We then study the fine structure of the wiggly energy landscape along the reaction coordinate linking the two coherent states and describing the optimal mechanism of cooperative unfolding. Quite remarkably, our study shows the fundamental difference in the size and structure of the folding-unfolding energy barriers in the hard (fixed displacements) and soft (fixed forces) loading devices which persists in the continuum limit. We argue that both, the synchronization and the non-equivalence of the mechanical responses in hard and soft devices, have their origin in the dominance of long-range interactions. We then apply our minimal model to skeletal muscles where the power-stroke in acto-myosin crossbridges can be interpreted as passive folding. A quantitative analysis of the muscle model shows that the relative rigidity of myosin backbone provides the long-range interaction mechanism allowing the system to effectively synchronize the power-stroke in individual crossbridges even in the presence of thermal fluctuations. In view of the prototypical nature of the proposed model, our general conclusions pertain to a variety of other biological systems where elastic interactions are mediated by effective backbones.

6.2.2. Thermodynamical framework for modeling chemical-mechanical coupling in muscle contraction - Formulation and validation

Participants: Matthieu Caruel, Dominique Chapelle [correspondant], Philippe Moireau.

Muscle contraction occurs at the nanoscale of a hierarchical multi-scale structure with the attachment of so-called cross-bridges within sarcomeres, namely, the creation of chemical bonds between myosin heads and specific sites on actin filaments. A cross-bridge in itself can be seen as a special chemical entity having internal mechanical variables - or degrees of freedom - pertaining to the actual geometric configuration, which implies that the free energy of the cross-bridge - whether in an attached or unattached state - must be made dependent on these internal variables (T.L. Hill, Free Energy Transduction And Biochemical Cycle Kinetics, Dover, 2004). This provides a thermodynamical basis for modeling the complex interplay of chemical and mechanical phenomena at the sarcomere level. Within this framework we propose a muscle model with two mechanical variables associated with a cross-bridge. For the action of individual cross-bridges occurring at the nanometer scale, the energy provided by the Langevin thermostat cannot be neglected, and we therefore propose to endow the internal mechanical variables with stochastic dynamics. Important motivations for this modeling choice include the ability to represent (i) the so-called power-stroke phenomenon and (ii) short-time responses of a muscle, e.g. to load steps. Our approach allows for systematic treatment of the model energetics, and in particular one goal of the proposed description is to investigate the potential benefit in mechanical efficiency with systems including - in addition to chemically-induced transformations - thermally-induced conformational changes such as the power-stroke.

6.2.3. Mechanical modeling and numerical methods for poromechanics: Applications to cardiac perfusion

Participants: Bruno Burtschell, Dominique Chapelle [correspondant], Philippe Moireau.

We have previously formulated a rather general modeling framework of poromechanics – formulations that combine solid and fluid components to represent the behavior of a porous medium – to take into account large deformations and rapid fluid flows, see [16]. This allows to consider, in particular, the application of blood perfusion within the cardiac tissue, which features these specific complex phenomena, out of the scope of classical poromechanical models. One of our major objectives now, within the PhD of Bruno Burtschell, is to propose and assess some associated relevant numerical schemes.

Some existing algorithms of fluid-structure interaction, with which our poromechanics formulations feature deep similarities, have been implemented – in FreeFEM++, both in axisymmetric configuration and in 3D – and compared. Their numerical and theoretical analysis – consistency, convergence – has been performed. Then, the adaptation of these algorithms to our poromechanics formulations enabled us to propose a time discretisation well-fitted to our framework, and to present its energy stability analysis. Further perspectives include implementation and numerical validation of this scheme, including special care regarding space discretisation, then integration into FELISCE ("HappyHeart" module).

6.2.4. Personalized modeling for cardiac amyloidosis diagnosis

Participants: Alessandro Felder, Dominique Chapelle, Philippe Moireau, Jean-François Deux [Hôpital Henri Mondor], Thibault Damy [Hôpital Henri Mondor].

Cardiac amyloidosis is a condition induced by pathological deposition of amyloid proteins within muscle tissue and nerves, thus severely impairing the cardiac function and often requiring cardiac transplantation as the only available treatment. Our objective here in a first stage is to use our previously developed patient-specific modeling methodologies to analyse some clinical cases – based on actual patient data – to better apprehend the impact of the pathology on biomechanical properties. Further perspectives include the modeling of the protein deposition and associated tissue remodeling in order to predict the disease evolution in a patient-specific context. This work is performed in collaboration with medical doctors from Hôpital Henri Mondor (Créteil).

6.3. Numerical Analysis

6.3.1. Fourth-order energy-preserving locally implicit discretization for linear wave equations

Participants: Juliette Chabassier [Magique-3d], Sébastien Imperiale [correspondant].

A family of fourth-order coupled implicit-explicit time schemes has been developed. The spatial coupling is done at the boundaries of several non conforming meshes of regions in which we want to simulate propagating waves. A global discrete energy is shown to be preserved and leads to global fourth-order consistency. Numerical results in 1D and 2D have been produced to illustrate the good behavior of the schemes and their potential for the simulation of realistic highly heterogeneous media and strongly refined geometries, for which using an explicit scheme everywhere can be extremely penalizing. Accuracy up to fourth order reduces the numerical dispersion inherent to implicit methods used with a large time step, and makes this family of schemes attractive compared to second order accurate methods in time.

6.4. Model-Data Interaction

6.4.1. A Luenberger observer for reaction-diffusion models with front position data

Participants: Dominique Chapelle, Annabelle Collin, Philipe Moireau [correspondant].

We propose a Luenberger observer for reaction-diffusion models with propagating front features, and for data associated with the location of the front over time. Such models are considered in various application fields, such as electrophysiology, wild-land fire propagation and tumor growth modeling. Drawing our inspiration from image processing methods, we start by proposing an observer for the eikonal-curvature equation that can be derived from the reaction-diffusion model by an asymptotic expansion. We then carry over this observer to the underlying reaction-diffusion equation by an "inverse asymptotic analysis", and we show that the associated correction in the dynamics has a stabilizing effect for the linearized estimation error. We also discuss the extension to joint state-parameter estimation by using the earlier-proposed ROUKF strategy. We then illustrate and assess our proposed observer method with test problems pertaining to electrophysiology modeling, including with a realistic model of cardiac atria. Our numerical trials show that state estimation is directly very effective with the proposed Luenberger observer, while specific strategies are needed to accurately perform parameter estimation – as is usual with Kalman filtering used in a nonlinear setting – and we demonstrate two such successful strategies.

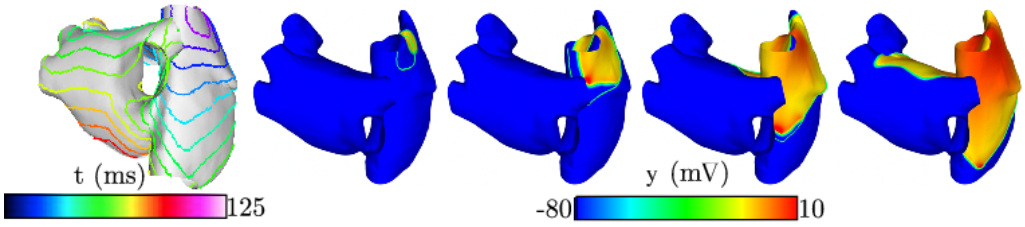

Figure 1. Collocated front data on an atria (left), and observer of the atrial electric activation pursuing the green front from a wrong initial condition (right, 4 time-steps)

6.4.2. Identification of weakly coupled multiphysics problems. Application to the inverse problem of electrocardiography

Participants: Cesare Corrado [Reo team], Jean-Frédéric Gerbeau [Reo team], Philippe Moireau [correspondant].

This work addresses the inverse problem of electrocardiography from a new perspective, by combining electrical and mechanical measurements. Our strategy relies on the definition of a model of the electromechanical contraction which is registered on ECG data but also on measured mechanical displacements of the heart tissue typically extracted from medical images. In this respect, we establish in this work the convergence of a sequential estimator which combines for such coupled problems various state of the art sequential data assimilation methods in a unified consistent and efficient framework. Indeed, we aggregate a Luenberger observer for the mechanical state and a Reduced-Order Unscented Kalman Filter applied on the parameters to be identified and a POD projection of the electrical state. Then using synthetic data we show the benefits of our approach for the estimation of the electrical state of the ventricles along the heart beat compared with more classical strategies which only consider an electrophysiological model with ECG measurements. Our numerical results actually show that the mechanical measurements improve the identifiability of the electrical problem allowing to reconstruct the electrical state of the coupled system more precisely. Therefore, this work is intended to be a first proof of concept, with theoretical justifications and numerical investigations, of the advantage of using available multi-modal observations for the estimation and identification of an electromechanical model of the heart.

6.4.3. Data assimilation for hyperbolic conservation laws. A Luenberger observer approach based on a kinetic description

Participants: Anne-Céline Boulanger [Ange team], Benoît Perthame [Mamba team], Philippe Moireau [correspondant], Jacques Sainte-Marie [Ange team].

Developing robust data assimilation methods for hyperbolic conservation laws is a challenging subject. Those PDEs indeed show no dissipation effects and the input of additional information in the model equations may introduce errors that propagate and create shocks. We propose a new approach based on the kinetic description of the conservation law. A kinetic equation is a first order partial differential equation in which the advection velocity is a free variable. In certain cases, it is possible to prove that the nonlinear conservation law is equivalent to a linear kinetic equation. Hence, data assimilation is carried out at the kinetic level, using a Luenberger observer also known as the nudging strategy in data assimilation. Assimilation then amounts to the handling of a BGK type equation. The advantage of this framework is that we deal with a single "linear" equation instead of a nonlinear system and it is easy to recover the macroscopic variables. The study is divided into several steps and essentially based on functional analysis techniques. First we prove the convergence of the model towards the data in case of complete observations in space and time. Second, we analyze the case of partial and noisy observations. To conclude, we validate our method with numerical results on Burgers equation and emphasize the advantages of this method with the more complex Saint-Venant system.

6.4.4. Optimal observer for parabolic problems

Participants: Karine Mauffrey, Philippe Moireau [correspondant].

We aim at proposing optimal observers strategies for reconstructing the solution of general systems of PDEs using available observations, including both wave-type equations and heat-like equations. The main objective of this work is to present a self-contained analysis. For a general parabolic system, we have established the exponential stability of the operator occurring in the equation satisfied by the error between the target and the optimal observer. The proof relies on two major hypotheses: an observability inequality satisfied by the observation operator and a controllability property for the modeling error operator by which model noises enter the dynamics (controllability property which is related to the invertibility of the solution of the associated infinite dimensional Riccati equation). Then we have tackled the discretisation questions and demonstrated that the discrete-time Kalman filter is an adequate discretization of the continuous-time Kalman filter. Finally we have also studied the strong formulation of the Kalman observer using a kernel representation of the Riccati operator.

6.4.5. Elastography by magnetic resonance imaging

Participants: Guillaume Bal [Columbia Unviersity], Cedric Bellis [LMA Marseille], Sébastien Imperiale [correspondant], Francois Monard [University of Washington- Seattle].

We have studied the potential application of elastography by Magnetic Resonance Imaging (MRI) within the framework of linear elasticity. We assume given internal full-field MRI measurements of the deformations of a non-homogeneous isotropic solid, and the aim is the quantitative reconstruction of the associated physical parameters. Upon using polluted measurements, a variational formulation is constructed, its inversion enabling the recovery of the parameters. The analysis of this inversion procedure provides existence and uniqueness results while the reconstruction stability with respect to the measurements is investigated. As the inversion procedure requires differentiating the measurements twice, a numerical differentiation scheme has been proposed and analyzed. It is based on a regularization that allows an optimally stable reconstruction of the sought parameters.

PARIETAL Project-Team

6. New Results

6.1. Highlights of the Year

- Congratulations also to Alex and Daniel Strohmeier for their best paper award at the PRNI 2014 conference: “Improved MEG/EEG source localization with reweighted mixed-norms“.
- Elvis Dohmatob got a honorable mention for the student paper award at PRNI 2014 for the work “Benchmarking solvers for TV-l1 least-squares and logistic regression in brain imaging”

6.2. Which fMRI clustering gives good brain parcellations?

Participants: Bertrand Thirion [Correspondant], Gaël Varoquaux, Elvis Dohmatob.

Analysis and interpretation of neuroimaging data often require one to divide the brain into a number of regions, or parcels, with homogeneous characteristics, be these regions defined in the brain volume or on on the cortical surface. While predefined brain atlases do not adapt to the signal in the individual subjects images, parcellation approaches use brain activity (e.g. found in some functional contrasts of interest) and clustering techniques to define regions with some degree of signal homogeneity. In this work, we address the question of which clustering technique is appropriate and how to optimize the corresponding model. We use two principled criteria: goodness of fit (accuracy), and reproducibility of the parcellation across bootstrap samples. We study these criteria on both simulated and two task-based functional Magnetic Resonance Imaging datasets for the Ward, spectral and K-means clustering algorithms. We show that in general Ward’s clustering performs better than alternative methods with regard to reproducibility and accuracy and that the two criteria diverge regarding the preferred models (reproducibility leading to more conservative solutions), thus deferring the practical decision to a higher level alternative, namely the choice of a trade-off between accuracy and stability.

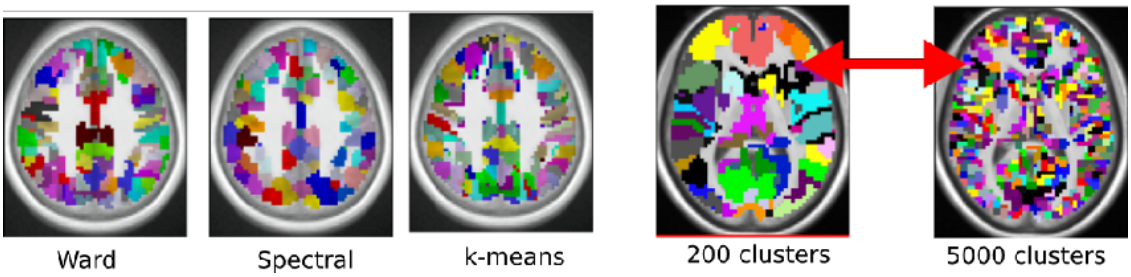

Figure 3. Practitioner have to decide which clustering method to use and how to select the number of clusters. In [21], we provide empirical guidelines and criteria to guide that choice in the context of functional brain imaging.

More details can be found in [21].

6.3. Principal Component Regression predicts functional responses across individuals

Participants: Bertrand Thirion [Correspondant], Gaël Varoquaux, Olivier Grisel.

Inter-subject variability is a major hurdle for neuroimaging group-level inference, as it creates complex image patterns that are not captured by standard analysis models and jeopardizes the sensitivity of statistical procedures. A solution to this problem is to model random subjects effects by using the redundant information conveyed by multiple imaging contrasts. In this paper, we introduce a novel analysis framework, where we estimate the amount of variance that is fit by a random effects subspace learned on other images; we show that a principal component regression estimator outperforms other regression models and that it fits a significant proportion (10% to 25%) of the between-subject variability. This proves for the first time that the accumulation of contrasts in each individual can provide the basis for more sensitive neuroimaging group analyzes.

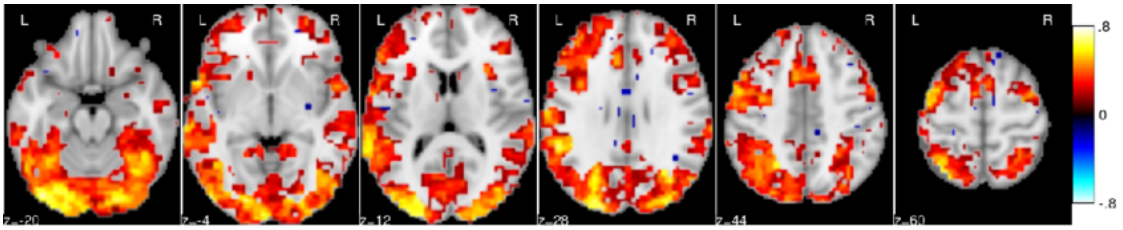

Figure 4. In most brain regions, knowing the amount of activation related to a set of reference contrasts yields an accurate prediction of the activation for a target contrast.

More details can be found in [36].

6.4. Deriving a multi-subject functional-connectivity atlas to inform connectome estimation

Participants: Ronald Phlypo [Correspondant], Bertrand Thirion, Gaël Varoquaux.

The estimation of functional connectivity structure from functional neuroimaging data is an important step toward understanding the mechanisms of various brain diseases and building relevant biomarkers. Yet, such inferences have to deal with the low signal-to-noise ratio and the paucity of the data. With at our disposal a steadily growing volume of publicly available neuroimaging data, it is however possible to improve the estimation procedures involved in connectome mapping. In this work, we propose a novel learning scheme for functional connectivity based on sparse Gaussian graphical models that aims at minimizing the bias induced by the regularization used in the estimation, by carefully separating the estimation of the model support from the coefficients. Moreover, our strategy makes it possible to include new data with a limited computational cost. We illustrate the physiological relevance of the learned prior, that can be identified as a functional connectivity atlas, based on an experiment on 46 subjects of the Human Connectome Dataset.

More details can be found in [35].

6.5. Machine Learning Patterns for Neuroimaging-Genetic Studies in the Cloud

Participants: Virgile Fritsch, Bertrand Thirion, Gaël Varoquaux.

Brain imaging is a natural intermediate phenotype to understand the link between genetic information and behavior or brain pathologies risk factors. Massive efforts have been made in the last few years to acquire high-dimensional neuroimaging and genetic data on large cohorts of subjects. The statistical analysis of such data is carried out with increasingly sophisticated techniques and represents a great computational challenge. Fortunately, increasing computational power in distributed architectures can be harnessed, if new neuroinformatics infrastructures are designed and training to use these new tools is provided. Combining a

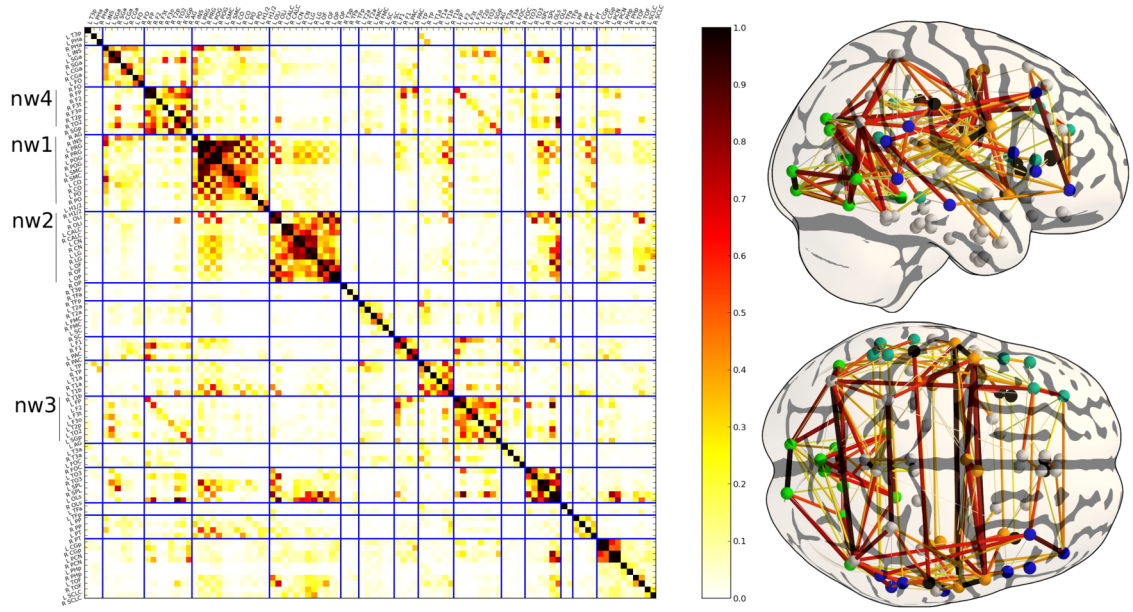

Figure 5. Prior on the functional connectivity: the coefficient of the matrix represent the frequency of an edge at each position. This model can be interpreted as a data-driven atlas of brain functional connections. In the current framework, it can easily be updated to take into account more data.

MapReduce framework (TomusBLOB) with machine learning algorithms (Scikit-learn library), we design a scalable analysis tool that can deal with non-parametric statistics on high-dimensional data. End-users describe the statistical procedure to perform and can then test the model on their own computers before running the very same code in the cloud at a larger scale. We illustrate the potential of our approach on real data with an experiment showing how the functional signal in subcortical brain regions can be significantly fit with genome-wide genotypes. This experiment demonstrates the scalability and the reliability of our framework in the cloud with a two weeks deployment on hundreds of virtual machines.

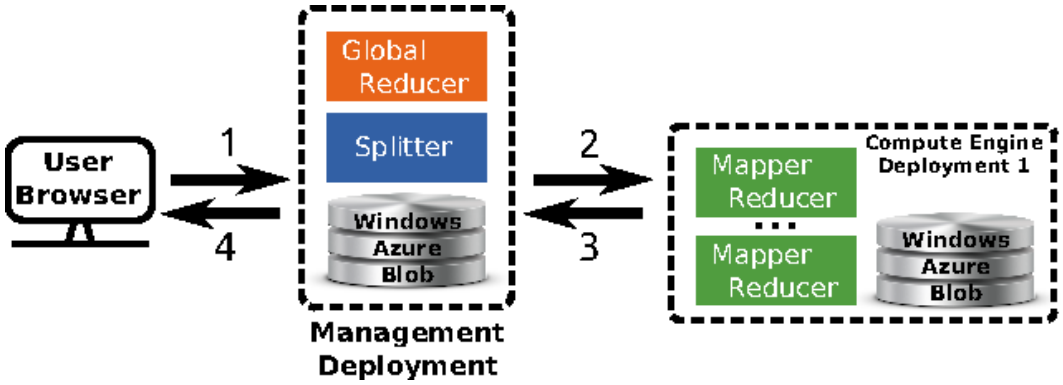

Figure 6. Overview of the multi site deployment of a hierarchical Tomus-MapReduce compute engine. 1) The end-user uploads the data and configures the statistical inference procedure on a webpage. 2) The Splitter partitions the data and manages the workload. The compute engines retrieves job information trough the Windows Azure Queues. 3) Compute engines perform the map and reduce jobs. The management deployment is informed of the progression via the Windows Azure Queues system and thus can manage the execution of the global reducer. 4) The user downloads the results of the computation on the webpage of the experiment.

More details can be found in [17].

6.6. Data-driven HRF estimation for encoding and decoding models

Participants: Fabian Pedregosa Izquierdo [correspondant], Michael Eickenberg, Alexandre Gramfort, Philippe Ciuciu, Bertrand Thirion, Gaël Varoquaux.

Despite the common usage of a canonical, data-independent, hemodynamic response function (HRF), it is known that the shape of the HRF varies across brain regions and subjects. This suggests that a data-driven estimation of this function could lead to more statistical power when modeling BOLD fMRI data. However, unconstrained estimation of the HRF can yield highly unstable results when the number of free parameters is large. We develop a method for the joint estimation of activation and HRF using a rank constraint causing the estimated HRF to be equal across events/conditions, yet permitting it to be different across voxels. Model estimation leads to an optimization problem that we propose to solve with an efficient quasi-Newton method exploiting fast gradient computations. This model, called GLM with Rank-1 constraint (R1-GLM), can be extended to the setting of GLM with separate designs which has been shown to improve decoding accuracy in brain activity decoding experiments. We compare 10 different HRF modeling methods in terms of encoding and decoding score in two different datasets. Our results show that the R1-GLM model significantly outperforms competing methods in both encoding and decoding settings, positioning it as an attractive method both from the points of view of accuracy and computational efficiency.

More details can be found in [19].

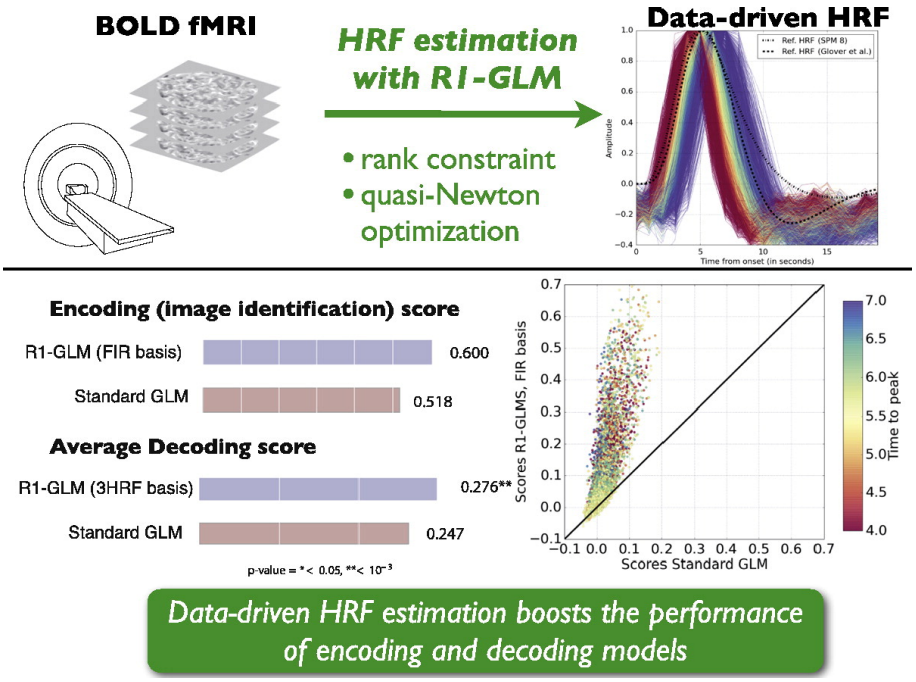

Figure 7. Illustration of the hemodynamic response function estimation framework introduced in [19].

6.7. Benchmarking solvers for TV-l1 least-squares and logistic regression in brain imaging

Participants: Elvis Dohmatob [correspondant], Michael Eickenberg, Gaël Varoquaux, Bertrand Thirion.

Learning predictive models from brain imaging data, as in decoding cognitive states from fMRI (functional Magnetic Resonance Imaging), is typically an ill-posed problem as it entails estimating many more parameters than available sample points. This estimation problem thus requires regularization. Total variation regularization, combined with sparse models, has been shown to yield good predictive performance, as well as stable and interpretable maps. However, the corresponding optimization problem is very challenging: it is non-smooth, non-separable and heavily ill-conditioned. For the penalty to fully exercise its structuring effect on the maps, this optimization problem must be solved to a good tolerance, resulting in a computational challenge. In this work, we explore a wide variety of solvers and exhibit their convergence properties on fMRI data. We introduce a variant of smooth solvers and show that it is a promising approach in these settings. Our findings show that care must be taken in solving TV-l1 estimation in brain imaging and highlight the successful strategies.

More details can be found in [30]

6.8. Interplay between functional connectivity and scale-free dynamics in intrinsic fMRI networks

Participant: Philippe Ciuciu [correspondant].

Studies employing functional connectivity-type analyses have established that spontaneous fluctuations in functional magnetic resonance imaging (fMRI) signals are organized within large-scale brain networks. Meanwhile, fMRI signals have been shown to exhibit 1/f-type power spectra – a hallmark of scale-free

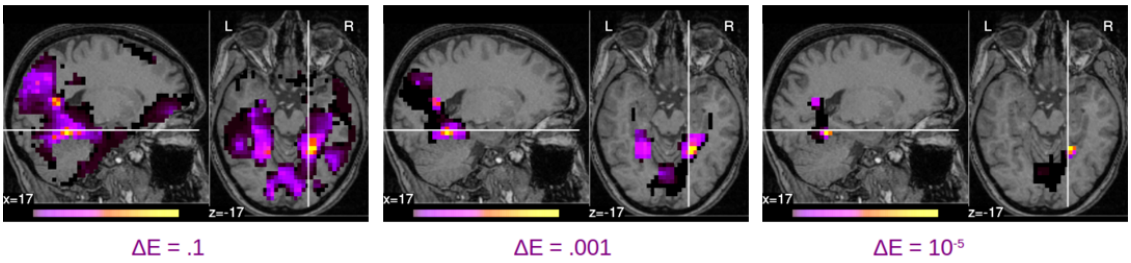

Figure 8. TV-l1 maps for a face-house discrimination task taken from a visual recognition dataset, with regularization parameters chosen by cross-validation, for different stopping criteria. Note that the stopping criterion is defined as a threshold on the energy decrease per one iteration of the algorithm. This figure shows the importance of convergence of the multivariate estimator, and motivates the need for a fast solver.

dynamics. We studied the interplay between functional connectivity and scale-free dynamics in fMRI signals, utilizing the fractal connectivity framework – a multivariate extension of the univariate fractional Gaussian noise model, which relies on a wavelet formulation for robust parameter estimation. We applied this framework to fMRI data acquired from healthy young adults at rest and performing a visual detection task. First, we found that scale-invariance existed beyond univariate dynamics, being present also in bivariate cross-temporal dynamics. Second, we observed that frequencies within the scale-free range do not contribute evenly to inter-regional connectivity, with a systematically stronger contribution of the lowest frequencies, both at rest and during task. Third, in addition to a decrease of the Hurst exponent and inter-regional correlations, task performance modified cross-temporal dynamics, inducing a larger contribution of the highest frequencies within the scale-free range to global correlation.

More details can be found in [16].

6.9. Supramodal processing optimizes visual perceptual learning and plasticity

Participants: Philippe Ciuciu [correspondant], Alexandre Gramfort.

Multisensory interactions are ubiquitous in cortex and it has been suggested that sensory cortices may be supramodal i.e. capable of functional selectivity irrespective of the sensory modality of inputs. Here, we asked whether learning to discriminate visual coherence could benefit from supramodal processing. To this end, three groups of participants were briefly trained to discriminate which of a red or green intermixed population of random-dot-kinematograms (RDKs) was most coherent in a visual display while being recorded with magnetoencephalography (MEG). During training, participants heard no sound (V), congruent acoustic textures (AV) or auditory noise (AVn); importantly, congruent acoustic textures shared the temporal statistics – i.e. coherence – of visual RDKs. After training, the AV group significantly outperformed participants trained in V and AVn although they were not aware of their progress. In pre- and post-training blocks, all participants were tested without sound and with the same set of RDKs. When contrasting MEG data collected in these experimental blocks, selective differences were observed in the dynamic pattern and the cortical loci responsive to visual RDKs. First and common to all three groups, vlPFC showed selectivity to the learned coherence levels whereas selectivity in visual motion area hMT+ was only seen for the AV group. Second and solely for the AV group, activity in multisensory cortices (mSTS, pSTS) correlated with post-training performances; additionally, the latencies of these effects suggested feedback from vlPFC to hMT+ possibly mediated by temporal cortices in AV and AVn groups. Altogether, we interpret our results in the context of the Reverse Hierarchy Theory of learning in which supramodal processing optimizes visual perceptual learning by capitalizing on sensory-invariant representations - here, global coherence levels across sensory modalities.

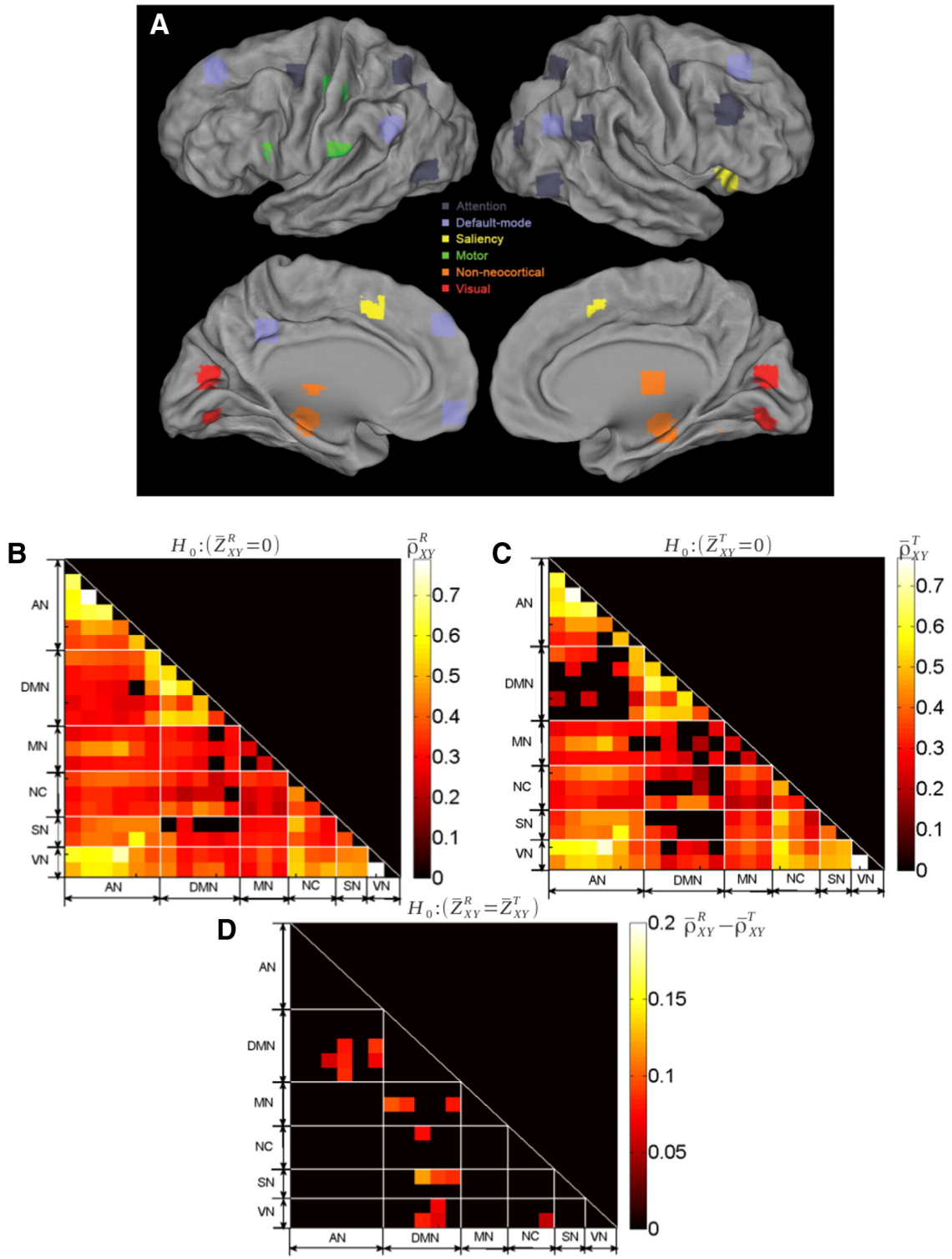

Figure 9. Networks definition and correlation structure. Top (A): ROIs mapped onto the cortical surface, with each color denoting a different network. Middle (B): Group-averaged inter-regional correlation matrix at rest ($p < 0.05$, Bonferroni corrected). Regions are grouped by network to ease visualization. Middle (C): Group-averaged inter-regional correlation matrix during the visual detection task ($p < 0.05$, Bonferroni corrected). Bottom (D): Difference of the correlation coefficients between rest and task (thresholded at $p < 0.01$, uncorrected, two-sample t-test for rest vs. task). The ROIs are grouped by networks; these networks correspond to the diagonal triangles surrounded by white dashed lines.

More details can be found in [25].

6.10. Variable density sampling with continuous trajectories. Application to MRI.

Participants: Nicolas Chauffert, Philippe Ciuciu [correspondant].

Reducing acquisition time is a crucial challenge for many imaging techniques. Compressed Sensing (CS) theory offers an appealing framework to address this issue since it provides theoretical guarantees on the reconstruction of sparse signals by projection on a low dimensional linear subspace. In this paper, we focus on a setting where the imaging device allows to sense a fixed set of measurements. We first discuss the choice of an optimal sampling subspace (smallest subset) allowing perfect reconstruction of sparse signals. Its standard design relies on the random drawing of independent measurements. We discuss how to select the drawing distribution and show that a mixed strategy involving partial deterministic sampling and independent drawings can help breaking the so-called "coherence barrier". Unfortunately, independent random sampling is irrelevant for many acquisition devices owing to acquisition constraints. To overcome this limitation, the notion of Variable Density Samplers (VDS) is introduced and defined as a stochastic process with a prescribed limit empirical measure. It encompasses samplers based on independent measurements or continuous curves. The latter are crucial to extend CS results to actual applications. Our main contribution lies in two original continuous VDS. The first one relies on random walks over the acquisition space whereas the second one is heuristically driven and rests on the approximate solution of a Traveling Salesman Problem. Theoretical analysis and retrospective CS simulations in magnetic resonance imaging highlight that the TSP-based solution provides improved reconstructed images in terms of signal-to-noise ratio compared to standard sampling schemes (spiral, radial, 3D iid...).

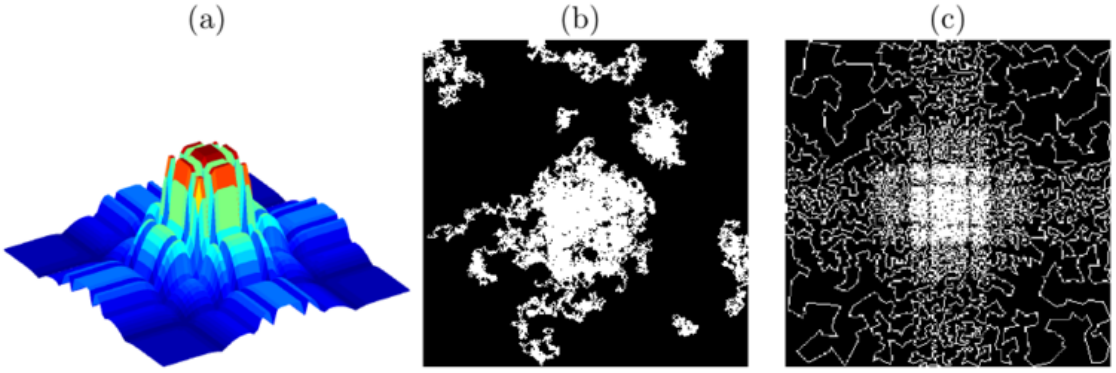

Figure 10. (a): Target distribution π to be approximated. Continuous random trajectories reaching distribution π based on Markov chains (b) and on a TSP solution (c). The latter is much more accurate.

More details can be found in [15].

POPIX Team

6. New Results

6.1. Highlights of the Year

Marc Lavielle published the book, *Mixed Effects Models for the Population Approach: Models, Tasks, Methods and Tools* (Chapman & Hall/CRC), which presents a rigorous framework for describing, implementing, and using mixed effects models. With these models, readers can perform parameter estimation and modeling across a whole population of individuals at the same time.

6.2. New result 1

We have proposed a nonlinear mixed-effects framework to jointly model longitudinal and repeated time-to-event data. A parametric nonlinear mixed-effects model is used for the longitudinal observations and a parametric mixed-effects hazard model for repeated event times. We have shown the importance for parameter estimation of properly calculating the conditional density of the observations (given the individual parameters) in the presence of interval and/or right censoring. Parameters are estimated by maximizing the exact joint likelihood with the Stochastic Approximation Expectation-Maximization algorithm. This workflow for joint models is now implemented in the Monolix software, and illustrated on several simulated and real data examples.

6.3. New result 2

We have succesfully extended the methodologies previously developed for ordinary differential equations (ODE) to delay differential equations (DDE). A C++ solver for DDE, and based on an explicit Runge-Kutta scheme, has been developed. This solver can now be used with Monolix, a platform for population modeling of longitudinal data, MlxPlore, a tool for the exploration of complex models and Simulx a R and Matlab function for the simulation of longitudinal data. We use.

INFINE Team

6. New Results

6.1. Highlights of the Year

- We proved a conjecture made in 2011 about the feasibility of non-trivial community detection just above a threshold below which it was known that only trivial detection could be done, see [13]. This was published in ACM STOC'14 and well-received, as the proof required the invention of new techniques to control the spectral properties of random matrices.
- The official opening of IoT-LAB of all sites through the "Workshop Internet Of Things/Equipex FIT IoT-LAB" held in Grenoble (on 6 and 6 november 2014), has been a major event for our team: it concludes several years of preparation of the IoT-LAB site located in Rocquencourt, currently managed by C. Adjih, E. Baccelli and I. Amdouni, which was itself opened the same month https://www.iot-lab.info/opening-of-the-paris-rocquencourt-site/.

6.2. Panorama

All the INFINE research activities encompass both theoretical or protocol designing research (to seek for conceptual advances or optimizations) and applied research (to validate and/or experiment the proposed concepts against real networking scenarios). The target applications range from Internet-based applications to mobile wireless networks. Taking an information- and user-centric perspective, we envision networks as means to convey relevant information to users, while adapting to customary practices (in terms of context, interests, or content demands) of such users. INFINE is thus organized along three main axes, namely Online Social Networks, Resource and Traffic Management, and Spontaneous Wireless Networks.

6.3. Online Social Networks (OSN)

Community detection; bandit algorithms; privacy preservation; reward mechanisms

6.3.1. Community detection

Participants: Laurent Massoulié, Marc Lelarge, Jiaming Xu.

We have progressed in the design of spectral methods for community detection and in the corresponding analysis (see above and references [3], [13], [22]).

6.3.2. Bandit algorithms for active learning of content type at low spam cost

Participants: Laurent Massoulié, Mesrob Ohanessian, Alexandre Proutière.

We developed a framework in which to cast the problem, and the so-called "greedy Bayes" algorithm to determine which user to expose to a given content. We proved corresponding optimality properties, and observed that "greedy Bayes" beats the so-called Thompson sampling approach, that is the state-of the-art method in bandit problems. Work currently under submission.

6.4. Resource and Traffic Management

Traffic offloading; infrastructure deployment; opportunistic routing; traffic modeling; intermittently connected networks.

6.4.1. From Routing to Network Deployment for Data Offloading in Metropolitan Areas

Participants: Eduardo Mucceli, Aline Carneiro Viana.

Smartphone sales are booming, nearly half billion were sold in 2011; more smartphones, more mobile data traffic, and Currently, 3G cellular networks in metropolitan areas are struggling to attend the recent boost up of mobile data consumption. Carefully deploying WiFi hotspots allow to maximize WiFi offloading and can both be cheaper than upgrade the cellular network structure and concede substantial improvement in the network capacity. In this context, in this work, we first propose a new way to map into a graph the *people behavior* (i.e., mobility context) in an urban scenario. Our proposed behavior-to-graph solution is simple, take into consideration the restrictions imposed by transportation modes to traffic demand, the space-time interaction between people and urban locations, and finally, is powerful to be used as input to any popular area identification problem (key points for an efficient network planning). Secondly, we propose a metric to identify locations more capable of providing coverage for people and consequently, more suitable for receiving hotspots. Deploying a small percentage of hotspots ranked by the herein proposed metric provides high percentages of coverage time for people moving around in the city. Using a real-life metropolitan trace, we show our routine-based strategy guarantees higher offload ratio than current approaches in the literature while using a realistic traffic model. Different parts of this work has been published in the international conferences IEEE SECON 2014 [14], IEEE WCNC 2014 [18] and IEEE WMNC 2014 [17], and in the international Student workshop IEEE Infocom [15]. An extended version of this paper is under submission in a trasaction. This version includes new characterization results of the used trace and new analysis of space-traffic correlation.

6.4.2. *Mobile Data Traffic Modeling: Revealing Hidden Facets*

Participants: Eduardo Mucceli, Aline Carneiro Viana, Kolar Purushothama Naveen, Carlos Sarraute.

Smartphone devices provide today the best means of gathering users information about content consumption behavior on a large scale. In this context, the literature is rich in work studying and modeling users mobility, but little is publicly known about users content consumption patterns. The *understanding of users' mobile data traffic demands* is of fundamental importance when looking for solutions to manage the recent boost up of mobile data usage [14] and to improve the quality of communication service provided. Hence, the definition of a *usage pattern* can allow telecommunication operators to better forseen future demanded traffic and consequently, to better (1) deploy data offloading hotspots or (2) timely plan network resources allocation and then, set subscription plans.

Using a large-scale dataset collected from a major 3G network in a big metropolitan area, in this work, we present the first detailed measurement-driven modeling of mobile data traffic usage of smartphone subscribers. Our main outcome is a synthetic measurement-based mobile data traffic generator, capable of imitating traffic-related activity patterns of different categories of subscribers and time periods of a routinary normal day in their lives. For this, we first characterize individual subscribers routinary behaviour, followed by the detailed investigation of subscribers' usage pattern (i.e., "when" and "how much" traffic is generated). Broadly, our observations bring important insights into network resource usage. We then classify the subscribers into six distinct profiles according to their usage pattern and model these profiles according to two different journey periods: peak and non-peak hours. We show that the synthetic trace generated by our data traffic model consistently imitates different subscriber profiles in two journey periods, when compared to the original dataset. We discuss relevant issues in traffic demands and describe implications in network planning and privacy. This work has been published in the international conference IEEE PERCOM 2014 [16]. An extended version of this paper is under submission in a trasaction and a technical report is available in [26]. This version includes new characterization results of the used trace, including analysis correlating age and gender to traffic demands, as well as new profiling results.

6.4.3. *On the Interaction between Content Caching and Routing*

Participants: Kolar Purushothama Naveen, Laurent Massoulié, Emmanuel Baccelli, Aline Carneiro Viana, Don Towsley.

Nowadays Internet users are mobile over 60% of their time online, and mobile data traffic is expected to increase by more than 60% annually to reach 15.9 exabytes per month by 2018. This evolution will likely incur durably congested wireless access at the edge despite progress in radio technologies. To alleviate congestion at the Internet edge, one promising approach is to target denser deployments of wireless access points. As a

result, mobile users are potentially within radio reach of several access points (AP) from which content may be directly downloaded. In this context, distinct AP's can have very different bandwidth and memory capacities. Such differences raise the following question: When requests can be sent to several such access points, how to optimize performance through both load balancing and content replication?

In this work, we introduce formal optimization models to address this question, where bandwidth availability is represented via a cost function, and content availability is represented either by a cost function or a sharp constraint. For both formulations we propose dynamic caching and request assignment algorithms. Crucially our request assignment scheme is based on a server price signal jointly reflecting content and bandwidth availability. Using mean field approximation and Lyapunov functions techniques, we prove that our algorithms are optimal and stable in a limiting fluid regime with large arrival rates and content chunking. Through simulations we exhibit the efficacy of our request assignment strategy in comparison to the common practices of assigning requests purely based on either bandwidth or content availability. Finally, using the popular LRU (Least Recently Used) strategy instead for cache replacements, we again demonstrate the superior performance of our request assignment strategies. This work in under submission in an international conference.

6.4.4. Data Delivery in Opportunistic and Intermittently Connected Networks

Participants: Ana Cristina Vendramin, Anelise Munaretto, Myriam Delgado, Aline Carneiro Viana, Mauro Fonseca.

The pervasiveness of computing devices and the emergence of new applications and cloud services are factors emphasizing the increasing need for adaptive networking solutions. In most cases, this adaptation requires the design of interdisciplinary approaches as those inspired by nature, social structures, games, and control systems. The approach presented in this work brings together solutions from different, yet complementary domains, i.e., networking, artificial intelligence, and complex networks, and is aimed at addressing the problem of efficient data delivery in intermittently connected networks.

As mobile devices become increasingly powerful in terms of communication capabilities, the appearance of opportunistic and intermittently connected networks referred to as Delay Tolerant Networks (DTNs) is becoming a reality. In such networks, contacts occur opportunistically in corporate environments such as conferences sites, urban areas, or university campuses. Understanding node mobility is of fundamental importance in DTNs when designing new communication protocols that consider opportunistic encounters among nodes. This work proposes the Cultural Greedy Ant (CGrAnt) protocol to solve the problem of data delivery in opportunistic and intermittently connected networks. CGrAnt is a hybrid Swarm Intelligence-based forwarding protocol designed to address the dynamic and complex environment of DTNs. CGrAnt is based on: (1) Cultural Algorithms (CA) and Ant Colony Optimization (ACO) and (2) operational metrics that characterize the opportunistic social connectivity between wireless users. The most promising message forwarders are selected via a greedy transition rule based on local and global information captured from the DTN environment. Using simulations, we first analyze the influence of the ACO operators and CA knowledge on the CGrAnt performance. We then compare the performance of CGrAnt with the PROPHET and Epidemic protocols (two well known related protocols in the literature) under varying networking parameters. The results show that CGrAnt achieves the highest delivery ratio (gains of 99.12% compared with PROPHET and 40.21% compared with Epidemic) and the lowest message replication (63.60% lower than PROPHET and 60.84% lower than Epidemic). This work is under submission to an international journal. Some parts of this work were previously published in the international conference ACM GECCO 2012 and in the Elsevier Computer Networks journal.

6.4.5. Vehicular Network under a Social Perception

Participants: Felipe D. Cunha, Aline Carneiro Viana, Raquel A. F. Mini, Antonio A.f. Loureiro.

Vehicular Mobility is strongly influenced by the speed limits, destinations, traffic conditions, period of the day, and direction of the public roads. At the same time, the driver's behavior produces great influences in vehicular mobility. People tend to go to the same places, at the same day period, through the same trajectories, which le ad them to the appearance of driver's daily routines. These routines lead us to the study of mobility

in VANETs under a social perspective and to investigate how effective is to explore social interactions in this kind of network. In this work, we thus characterize and evaluate social properties of a realistic vehicular trace found in literature. Our aim is to study the vehicles' mobility in accordance to social behaviors. Social metrics are computed and the obtained results are compared to random graphs. With our analysis, we could verify the existence of regularity and common interests among the drivers in vehicular networks. This work was published in the international conference IEEE ISCC 2014 [10], in the international Student workshop of IEEE Infocom 2014 [9], and at the internation workshop Internet of Things Communications and Technologies (IoT 2013) held in conjunction with IEEE WiMob 2013.

After having identified routine in vehicles mobility patterns and their correlation with the period of the day, we then leverage the identified social aspects to design a *Socially Inspired Broadcast Data Dissemination* for VANETs. We claim that protocols and applications designed for Vehicular Ad Hoc Networks need to adapt to vehicles routines in order to provide better services. With this issue in mind, we designed a data dissemination solution for these networks that considers the daily road traffic variation of large cities and the relationship among vehicles. The focus of our approach is to select the best vehicles to rebroadcast data messages according to social metrics, in particular, the clustering coefficient and the node degree. Moreover, our solution is designed in such a way that it is completely independent of the perceived road traffic density. Simulation results show that, when compared to related protocols, our proposal provides better delivery guarantees, reduces the network overhead and possesses an acceptable delay. This work was published as a short paper at the international conference ACM MSWiM 2014 [8]

6.4.6. Design and Analysis of an Efficient Friend-to-Friend Content Dissemination System

Participants: Kanchana Thilakarathna, Aline Carneiro Viana, Aruna Seneviratne, Henrik Petander.

In this work, we focus on dissemination of content for delay tolerant applications/services, (i.e. content sharing, advertisement propagation, etc.) where users are geographically clustered into communities. Due to emerging security and privacy concerns, majority of users are becoming more reluctant to interact with strangers and are only willing to share information/content with the users who are previously identified as friends. As a result, despite its promise, opportunistic communications systems have not been widely adopted. In addition, in this environment, opportunistic communication will not be effective due to the lack of known friends within the communication range. We thus propose a novel architecture which combines the advantages of distributed decentralized storage and opportunistic communications. The proposed system addresses the trust and privacy concerns of opportunistic communications systems, and enables the provision of efficient distributed mobile social networking services. We exploit the fact that users will trust their friends, and the friends will help in disseminating content by temporarily storing and forwarding content. This can be done by replicating content on friends' devices who are likely to consume that content and provide the content to other friends when the device has access to low cost networks. The fundamental challenge then is to minimize the number of replicas, to ensure high and timely availability. We provide a formal definition of this content replication problem, and show that it is NP hard. Then, we propose a community based greedy heuristic algorithm with novel dynamic centrality metrics that replicates the content on a minimum number of friends' devices, and maximizes the availability of content. Using both real world and synthetic traces, we validate effectiveness of the proposed scheme. In addition, we demonstrate the practicality of the the proposed system, through an implementation on Android smartphones. This work is under submission in an international transaction. An initial version of this work was published at the international conference ACM MobiHoc 2013, and an extended version is under submission in an international trasaction.

6.4.7. Telling Apart Social and Random Relationships in Dynamic Networks

Participants: Pedro Olmo Vaz de Melo, Aline Carneiro Viana, Marco Fiore, Katia Jaffrès-Runser, Frédéric Le Mouël, Antonio A. F. Loureiro, Lavanya Addepalli, Guangshuo Chen.

Recent studies have analyzed data generated from mobile individuals in urban regions, such as cab drivers or students in large campuses. Particular attention has been paid to the dynamics of user movement, whose real-world complexity cannot be fully captured through synthetic models. Indeed, understanding user mobility is of fundamental importance when designing new communication protocols that exploit opportunistic encounters

among users. In this case, the problem mainly lies in correctly forecasting future contacts. To that end, the regularity of daily activities comes in handy, as it enforces periodic (and thus predictable) space-time patterns in human mobility. Although human behavior is characterized by an elevated rate of regularity, random events are always possible in the routines of individuals. Those are hardly predictable situations that deviate from the regular pattern and are unlikely to repeat in the future.

We argue that the ability to accurately spot random and social relationships in dynamic networks is essential to network applications that rely on a precise description of human routines, such as recommendation systems, forwarding strategies and opportunistic dissemination protocols. We thus propose a strategy to analyze users' interactions in mobile networks where users act according to their interests and activity dynamics. Our strategy, named *Random rElationship ClASsifier sTrategy (RECAST)*, allows classifying users' wireless interactions, separating random interactions from different kinds of social ties. To that end, RECAST observes how the real system differs from an equivalent one where entities' decisions are completely random. We evaluate the effectiveness of the RECAST classification on five real-world user contact datasets collected in diverse networking contexts. Our analysis unveils significant differences among the dynamics of users' wireless interactions in the datasets, which we leverage to unveil the impact of social ties on opportunistic routing. We show that, for such specific purpose, the relationships inferred by classifier are more relevant than, e.g., self-declared friendships on Facebook. An initial version of this work was published in the international conference ACM MSWiM 2013 (selected as one of the five better papers of this venue) and an extented version bringing new analysis (e.g., the contact duration-related analysis performed by the internship Lavanya Addepalli and the PhD student Guangshuo Chen) was accepted to be published in 2015 at the Performance Evaluation Elsevier Journal [2].

6.5. Spontaneous Wireless Networks and Internet of Things

internet of things; wireless sensor networks; dissemination; resource management

6.5.1. Network Coding in Large Scale IoT Networks

Participants: Cedric Adjih, Ichrak Amdouni, Hana Baccouch, Antonia Masucci.

We had designed a generic broadcast protocol, called DragonNet, based on network coding and designed for constrained networks such as wireless sensor networks and internet of things. It minimizes the assumptions made of the networks. A variant of this protocol was implemented and run on IoT-LAB: some results were initially presented at IRTF, and a live demo was presented in MASS in october 2014.

6.5.2. Information-Centric Networking in the Internet of Things

Participants: Emmanuel Baccelli, Oliver Hahm, Matthias Waehlisch, Thomas Schmidt, Christian Mehlis.

Within this activity, we explored the feasibility, advantages, and challenges of an ICN-based approach in the Internet of Things. We report on the first NDN experiments in a life-size IoT deployment, spread over tens of rooms on several floors of a building. Based on the insights gained with these experiments, we have analysed the shortcomings of CCN applied to IoT. Several interoperable CCN enhancements are then proposed and evaluated. We significantly decreased control traffic (i.e., interest messages) and leverage data path and caching to match IoT requirements in terms of energy and bandwidth constraints. Our optimizations increase content availability in case of IoT nodes with intermittent activity. Within this activity, we also provided the first experimental comparison of CCN with the common IoT standards 6LoWPAN/RPL/UDP.

AVIZ Project-Team

6. New Results

6.1. Highlights of the Year

We had a number of highlights this year:

- Jean-Daniel Fekete was General Chair of the IEEE VIS 2014 conference, organized for the first time ever outside of the USA, in Paris, with a record attendance.
- Aviz presented 7 articles at the IEEE VIS 2014 conference, and co-organized 3 workshops.
- Five PhD students defended this year.
- Benjamin Bach was awarded the second price in the IEEE VGTC Doctoral Dissertation Competition for his thesis "Connections, Changes, Cubes: Unfolding Dynamic Networks for Visual Exploration" [10].
- Yvonne Jansen was awarded the second price for the Gilles Kahn dissertation award for her thesis "Physical and Tangible Information Visualization" [11].
- Samuel Huron received the best paper honorable mention award at DIS 2014 for the paper “Constructive Visualization” [28].

6.2. Effectiveness of Staggered Animations

Participants: Fanny Chevalier, Pierre Dragicevic [correspondant], Steven Franconeri.

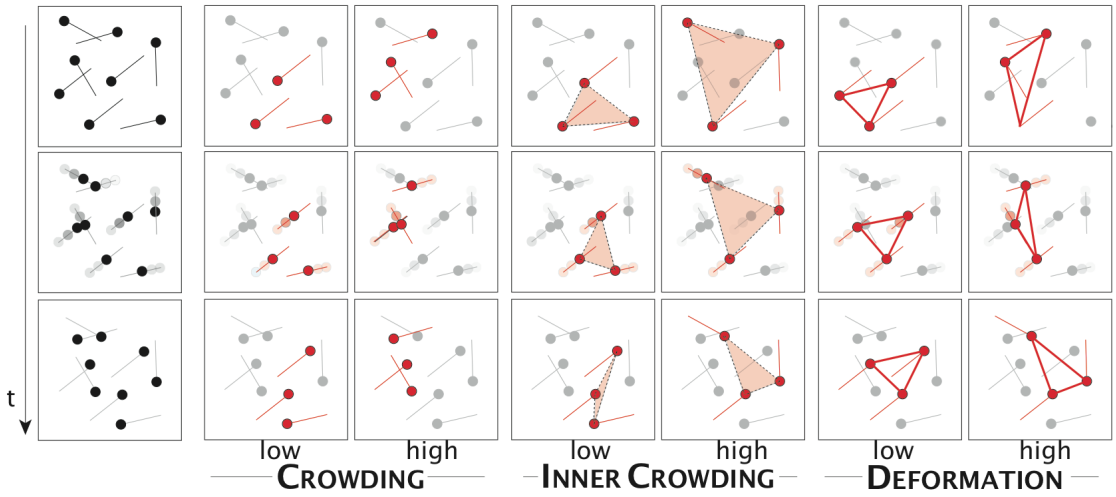

Figure 7. Illustration of the complexity metrics used in the study.

Interactive visual applications often rely on animation to transition from one display state to another. There are multiple animation techniques to choose from, and it is not always clear which should produce the best visual correspondences between display elements. One major factor is whether the animation relies on staggering—an incremental delay in start times across the moving elements. It has been suggested that staggering may reduce occlusion, while also reducing display complexity and producing less overwhelming animations, though no empirical evidence has demonstrated these advantages. Work in perceptual psychology does show that reducing occlusion, and reducing inter-object proximity (crowding) more generally, improves performance in multiple object tracking.

We empirically investigated the effectiveness of staggering [15]. We ran simulations confirming that staggering can in some cases reduce crowding in animated transitions involving dot clouds (as found in, e.g., animated 2D scatterplots). We empirically evaluated the effect of two staggering techniques on tracking tasks, focusing on cases that should most favour staggering. We found that introducing staggering has a negligible, or even negative, impact on multiple object tracking performance. The potential benefits of staggering may be outweighed by strong costs: a loss of common-motion grouping information about which objects travel in similar paths, and less predictability about when any specific object would begin to move. Staggering may be beneficial in some conditions, but they have yet to be demonstrated. Our results are a significant step toward a better understanding of animation pacing, and provide direction for further research.

More on the project Web page: fannychevalier.net/animations

6.3. Tablet-Based Interaction for Immersive 3D Data Exploration

Participants: David López, Lora Oehlberg, Candemir Doger, Tobias Isenberg [correspondant].

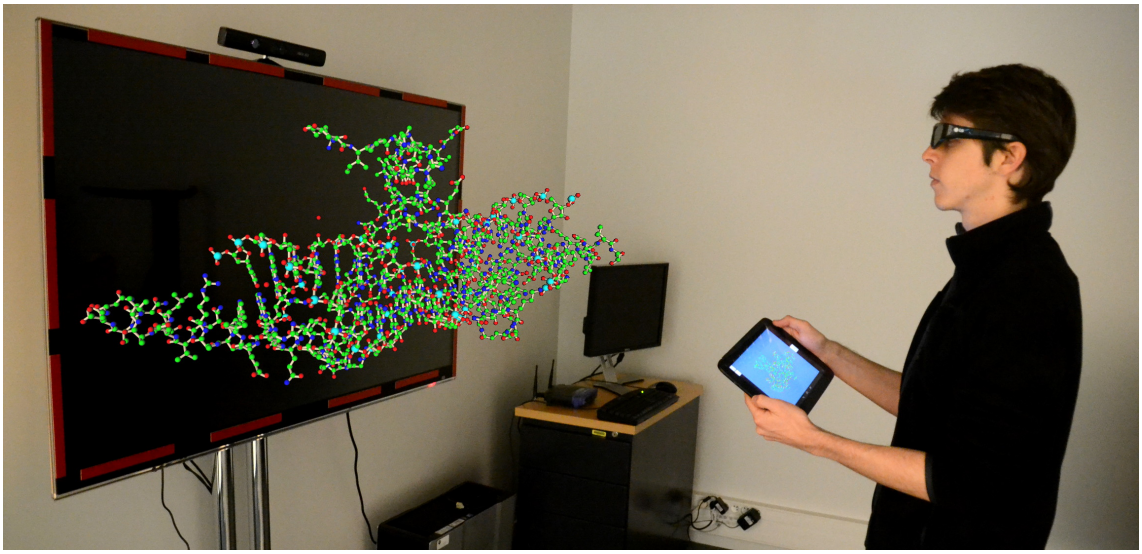

Figure 8. Illustration of the interaction setup for a combined touch-based navigation and stereoscopic viewing of 3D data.

We examined touch-based navigation of 3D visualizations in a combined monoscopic and stereoscopic viewing environment [32] (see Figure 8). We identified a set of interaction modes, and a workflow that helps users transition between these modes to improve their interaction experience. For this purpose we analyzed, in particular, the control-display space mapping between the different reference frames of the stereoscopic and

monoscopic displays. We showed how this mapping supports interactive data exploration, but may also lead to conflicts between the stereoscopic and monoscopic views due to users' movement in space; we resolved these problems through synchronization. To support our discussion, we conducted an exploratory observational evaluation with domain experts in fluid mechanics and structural biology. These experts explored domain-specific datasets using variations of a system that embodies the interaction modes and workflows; we could report on their interactions and qualitative feedback on the system and its workflow.

6.4. Understanding the Perception of Star Glyphs

Participants: Johannes Fuchs, Petra Isenberg [correspondant], Anastasia Bezerianos, Fabian Fischer, Enrico Bertini.

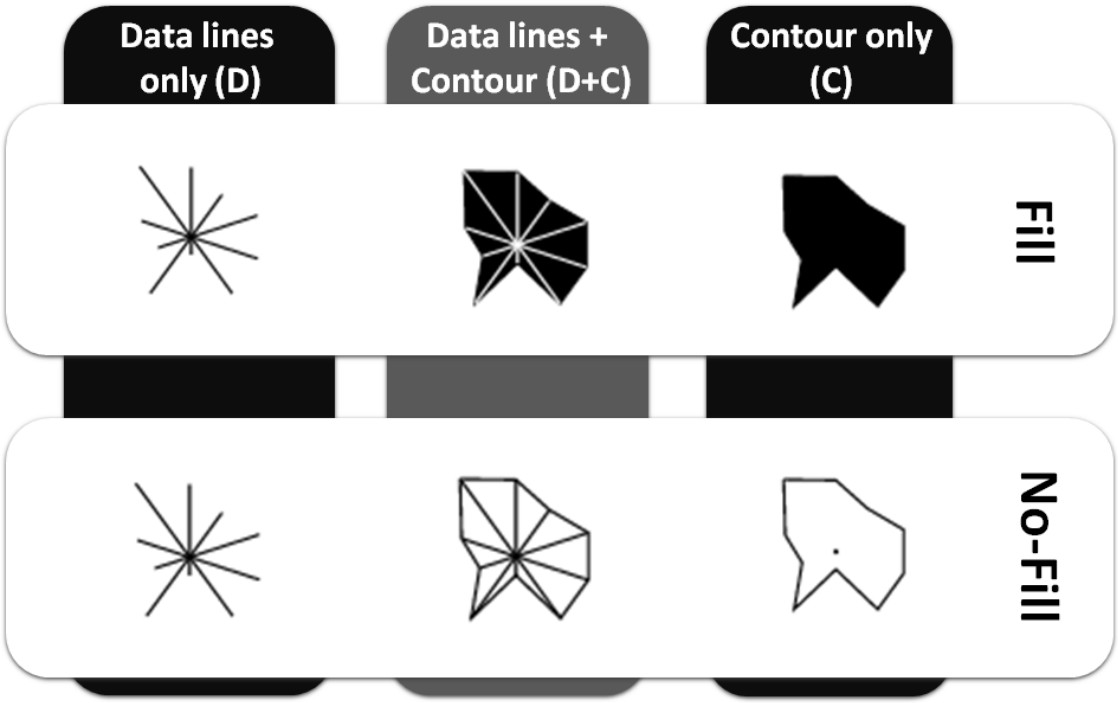

Figure 9. Illustration of the design space of the perception study.

We conducted three experiments to investigate the effects of contours on the detection of data similarity with star glyph variations [16]. A star glyph is a small, compact, data graphic that represents a multi-dimensional data point. Star glyphs are often used in small-multiple settings, to represent data points in tables, on maps, or as overlays on other types of data graphics. In these settings, an important task is the visual comparison of the data points encoded in the star glyph, for example to find other similar data points or outliers. We hypothesized that for data comparisons, the overall shape of a star glyph—enhanced through contour lines—would aid the viewer in making accurate similarity judgments. To test this hypothesis, we conducted three

experiments. In our first experiment, we explored how the use of contours influenced how visualization experts and trained novices chose glyphs with similar data values. Our results showed that glyphs without contours make the detection of data similarity easier. Given these results, we conducted a second study to understand intuitive notions of similarity. Star glyphs without contours most intuitively supported the detection of data similarity. In a third experiment, we tested the effect of star glyph reference structures (i.e., tickmarks and gridlines) on the detection of similarity. Surprisingly, our results show that adding reference structures does improve the correctness of similarity judgments for star glyphs with contours, but not for the standard star glyph. As a result of these experiments, we conclude that the simple star glyph without contours performs best under several criteria, reinforcing its practice and popularity in the literature. Contours seem to enhance the detection of other types of similarity, e.g., shape similarity and are distracting when data similarity has to be judged. Based on these findings we provide design considerations regarding the use of contours and reference structures on star glyphs.

6.5. Constructive Visualization

Participants: Samuel Huron [correspondant], Yvonne Jansen, Sheelagh Carpendale.

Figure 10. Constructing a visualization with tokens: right hand positions tokens, left hand points to the corresponding data.

The accessibility of infovis authoring tools to a wide audience has been identified as one of the major research challenges. A key task of the authoring process is the development of visual mappings. While the infovis community has long been deeply interested in finding effective visual mappings, comparatively little attention has been placed on how people construct visual mappings. We conducted a study designed to shed light on how people spontaneously transform data into visual representations [18]. We asked people to create, update and explain their own information visualizations using simple materials such as tangible building blocks. We learned that all participants, most of whom had no experience in visualization, were readily able to create and talk about their own visualizations. On the basis of our observations, we discussed the actions of our participants in the context of the development of their visual representations and their analytic activities. From this we suggested implications for tool design that can enable broader support for infovis authoring.

More on the project Web page: constructive.gforge.inria.fr

6.6. Multi-touch Gestures for Data Graphics

Participants: Wesley Willett, Qi Lan, Petra Isenberg.

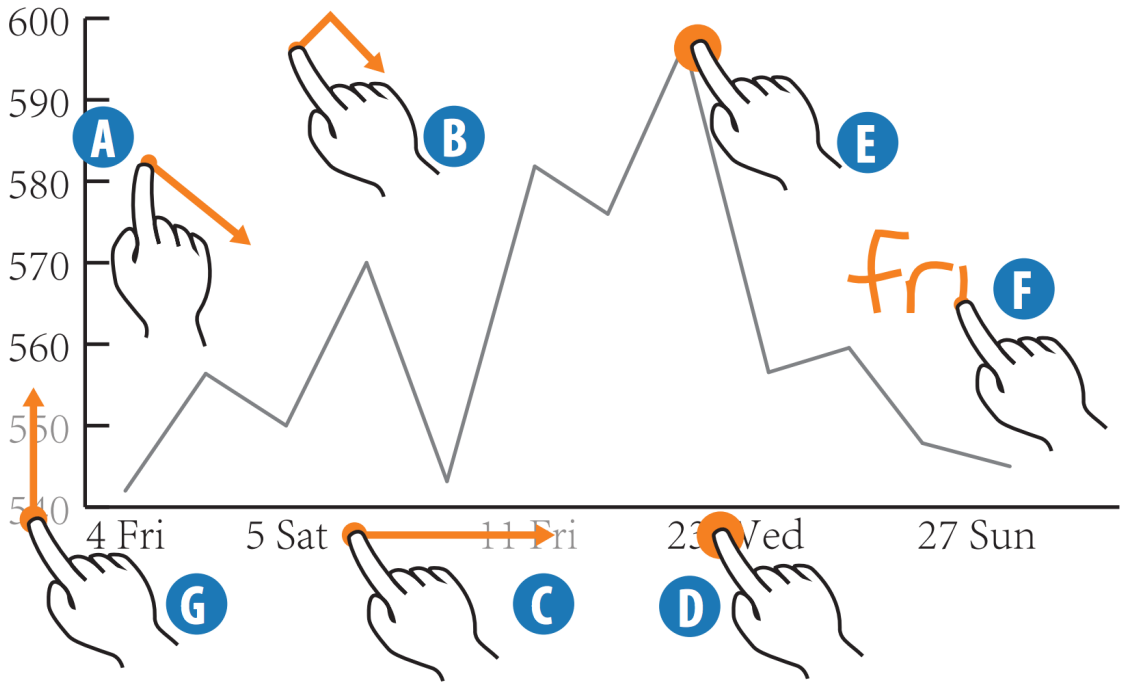

Figure 11. The most common gestures used for selecting (a) downward trends, (b) peaks, (c) ordinal ranges, (d) non-contiguous items, (e) highest points, (f) repeating dates, and (g) the lowest three points in a time series chart.

Selecting data items is a common and extremely important form of interaction with data graphics, and serves as the basis for many other data interaction techniques. However, interactive charting tools for multi-touch displays typically only provide dedicated multi-touch gestures for single-point selection or zooming. We conducted a study in which we used gesture elicitation to explore a wider range of possible selection interactions for multi-touch data graphics [35]. The results show a strong preference for simple, one-handed selection gestures. They also show that users tend to interact with chart axes and make figurative selection gestures outside the chart, rather than interact with the visual marks themselves. Finally, we found strong consensus around several unique selection gestures related to visual chart features.

6.7. Exploring Word-Scale Visualizations

Participants: Pascal Goffin, Wesley Willett, Jean-Daniel Fekete, Petra Isenberg.

Figure 12. Four examples of the integration of word-scale visualizations into HTML documents

We presented an exploration and a design space that characterize the usage and placement of word-scale visualizations within text documents [17]. Word-scale visualizations are a more general version of sparklines—small, word-sized data graphics that allow meta-information to be visually presented in-line with document text. In accordance with Edward Tufte's definition, sparklines are traditionally placed directly before or after words in the text. We described alternative placements that permit a wider range of word-scale graphics and more flexible integration with text layouts. These alternative placements include positioning visualizations between lines, within additional vertical and horizontal space in the document, and as interactive overlays on top of the text. Each strategy changes the dimensions of the space available to display the visualizations, as well as the degree to which the text must be adjusted or reflowed to accommodate them. We provided an illustrated design space of placement options for word-scale visualizations and identify six important variables that control the placement of the graphics and the level of disruption of the source text. We also contributed a quantitative analysis that highlights the effect of different placements on readability and text disruption. Finally, we used this analysis to propose guidelines to support the design and placement of word-scale visualizations.

More on the project Web page: www.aviz.fr/sparklificator

6.8. Assessing Visualization Literacy

Participants: Jeremy Boy, Ronald A. Rensink, Enrico Bertini, Jean-Daniel Fekete.

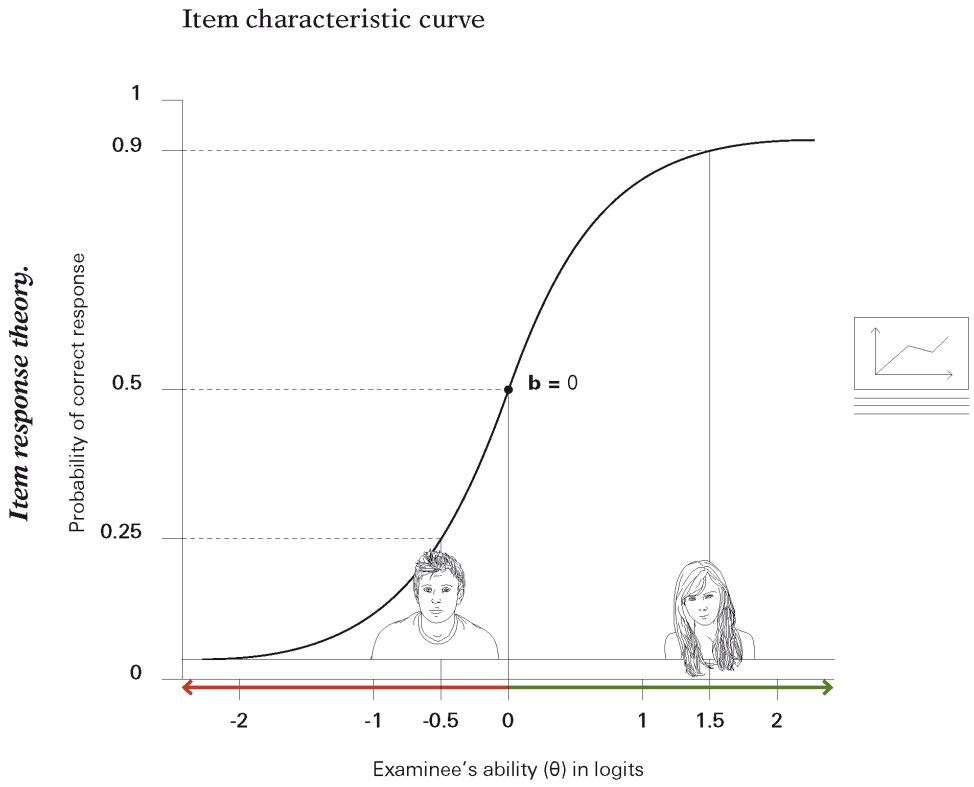

Figure 13. Example of an Item Characteristic Curve, and how people's abilities are plotted against a test-item's difficulty to determine probability of success.

We described a method for assessing the visualization literacy (VL) of a user [14]. Assessing how well people understand visualizations has great value for research (e.g., to avoid confounds), for design (e.g., to best determine the capabilities of an audience), for teaching (e. g., to assess the level of new students), and for recruiting (e. g., to assess the level of interviewees). In this project we proposed a method for assessing VL based on Item Response Theory. We described the design and evaluation of two VL tests for line graphs, and presents the extension of the method to bar charts and scatterplots. Finally, we discussed the reimplementation of these tests for fast, effective, and scalable web-based use.

More on the project Web page: peopleviz/vLiteracy/home.

DAHU Project-Team

6. New Results

6.1. Highlights of the Year

Victor Vianu was elected member of Academia Europaea.
BEST PAPER AWARD :
[21] **Joint 25th International Conference on Rewriting Techniques and Applications and 12th International Conference on Typed Lambda Calculi and Applications**. S. SCHMITZ.

6.2. Distributed data management

Participants: Serge Abiteboul, Émilien Antoine, Victor Vianu.

The management of Web users' personal information is increasingly distributed across a broad array of applications and systems, including online social networks and cloud-based services. While users wish to share and integrate data using these systems, it is increasingly difficult to avoid the risks of unintended disclosures or unauthorized access by applications.

In [15], we study deduction in the presence of inconsistencies. Following previous works, we capture deduction via datalog programs and inconsistencies through violations of functional dependencies (FDs). We study and compare two semantics for datalog with FDs: the first, of a logical nature, is based on inferring facts one at a time, while never violating the FDs; the second, of an operational nature, consists in a fixpoint computation in which maximal sets of facts consistent with the FDs are inferred at each stage. Both semantics are nondeterministic, yielding sets of possible worlds. We introduce a PTIME (in the size of the extensional data) algorithm, that given a datalog program, a set of FDs and an input instance, produces a c-table representation of the set of possible worlds. Then, we propose to quantify nondeterminism with probabilities, by means of a probabilistic semantics. We consider the problem of capturing possible worlds along with their probabilities via probabilistic c-tables. We then study classical computational problems in this novel context. We consider the problems of computing the probabilities of answers, of identifying most likely supports for answers, and of determining the extensional facts that are most influential for deriving a particular fact. We show that the interplay of recursion and FDs leads to novel technical challenges in the context of these problems.

Jakub Kalas (ENS Cachan) spent 4 months in the team working on Personal Information Management Systems, using primarily positioning from data mobile phone and data from search engines.

6.3. Query Processing for the Web

Participants: Johann Brault-Baron, Arnaud Durand, Nadime Francis, Luc Segoufin, Cristina Sirangelo.

In many applications the output of a query may have a huge size and enumerating all the answers may already consume too many of the allowed resources. In this case it may be appropriate to first output a small subset of the answers and then, on demand, output a subsequent small numbers of answers and so on until all possible answers have been exhausted. To make this even more attractive it is preferable to be able to minimize the time necessary to output the first answers and, from a given set of answers, also minimize the time necessary to output the next set of answers - this second time interval is known as the *delay*. We have shown that this was doable with a almost linear preprocessing time and constant enumeration delay for first-order queries over structures of low degree [18]. We also presented a survey about this work at the Intl. Symp. on Theoretical Aspects of Computer Science (STACS) [22].

We have also been interested in querying data structured as graphs, which is nowadays spreading on the Web. Examples are social networks, linked data and the semantic web, via the RDF format. We have tackled the problem of answering queries over graph databases which are available only trough a given set of views. This is a common situation in many applications where access to data needs to be either controlled or optimized. In [19] we have studied when it is possible to rewrite over the views queries issued on the original data, and which query languages are needed for this purpose. We have considered views and queries expressed as Regular path queries, a very common graph query language.

6.4. Complexity in Counter Systems and Substructural Logics

Participant: Sylvain Schmitz.

The ties between propositional substructural logics (like linear logic, relevance logic, affine logic, etc.) on the one hand and extensions of vector addition systems on the other hand have long been known, as they lie for instance at the heart of undecidability proof of provability in linear logic. In a series a papers we recently revisited these connections with an eye on complexity issues. This allowed us to prove tight complexity bounds on provability in affine and contractive fragments of linear logic [20], in affine $(!, \oplus)$-Horn linear logic [16], and in implicational relevance logic [21] (an open problem for more than 25 years, with consequences on type inhabitation in the λI-calculus). Our work also yields a new Tower lower bound on reachability in branching vector addition systems [20], which entails the same lower bound for logics on XML trees [4], for which decidability is still open.

Although the connection with data logics might not seem obvious at first, the models of counter systems considered in these papers are tightly connected with logics for XML processing [5], [4]. Further investigations in the relationships between data logics, substructural logics, and counter systems are the main thrust behind the just accepted ANR PRODAQ project (see Section 8.1.1).

6.5. Incomplete Databases

Participants: Nadime Francis, Cristina Sirangelo.

Incomplete databases appear in several different scenarios. Intuitively, pieces of information might not be available, or can get lost due to failures in storage or transmission. Alternatively, some complex data managements tasks, such as data integration or data exchange, use incomplete databases as a model for databases with missing or unspecified information. In the context of the Web, these tasks have become even more crucial, which increased the need to handle incomplete databases. Given an incomplete database, one of the first question to answer is that of consistency: can we make sure that the incomplete database can be completed as a real database conforming to some specified schema.

Together with Claire David and Filip Murlak, we have considered this problem when incomplete instances are represented as incomplete XML documents, where labels and nodes might be missing, and we additionally assume the DOM semantics, meaning that nodes never lose their identity (otherwise, they are considered completely lost). These are further modeled as injective tree patterns using child and descendant relations. In [17], we close the question of the complexity of checking the consistency of such patterns with regards to a fixed regular tree language: it is polynomial for patterns that do not use child edges, and for patterns that use both, it is already NP-complete for patterns using at most two descendant edges per branch, the case for at most one descendant edge being already known to be polynomial.

In [12] we have studied the feasibility of query answering in the presence of incomplete information in data. In particular we have generalized conditions allowing classical query evaluation techniques to be applicable also in the presence of incompleteness. Our results show that conditions found in some of our previous work can be significantly relaxed so as to account for more complex semantics of incompleteness, originating in the fields of logic programming, programming semantics and data exchange.

IN-SITU Project-Team

6. New Results

6.1. Highlights of the Year

Wendy Mackay received the *ACM SIGCHI Lifetime Service Award*.

BEST PAPERS AWARDS :
[22] **CHI '14**. C. LIU, O. CHAPUIS, M. BEAUDOUIN-LAFON, É. LECOLINET, W. E. MACKAY.

6.2. Interaction Techniques

Participants: Caroline Appert, Michel Beaudouin-Lafon, Anastasia Bezerianos, David Bonnet, Olivier Chapuis [correspondant], Cédric Fleury, Stéphane Huot, Can Liu, Justin Mathew, Wendy Mackay, Halla Olafsdottir, Theophanis Tsandilas, Oleksandr Zinenko.

InSitu explores interaction and visualization techniques in a variety of contexts, including individual interaction techniques on different display surfaces that range from mobile devices to very large wall-sized displays, including standard desktop systems and tabletops.

This year, we investigated multi-touch gestures on tabletop [26], we considered the combination of Tilt and Touch on smartphone [29], we proposed novel bi-manual interaction techniques for tablets [18], we introduced a novel focus+context technique to facilitate route following [14], we introduced the *GlideCursor* to facilitate pointing on large display [15], we compared physical navigation in front of a wall-size display with virtual navigation on the desktop [22], we studied users' behavior in immersive Virtual Environments [12], we built a tool to ease the extraction and the expression of parallelism in programs [30] and we investigated the effect of contours on star glyphs [13].

In addition to providing knowledge for designers and practitioners, this set of remarkable results advances our overall knowledge regarding basic interactive phenomena, and allows to better understand how user practices will change.

Multitouch on Tabletop – We systematically studied how users adapt their grasp when asked to translate and rotate virtual objects on a multitouch tabletop [26]. We have shown that users choose a grip orientation that is influenced by three factors: (1) a preferred orientation defined by the start object position, (2) a preferred orientation defined by the target object position, and (3) the anticipated object rotation. We have examined these results in the light of the most recent models of planning for manipulating physical objects and explored how these results can inform the design of tabletop applications.

Tilt & Touch – We studied the combination of tilt and touch when interacting with mobile devices [29]. We conducted an experiment to explore the effectiveness of *TilTouch* gestures for both one-handed and two-handed use. Our results indicate the best combinations of *TilTouch* gestures in terms of performance, motor coordination, and user preferences.

SPad – We created *SPad* [18], a new bimanual interaction technique designed to improve productivity on multi-touch tablets: the user activates quasimodes with the thumb of the non-dominant hand while holding the device with that hand and interacts with the content with the dominant hand (figure 3). We conducted an iterative design process and created a tablet application that demonstrates how SPad enables faster, more direct and more powerful interaction without increasing complexity.

Figure 3. Left: SPad control accessible with the thumb. Three commands are accessible with a tap of the thumb, 4 menus are accessible with swipes. Right: SPad in use to paste and move objects that have just been copied.

RouteLenses – Millions of people go to the Web to search for geographical itineraries. Inspecting those map itineraries remains tedious because they seldom fit on screen, requiring much panning & zooming to see details. Focus+context techniques address this problem by displaying routes at a scale that allows them to fully fit on screen: users see the entire route at once, and perform magnified steering using a lens to navigate along the path, revealing additional detail. We created *RouteLenses* [14], a type of lenses that automatically adjusts their position based on the geometry of the path that users steer through (figure 4 . RouteLenses make it easier for users to follow a route, yet do not constrain movements too strictly, leaving them free to move the lens away from the path to explore its surroundings.

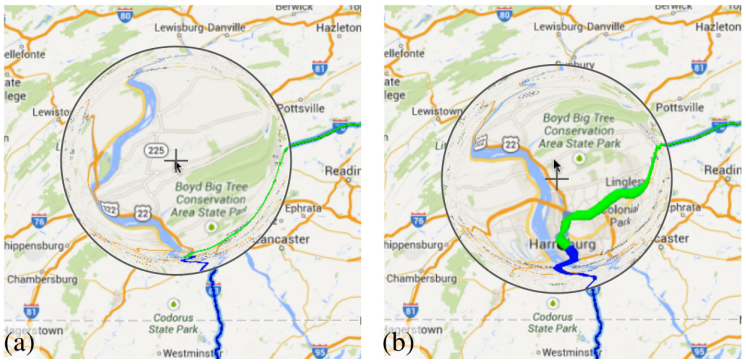

Figure 4. Following an itinerary. (a) Conventional lens: the user overshoots at a right turn in Harrisburg; losing the route that falls in the distorted region. (b) RouteLens: the route's attraction compensates the overshoot; the lens remains closer to the route, which remains in focus.

GlideCursor – Pointing on large displays with an indirect, relative pointing device such as a touchpad often requires clutching. We designed and evaluated *GlideCursor* [15], which lets the cursor continues to move during clutching gestures. The effect is that of controlling the cursor as a detached object that can be pushed, with inertia and friction similar to a puck being pushed on a table. We analyzed gliding from a practical and a theoretical perspective and conducted two studies. The first controlled experiment established that gliding

reduces clutching and can improve pointing performance for large distances. We introduced a measure called *cursor efficiency* to capture the effects of gliding on clutching. The second experiment demonstrated that participants use gliding even when an efficient acceleration function lets them perform the task without it, without degrading performance.

Wall vs. Desktop – The advent of ultra-high resolution wall-size displays and their use for complex tasks require a more systematic analysis and deeper understanding of their advantages and drawbacks compared with desktop monitors. While previous work has mostly addressed search, visualization and sense-making tasks, we have designed and evaluated an abstract classification task that involves explicit data manipulation [22]. Based on our observations of real uses of a wall display (figure 5 -left), this task represents a large category of applications. We conducted a controlled experiment that uses this task to compare physical navigation in front of a wall-size display (figure 5 -right) with virtual navigation using pan-and-zoom on the desktop. Our main finding is a robust interaction effect between display type and task difficulty: while the desktop can be faster than the wall for simple tasks, the wall gains a sizable advantage as the task becomes more difficult.

Figure 5. Left: Fine tuning the CHI 2013 conference schedule on the WILD display. Center: Classification task inspired by the CHI scheduling task conducted on the wall-sized display. Subjects must move misclassified red disks into containers (represented by the individual screens) of the same class. The class is represented by a very small letter at the center of each disk, forcing subjects to physically move in front of the display.

Immersive VE – The feeling of presence is essential for efficient interaction within Virtual Environments (VEs). When a user is fully immersed within a VE through a large immersive display system, her feeling of presence can be altered because of disturbing interactions with her physical environment. This alteration can be avoided by taking into account the physical features of the user as well as those of the system hardware. Moreover, the 3D abstract representation of these physical features can also be useful for collaboration between distant users. In [12] we presented how we use the Immersive Interactive Virtual Cabin (IIVC) model to obtain this virtual representation of the user's physical environment and we illustrated how this representation can be used in a collaborative navigation task in a VE. We also presented how we can add 3D representations of 2D interaction tools in order to cope with asymmetrical collaborative configurations, providing 3D cues for a user to understand the actions of others even if he/she is not fully immersed in the shared VE.

Clint – We created *Clint*, a direct manipulation tool to ease the extraction and the expression of parallelism in existing programs [30]. Clint is built on top of state-of-the-art compilation tools (polyhedral representation of programs) in order to give a visual representation of the code, perform automatic data dependence analysis and to ensure the correctness of code transformations (figure 6). It can be used to rework and improve automatically generated optimizations and to make manual program transformation faster, safer and more efficient.

Start Glyphs – We conducted three studies using crowd-sourcing on Amazon mechanical Turk, to determine the effect of using contours on data glyphs such as star glyphs [13]. Our results indicate that glyphs without

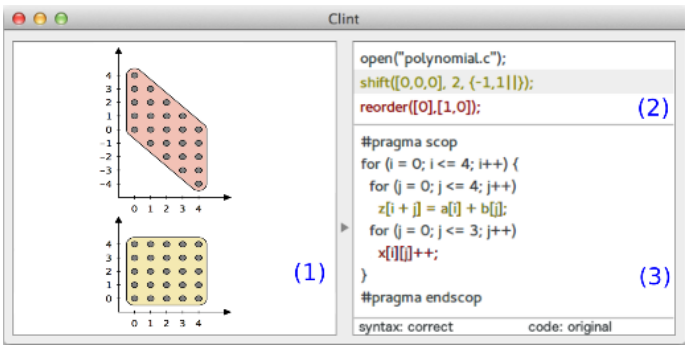

Figure 6. Clint interface includes: (1) interactive visualization, (2) editable history view, and (3) source code editor.

contours lead viewers to naturally make judgements that are data-driven. Whereas adding contours encourages shape similarity, e.g. perceiving rotated variations of glyphs as similar (even though they are not similar in data space).

6.3. Research Methods

Participants: Michel Beaudouin-Lafon, Anastasia Bezerianos, Jérémie Garcia, Stéphane Huot, Ilaria Liccardi, Wendy Mackay [correspondant], Justin Mathew.

Conducting empirical research is a fundamental part of InSitu's research activities, including observation of users in field and laboratory settings to discover problems faced by users, controlled laboratory experiments to evaluate the effectiveness of the technologies we develop, longitudinal field studies to determine how our technologies work in the real world, and participatory design, to explore design possibilities with users throughout the design process.

Computer-aided Composition – We designed *Polyphony* [20], a novel interface for systematically studying all phases of computer-aided composition, and then used it to observe expert creative behavior. *Polyphony* is a unified user interface that integrates interactive paper and electronic user interfaces for composing music. We asked 12 composers to use it (figure 7 -left) to compose an electronic accompaniment to a 20-second instrumental composition by Anton Webern. The resulting dozen comparable snapshots of the composition process reveal how composers both adapt and appropriate tools in their own way. In collaboration with IRCAM, we also conducted a longitudinal study where we closely collaborated with composer Philippe Leroux [19] in the creation of his piece *Quid sit musicus*. The composer used our interfaces based on interactive paper along with an OpenMusic library to generate compositional material for this work (figure 7 -right).

Multitouch Gestures – We created a design space of simple multitouch gestures that designers of user interfaces can systematically explore to propose more gestures to users [27]. We further considered a set of 32 gestures for tablet-sized devices, by developing an incremental recognition engine that works with current hardware technology, and empirically testing the usability of those gestures. In our experiment, individual gestures were recognized with an average accuracy of $\sim$90%, and users successfully achieved some of the transitions between gestures without the use of explicit delimiters. The goal of this work is to assist designers in optimizing the use of the rich multi-touch input channel for the activation of discrete and continuous controls, and to enable fluid transitions between controls, e.g. when selecting text over multiple views, manipulating different degrees of freedom of a graphical object or invoking a command and setting its parameter values in a row.

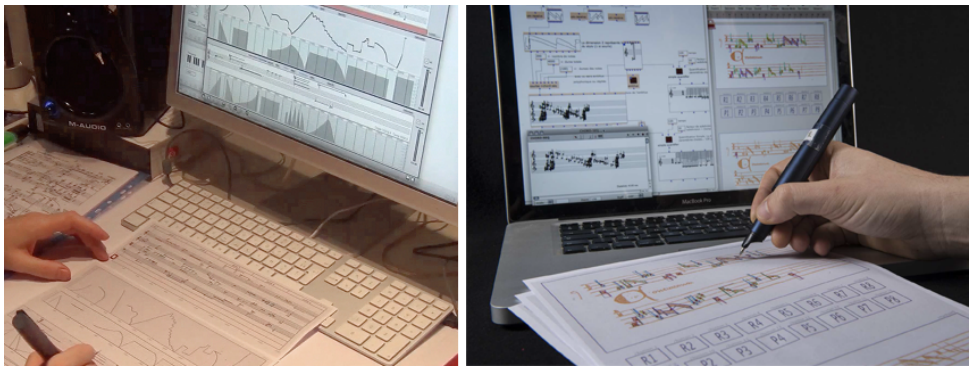

Figure 7. Left: A composer completes the composition task using Polyphony. Right: Set of tools created for Philippe Leroux' piece Quid sit musicus. Photo by H. Raguet, Inria.

Spatial Audio – We investigated the issues of spatialization techniques for object-based audio production and introduced the Spatial Audio Design Spaces framework (SpADS) [25], which describes the spatial manipulation of object-based audio. These design spaces are based on interviews with professional sound engineers and on a morphological analysis of 3D audio objects that clarifies the relationships between recording and rendering techniques that define for 3D speaker configurations. This will allow us to analyze and design novel advanced object-based controllers.

Physical Visualizations – We studied the design process of physical visualizations. An increasing variety of such visualizations are being built, for purposes ranging from art and entertainment to business analytics and scientific research. However, crafting them remains a laborious process and demands expertise in both data visualization and digital fabrication. We analyzed the limitations of current workflows through three real case studies and created *MakerVis*, the first tool that integrates the entire workflow, from data filtering to physical fabrication (figure 8). Design sessions with three end users showed that tools such as *MakerVis* can dramatically lower the barriers behind producing physical visualizations. Observations and interviews also revealed important directions for future research. These include rich support for customization, and extensive software support for materials that accounts for their unique physical properties as well as their limited supply.

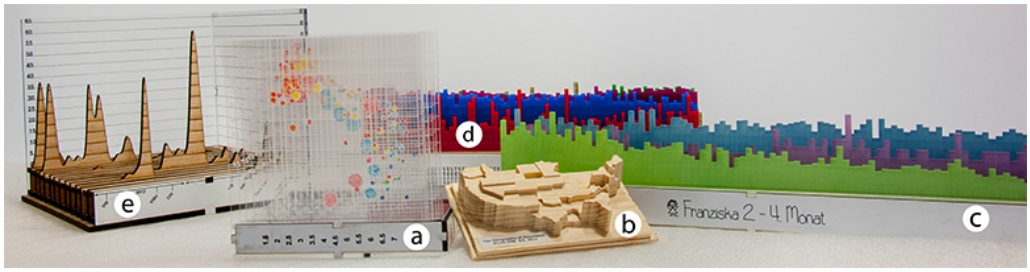

Figure 8. Physical visualizations created with MakerVis: a) a scatterplot created after Hans Rosling's TED talk, b) a prism map showing happiness across the US computed from Twitter sentiments, c),d),e) visualizations created by users during design sessions.

6.4. Engineering of interactive systems

Participants: Caroline Appert, Michel Beaudouin-Lafon [correspondant], Olivier Chapuis, Cédric Fleury, Stéphane Huot, Theophanis Tsandilas, Wendy Mackay.

InSitu has a long tradition of developing software tools and user interface toolkits to facilitate the creation of interactive systems. These tools allow us to better experiment with our ideas and are therefore an integral part of our research methodology. Most of them are freely available and some are used outside InSitu for research or teaching.

Interactive Paper – We created *PaperComposer* [31], a graphical interface builder for creating personal interactive-paper applications for musical creation. We also built an API that facilitates the development of interactive paper components for PaperComposer. The API enables developers to define new paper components that accept additional musical data with their own representation structures and interactions.

3D Telepresence – In the context of 3D telepresence, we studied how to transmit a 3D model of the users to a remote location. In [17] we present a 3D head reconstruction method for low cost 3D telepresence systems that uses only a single consumer level hybrid sensor (color+depth) located in front of the users. Our method fuses the real-time, noisy and incomplete output of a hybrid sensor with a set of static, high-resolution textured models acquired in a calibration phase (figure 9). A complete and fully textured 3D model of the users' head can thus be reconstructed in real-time, accurately preserving the facial expression of the user. The main features of our method are a mesh interpolation and a fusion of a static and a dynamic textures to combine respectively a better resolution and the dynamic features of the face.

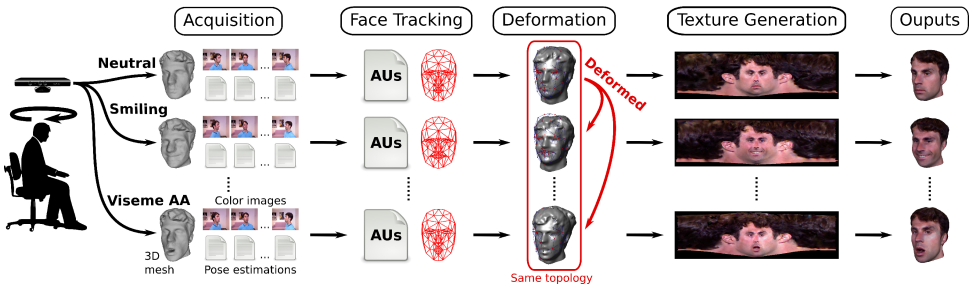

Figure 9. Acquisition step of the 3D face reconstruction: data are processed to create a complete and fully textured 3D head model for each facial expression. This set of head models are then used to improve the real-time reconstruction of the user's head during a 3D telepresence session.

Wall-sized displays – We developed Smarties [16], a system that allows developers to easily add interactive support to their wall-sized display applications by using mobile devices such as tablets. The system includes an original mobile interface that can be customized by the application itself (without programming the mobile device), a communication protocol between the mobile devices and the application running on the wall-sized display, and libraries in different programming languages that implement the protocol and handle synchronization, locking and input conflicts. Synchronization between multiple mobile devices is handled by the libraries, and thus the system supports free collaboration. The mobile devices come with multiple cursor controllers, also associated with keyboards, widgets and clipboards.

OAK Project-Team

6. New Results

6.1. Highlights of the Year

The year has allowed reaching important results in four research areas of the group: query-based why-not provenance with explanations , minimal query reformulations under constraints [14], Linked Open Data analytics , and RDF data management in the cloud .
BEST PAPERS AWARDS :
[6] **Extending Database Technology (EDBT)**. N. BIDOIT, M. HERSCHEL, K. TZOMPANAKI.
[10] **23rd International World Wide Web Conference**. D. COLAZZO, F. GOASDOUÉ, I. MANOLESCU, A. ROATIS.
[], [23] **The International Journal on Very Large Databases**. Z. KAOUDI, I. MANOLESCU.

6.2. Scalable and Expressive Techniques for the Semantic Web

A main scientific topic of the team is the design of expressive and efficient tools for analyzing and manipulating Semantic Web data, in particular RDF. Our 2014 results in this area follow three complementary directions.

First, we have finalized our model for RDF analytics and proposed a full framework in which we fully redesign, from the bottom up, core data analytics concepts and tools in the context of RDF data, leading to the first complete formal framework for warehouse-style RDF analytics. Notably, we defined (i)*analytical schemas* tailored to heterogeneous, semantics-rich RDF graph, (ii)*analytical queries* which (beyond relational cubes) allow flexible querying of the data and the schema as well as powerful aggregation and (iii)*OLAP-style operations*. We implemented our RDF analytics platform on top of the KDB system and ported it on Postgres as well [10], [28]; work is ongoing to adapt it on a massively parallel RDF query evaluation platform, namely CliqueSquare (see below). In [24], we describe novel techniques for optimizing the evaluation of RDF analytical queries based on previously computed analytical query results.

Second, we continued our work on efficient evaluation of queries on RDF data, in the presence of constraints. *Reformulation-based query answering* is a query processing technique aiming at answering queries against data, under constraints. It consists of reformulating the query based on the constraints, so that evaluating the reformulated query directly against the data (i.e. without considering any more the constraints) produces the correct answer set. We have show how to optimize reformulation-based query answering in the setting of *ontology-based data access*, where SPARQL conjunctive queries are posed against RDF facts on which constraints expressed by an RDF Schema hold. The literature provides solutions for various fragments of RDF, aiming at computing the equivalent union of maximally-contained conjunctive queries w.r.t. the constraints. However, in general, such a union is large, thus it cannot be efficiently processed by a query engine. In this context, we have shown that generalizing the query reformulation language allows considering a space of reformulated queries (instead of a single possible choice), and selecting tthe reformulated query with lower estimated evaluation cost. We have shown experimentally that our technique enables reformulation-based query answering where the state-of-the-art approaches are simply unfeasible, while it may decrease their costs by orders of magnitude in other cases [20], [26].

Third, we have continued our work on cloud-based RDF data management. In [22], we have demonstrated CliqueSquare, a platform we developed in the team for the massively parallel processing of RDF queries. CliqueSquare enjoys the benefits of a query optimization algorithm which creates query plans as flat as possible, which in turn translates into massive opportunities for parallel processing. In [23], we have finalized our work on managing RDF data within the Amazon Web Services cloud. Finally, we have conducted a study of the existing models and algorithms published so far for the massively parrallel processing of RDF queries, which appeared as a survey in the VLDB Journal [] and was also the basis of a tutorial at the ACM SIGMOD conference.

6.3. Massively Distributed Data Management Systems

Work in this area concerning the massively parallel processing of Semantic Web data was covered within the respective module.

We have finalized our work on massively parallel processing of XML queries based on the Apache Flink framework, formerly known as Stratosphere from the Technical University of Berlin, which implements the PACT model (an expressive extension of MapReduce). In [21], we have addressed the problem of efficiently parallelizing the execution of complex nested data processing, expressed in XQuery. We provided novel algorithms showing how to translate such queries into PACT, a recent framework generalizing MapReduce in particular by supporting many-input tasks. We presented the first formal translation of complex XQuery algebraic expressions into PACT plans, and demonstrated experimentally the efficiency and scalability of our approach. The work has recently been accepted for publication to IEEE TKDE (to appear in 2015),

Finally, we have considered improving the performance of massively parallel data processing programs expressed using the PigLatin language. PigLatin is a popular language within the data management community interested in the efficient parallel processing of large data volumes. The dataflow-style primitives of PigLatin provide an intuitive way for users to write complex analytical queries, which are in turn compiled into MapReduce jobs. Currently, subexpressions occurring repeatedly in PigLatin scripts are executed as many times as they occur, leading to avoidable MapReduce jobs. The current PigLatin optimizer is not capable of recognizing, and thus optimizing, such repeated subexpressions. In [18], we have presented We present a novel approach for identifying and reusing common subexpressions occurring in PigLatin scripts. In particular, we lay the foundation of our reuse-based algorithms by formalizing the semantics of the PigLatin query language with extended nested relational algebra for bags. Our algorithm, named PigReuse, operates on the algebraic representations of PigLatin scripts, identifies subexpression merging opportunities, selects the best ones to execute based on a cost function, and merges other equivalent expressions to share its result; our experiments have confirmed the efficiency and effectiveness of our reuse-based algorithms and optimization strategies.

6.4. Advanced Algorithms for Data Querying and Transformation

We revisit in [14] the Chase&Backchase (C&B) algorithm for query reformulation under constraints. For an important class of queries and constraints, C&B has been shown to be complete, i.e. guaranteed to find all (join-)minimal reformulations under constraints. C&B is based on constructing a canonical rewriting candidate called a universal plan, then inspecting its exponentially many sub-queries in search for minimal reformulations, essentially removing redundant joins in all possible ways. This inspection involves chasing the subquery. Because of the resulting exponentially many chases, the conventional wisdom has held that completeness is a concept of mainly theoretical interest. We show that completeness can be preserved at practically relevant cost by introducing a novel reformulation algorithm that instruments the chase to maintain provenance information connecting the joins added during the chase to the universal plan subqueries responsible for adding these joins. This allows it to directly "read off" the minimal reformulations from the result of a single chase of the universal plan, saving exponentially many chases of its subqueries. We exhibit natural scenarios yielding speedups of over two orders of magnitude between the execution of the best view-based rewriting found by a commercial query optimizer and that of the best rewriting found by our algorithm.

Different types of explanations that serve as Why-Not answers have been proposed in the past and are either based on the available data, the query tree, or both. A first approach to this so called why-not provenance has been recently proposed. In [6], we show that this first approach has some shortcomings. To overcome these shortcomings, we propose Ned, an algorithm to explain data missing from a query result. NedExplain computes the why-not provenance for monotone relational queries with aggregation. This work contributes to providing necessary formalization in which the new algorithm is build. It also develops a comparative evaluation showing that it is both more efficient and effective than the state-of-the-art approach.

Solutions to answering Why-Not questions are generally more efficient and easier to interpret by developers than solutions solely based on data. However, algorithms producing such query-based explanations including ours ([6]) so far may return different results for reordered conjunctive query trees, and even worse, these results

may be incomplete. Clearly, this represents a significant usability problem, as the explanations developers get may be partial and developers have to worry about the query tree representation of their query, losing the advantage of using a declarative query language. As remedy to this problem, in [5][17], we propose to capture query based answers of Why-Not questions through operator polynomial and we devised an algorithm called Ted that produces the same complete query-based explanations for reordered conjunctive query trees.

6.5. Social Data Management and Crowdsourcing

In [12], we focused on the issue of defining models and metrics for reciprocity in signed graphs. In unsigned directed networks, reciprocity quantifies the predisposition of network members in creating mutual connections. On the other hand, this concept has not yet been investigated in the case of signed graphs. We capitalize on the graph degeneracy concept to identify subgraphs of the signed network in which reciprocity is more likely to occur. This enables us to assess reciprocity at a global level, rather than at an exclusively local one as in existing approaches. The large scale experiments we perform on real world data sets of trust networks lead to both interesting and intuitive results. We believe these reciprocity measures can be used in various social applications such as trust management, community detection and evaluation of individual nodes. The global reciprocity we define in this paper is closely correlated to the clustering structure of the graph, more than the local reciprocity as it is indicated by the experimental evaluation we conducted.

As initial step towards better answering information needs in applications managing social content that is structured and possibly enriched with semantic annotations, in [19], we present a preliminary data model and an approach for answering queries over structured, social and semantic-rich content, taking into account all dimensions of the data in order to return the most meaningful results.