



RESEARCH CENTER
Saclay - Île-de-France

FIELD

Activity Report 2015

Section New Results

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COMETE Project-Team

7. New Results

7.1. Foundations of information hiding

Information hiding refers to the problem of protecting private information while performing certain tasks or interactions, and trying to avoid that an adversary can infer such information. This is one of the main areas of research in Comète; we are exploring several topics, described below.

7.1.1. *On the information leakage of differentially-private mechanisms*

Differential privacy aims at protecting the privacy of participants in statistical databases. Roughly, a mechanism satisfies differential privacy if the presence or value of a single individual in the database does not significantly change the likelihood of obtaining a certain answer to any statistical query posed by a data analyst. Differentially-private mechanisms are often oblivious: first the query is processed on the database to produce a true answer, and then this answer is adequately randomized before being reported to the data analyst. Ideally, a mechanism should minimize leakage, i.e., obfuscate as much as possible the link between reported answers and individuals' data, while maximizing utility, i.e., report answers as similar as possible to the true ones. These two goals, however, are in conflict with each other, thus imposing a trade-off between privacy and utility.

In [12] we used quantitative information flow principles to analyze leakage and utility in oblivious differentially-private mechanisms. We introduced a technique that exploits graph symmetries of the adjacency relation on databases to derive bounds on the min-entropy leakage of the mechanism. We considered a notion of utility based on identity gain functions, which is closely related to min-entropy leakage, and we derived bounds for it. Finally, given some graph symmetries, we provided a mechanism that maximizes utility while preserving the required level of differential privacy.

7.1.2. *Geo-indistinguishability: A Principled Approach to Location Privacy*

With the increasing popularity of handheld devices, location-based applications and services have access to accurate and real-time location information, raising serious privacy concerns for their users. In [17] we reported on our ongoing project aimed at protecting the privacy of the user when dealing with location-based services. The starting point of our approach is the principle of geo-indistinguishability, a formal notion of privacy that protects the user's exact location, while allowing approximate information – typically needed to obtain a certain desired service – to be released. We then presented two mechanisms for achieving geo-indistinguishability, one generic to sanitize locations in any setting with reasonable utility, the other custom-built for a limited set of locations but providing optimal utility. Finally we extended our mechanisms to the case of location traces, where the user releases his location repeatedly along the day and we provide a method to limit the degradation of the privacy guarantees due to the correlation between the points. All the mechanisms were tested on real datasets and compared both among themselves and with respect to the state of the art in the field.

7.1.3. *Constructing elastic distinguishability metrics for location privacy*

The recently introduced notion of geo-indistinguishability tries to address the problem of accessing location-aware services in a privacy-friendly way by adapting the well-known concept of differential privacy to the area of location-based systems. Although geo-indistinguishability presents various appealing aspects, it has the problem of treating space in a uniform way, imposing the addition of the same amount of noise everywhere on the map.

In [13] we proposed a novel elastic distinguishability metric that warps the geometrical distance, capturing the different degrees of density of each area. As a consequence, the obtained mechanism adapts the level of noise while achieving the same degree of privacy everywhere. We also showed how such an elastic metric can easily incorporate the concept of a "geographic fence" that is commonly employed to protect the highly recurrent locations of a user, such as his home or work. We performed an extensive evaluation of our technique by building an elastic metric for Paris' wide metropolitan area, using semantic information from the OpenStreetMap database. We compared the resulting mechanism against the Planar Laplace mechanism satisfying standard geo-indistinguishability, using two real-world datasets from the Gowalla and Brightkite location-based social networks. The results showed that the elastic mechanism adapts well to the semantics of each area, adjusting the noise as we move outside the city center, hence offering better overall privacy.

7.1.4. Quantitative Information Flow for Scheduler-Dependent Systems

Quantitative information flow analyses measure how much information on secrets is leaked by publicly observable outputs. One area of interest is to quantify and estimate the information leakage of composed systems. Prior work has focused on running disjoint component systems in parallel and reasoning about the leakage compositionally, but has not explored how the component systems are run in parallel or how the leakage of composed systems can be minimised.

In [23] we considered the manner in which parallel systems can be combined or scheduled. This considers the effects of scheduling channels where resources may be shared, or whether the outputs may be incrementally observed. We also generalised the attacker's capability, of observing outputs of the system, to consider attackers who may be imperfect in their observations, e.g. when outputs may be confused with one another, or when assessing the time taken for an output to appear. Our main contribution was to present how scheduling and observation affect information leakage properties. In particular, that scheduling can hide some leaked information from perfect observers, while some scheduling may reveal secret information that is hidden to imperfect observers. In addition we presented an algorithm to construct a scheduler that minimises the min-entropy leakage and min-capacity in the presence of any observer.

7.2. Foundations of Concurrency

Distributed systems have changed substantially in the recent past with the advent of phenomena like social networks and cloud computing. In the previous incarnation of distributed computing the emphasis was on consistency, fault tolerance, resource management and related topics; these were all characterized by *interaction between processes*. Research proceeded along two lines: the algorithmic side which dominated the Principles Of Distributed Computing conferences and the more process algebraic approach epitomized by CONCUR where the emphasis was on developing compositional reasoning principles. What marks the new era of distributed systems is an emphasis on managing access to information to a much greater degree than before.

7.2.1. An Algebraic View of Space/Belief and Extrusion/Utterance for Concurrency/Epistemic Logic

The notion of constraint system (cs) is central to declarative formalisms from concurrency theory such as process calculi for concurrent constraint programming (ccp). Constraint systems are often represented as lattices: their elements, called constraints, represent partial information and their order corresponds to entailment. Recently a notion of n-agent spatial cs was introduced to represent information in concurrent constraint programs for spatially distributed multi-agent systems. From a computational point of view a spatial constraint system can be used to specify partial information holding in a given agent's space (local information). From an epistemic point of view a spatial cs can be used to specify information that a given agent considers true (beliefs). Spatial constraint systems, however, do not provide a mechanism for specifying the mobility of information/processes from one space to another. Information mobility is a fundamental aspect of concurrent systems.

In the poster paper [24] we discussed using constraint systems with an algebraic operator that correspond to moving information in-between spaces as to mimic the mobility of data of distributed systems such as posting opinions/lies to other spaces or publicly disclosing data. In the conference paper [22] we enriched spatial constraint systems with operators to specify information and processes moving from a space to another. We referred to these news structures as spatial constraint systems with extrusion. We investigated the properties of this new family of constraint systems and illustrated their applications. From a computational point of view the new operators provide for process/information extrusion, a central concept in formalisms for mobile communication. From an epistemic point of view extrusion corresponds to a notion we called utterance; a piece of information that an agent communicates to others but that may be inconsistent with the agent's beliefs. Utterances can then be used to express instances of epistemic notions, which are commonplace in social media, such as hoaxes or intentional lies. Spatial constraint systems with extrusion can be seen as complete Heyting algebras equipped with maps to account for spatial and epistemic specifications. In the journal paper [28] we extended our work in [22] by showing that spatial constraint systems can also express the epistemic notion of knowledge by means of a derived spatial operator that specifies global information.

7.2.2. A Labelled Semantics for Soft Concurrent Constraint Programming

In [21] we presented a labelled semantics for Soft Concurrent Constraint Programming (SCCP), a language where concurrent agents may synchronize on a shared store by either posting or checking the satisfaction of (soft) constraints. SCCP generalizes the classical formalism by parametrising the constraint system over an order-enriched monoid: the monoid operator is not required to be idempotent, thus adding the same information several times may change the store. The novel operational rules are shown to offer a sound and complete co-inductive technique to prove the original equivalence over the unlabelled semantics.

7.2.3. Verification methods for concurrent Constraint Programming

Concurrent Constraint Programming (CCP) is a well-established declarative framework from concurrency theory. Its foundations and principles e.g., semantics, proof systems, axiomatizations, have been thoroughly studied for over the last two decades. In contrast, the development of algorithms and automatic verification procedures for CCP have hitherto been far too little considered.

To the best of our knowledge there is only one existing verification algorithm for the standard notion of CCP program (observational) equivalence. In [16] we first showed that this verification algorithm has an exponential-time complexity even for programs from a representative sub-language of CCP; the summation-free fragment (CCP+). We then significantly improved on the complexity of this algorithm by providing two alternative polynomial-time decision procedures for CCP+ program equivalence. Each of these two procedures has an advantage over the other. One has a better time complexity. The other can be easily adapted for the full language of CCP to produce significant state space reductions. The relevance of both procedures derives from the importance of CCP+. This fragment, which has been the subject of many theoretical studies, has strong ties to first-order logic and an elegant denotational semantics, and it can be used to model real-world situations. Its most distinctive feature is that of confluence, a property we exploit to obtain our polynomial procedures.

Bisimilarity is a standard behavioral equivalence in concurrency theory. However, only recently a well-behaved notion of bisimilarity for CCP, and a CCP partition refinement algorithm for deciding the strong version of this equivalence have been proposed. Weak bisimilarity is a central behavioral equivalence in process calculi and it is obtained from the strong case by taking into account only the actions that are observable in the system. Typically, the standard partition refinement can also be used for deciding weak bisimilarity simply by using Milner's reduction from weak to strong bisimilarity; a technique referred to as saturation. In [15] we demonstrated that, because of its involved labeled transitions, the above-mentioned saturation technique does not work for CCP. We also gave an alternative reduction from weak CCP bisimilarity to the strong one that allows us to use the CCP partition refinement algorithm for deciding this equivalence. We also proved that due to distinctive nature of CCP, the new method does not introduce infinitely-branching in the resulting transition systems. Finally, we derived an algorithm to automatically verify the notion of weak bisimilarity in CCP.

The ntcc calculus extends CCP with the notion of discrete time-units for the specification of reactive systems. Moreover, ntcc features constructors for non-deterministic choices and asynchronous behavior, thus allowing for (1) synchronization of processes via constraint entailment during a time-unit and (2) synchronization of processes along time-intervals. In [20] we developed the techniques needed for the automatic verification of ntcc programs based on symbolic model checking. We showed that the internal transition relation, modeling the behavior of processes during a time-unit (1 above), could be symbolically represented by formulas in a suitable fragment of linear time temporal logic. Moreover, by using standard techniques as difference decision diagrams, we provided a compact representation of these constraints. Then, relying on a fixpoint characterization of the timed constructs, we obtained a symbolic model of the observable transition (2 above). We proved that our construction is correct with respect to the operational semantics. Finally, we introduced a prototypical tool implementing our method.

GEOMETRICA Project-Team

7. New Results

7.1. Mesh Generation and Geometry processing

7.1.1. *Discrete Derivatives of Vector Fields on Surfaces An Operator Approach*

Participants: Frédéric Chazal, Maksim Ovsjanikov.

In collaboration with O. Azencot, M. Ben Chen (Technion, Israel Institute of Technology).

Vector fields on surfaces are fundamental in various applications in computer graphics and geometry processing. In many cases, in addition to representing vector fields, the need arises to compute their derivatives, for example, for solving partial differential equations on surfaces or for designing vector fields with prescribed smoothness properties. In this work, we consider the problem of computing the Levi-Civita covariant derivative, that is, the tangential component of the standard directional derivative, on triangle meshes. This problem is challenging since, formally, tangent vector fields on polygonal meshes are often viewed as being discontinuous, hence it is not obvious what a good derivative formulation would be. We leverage the relationship between the Levi-Civita covariant derivative of a vector field and the directional derivative of its component functions to provide a simple, easy-to-implement discretization for which we demonstrate experimental convergence. In addition, we introduce two linear operators which provide access to additional constructs in Riemannian geometry that are not easy to discretize otherwise, including the parallel transport operator which can be seen simply as a certain matrix exponential. Finally, we show the applicability of our operator to various tasks, such as fluid simulation on curved surfaces and vector field design, by posing algebraic constraints on the covariant derivative operator.

7.1.2. *Isotopic Meshing within a Tolerance Volume*

Participant: David Cohen-Steiner.

In collaboration with M. Mandad, P. Alliez (Titane Project-team).

We give an algorithm [22] that generates from an input tolerance volume a surface triangle mesh guaranteed to be within the tolerance, intersection free and topologically correct. A pliant meshing algorithm is used to capture the topology and discover the anisotropy in the input tolerance volume in order to generate a concise output. We first refine a 3D Delaunay triangulation over the tolerance volume while maintaining a piecewise-linear function on this triangulation, until an isosurface of this function matches the topology sought after. We then embed the isosurface into the 3D triangulation via mutual tessellation, and simplify it while preserving the topology. Our approach extends to surfaces with boundaries and to non-manifold surfaces. We demonstrate the versatility and efficacy of our approach on a variety of data sets and tolerance volumes.

7.1.3. *CGALmesh: A Generic Framework for Delaunay Mesh Generation*

Participants: Jean-Daniel Boissonnat, Clément Jamin, Mariette Yvinec.

In collaboration with P. Alliez (Titane Project-team).

CGALmesh [21] is the mesh generation software package of the Computational Geometry Algorithm Library (CGAL). It generates isotropic simplicial meshes—surface triangular meshes or volume tetrahedral meshes—from input surfaces, 3D domains, and 3D multidomains, with or without sharp features. The underlying meshing algorithm relies on restricted Delaunay triangulations to approximate domains and surfaces and on Delaunay refinement to ensure both approximation accuracy and mesh quality. CGALmesh provides guarantees on approximation quality and on the size and shape of the mesh elements. It provides four optional mesh optimization algorithms to further improve the mesh quality. A distinctive property of CGALmesh is its high flexibility with respect to the input domain representation. Such a flexibility is achieved through a careful software design, gathering into a single abstract concept, denoted by the oracle, all required interface features between the meshing engine and the input domain. We already provide oracles for domains defined by polyhedral and implicit surfaces.

7.2. Topological and Geometric Inference

7.2.1. *Subsampling Methods for Persistent Homology*

Participants: Frédéric Chazal, Bertrand Michel.

In collaboration with B.T. Fasy, F. Lecci, A. Rinaldo and L. Wasserman (Carnegie Mellon University).

Persistent homology is a multiscale method for analyzing the shape of sets and functions from point cloud data arising from an unknown distribution supported on those sets. When the size of the sample is large, direct computation of the persistent homology is prohibitive due to the combinatorial nature of the existing algorithms. We propose to compute the persistent homology of several subsamples of the data and then combine the resulting estimates. We study the risk of two estimators and we prove that the subsampling approach carries stable topological information while achieving a great reduction in computational complexity.

7.2.2. *Efficient and Robust Persistent Homology for Measures*

Participants: Frédéric Chazal, Steve Oudot.

In collaboration with M. Buchet (Ohio State University) and Donald Sheehy (University of Connecticut).

A new paradigm for point cloud data analysis has emerged recently, where point clouds are no longer treated as mere compact sets but rather as empirical measures. A notion of distance to such measures has been defined and shown to be stable with respect to perturbations of the measure. This distance can easily be computed pointwise in the case of a point cloud, but its sublevel-sets, which carry the geometric information about the measure, remain hard to compute or approximate. This makes it challenging to adapt many powerful techniques based on the Euclidean distance to a point cloud to the more general setting of the distance to a measure on a metric space. We propose [28] an efficient and reliable scheme to approximate the topological structure of the family of sublevel-sets of the distance to a measure. We obtain an algorithm for approximating the persistent homology of the distance to an empirical measure that works in arbitrary metric spaces. Precise quality and complexity guarantees are given with a discussion on the behavior of our approach in practice.

7.2.3. *Topological analysis of scalar fields with outliers*

Participants: Frédéric Chazal, Steve Oudot.

In collaboration with M. Buchet, T.K. Dey, F. Fan, Y. Wang (Ohio State University).

Given a real-valued function f defined over a manifold M embedded in Euclidean space, we are interested in recovering structural information about f from the sole information of its values on a finite sample P [27]. Existing methods provide approximation to the persistence diagram of f when the noise is bounded in both the functional and geometric domains. However, they fail in the presence of aberrant values, also called outliers, both in theory and practice. We propose a new algorithm that deals with outliers. We handle aberrant functional values with a method inspired from the k -nearest neighbors regression and the local median filtering, while the geometric outliers are handled using the distance to a measure. Combined with topological results on nested filtrations, our algorithm performs robust topological analysis of scalar fields in a wider range of noise models than handled by current methods. We provide theoretical guarantees on the quality of our approximation and some experimental results illustrating its behavior.

7.2.4. *Zigzag Persistence via Reflections and Transpositions*

Participants: Clément Maria, Steve Oudot.

We introduce [33] a simple algorithm for computing zigzag persistence, designed in the same spirit as the standard persistence algorithm. Our algorithm reduces a single matrix, maintains an explicit set of chains encoding the persistent homology of the current zigzag, and updates it under simplex insertions and removals. The total worst-case running time matches the usual cubic bound.

A noticeable difference with the standard persistence algorithm is that we do not insert or remove new simplices "at the end" of the zigzag, but rather "in the middle". To do so, we use arrow reflections and transpositions, in the same spirit as reflection functors in quiver theory. Our analysis introduces a new kind of reflection called the "weak-diamond", for which we are able to predict the changes in the interval decomposition and associated compatible bases. Arrow transpositions have been studied previously in the context of standard persistent homology, and we extend the study to the context of zigzag persistence. For both types of transformations, we provide simple procedures to update the interval decomposition and associated compatible homology basis.

7.2.5. *Stable Topological Signatures for Points on 3D Shapes*

Participants: Mathieu Carrière, Steve Oudot, Maksims Ovsjanikovs.

Comparing points on 3D shapes is among the fundamental operations in shape analysis. To facilitate this task, a great number of local point signatures or descriptors have been proposed in the past decades. However, the vast majority of these descriptors concentrate on the local geometry of the shape around the point, and thus are insensitive to its connectivity structure. By contrast, several *global* signatures have been proposed that successfully capture the overall topology of the shape and thus characterize the shape as a whole. We propose [29], [43] the first point descriptor that captures the topology structure of the shape as 'seen' from a single point, in a multiscale and provably stable way. We also demonstrate how a large class of topological signatures, including ours, can be mapped to vectors, opening the door to many classical analysis and learning methods. We illustrate the performance of this approach on the problems of supervised shape labeling and shape matching. We show that our signatures provide complementary information to existing ones and allow to achieve better performance with less training data in both applications.

7.2.6. *Structure and Stability of the 1-Dimensional Mapper*

Participants: Mathieu Carrière, Steve Oudot.

Given a continuous function $f : X \rightarrow \mathbb{R}$ and a cover I of its image by intervals, the Mapper is the nerve of a refinement of the pullback cover $f^{-1}(I)$. Despite its success in applications, little is known about the structure and stability of this construction from a theoretical point of view. As a pixelized version of the Reeb graph of f , it is expected to capture a subset of its features (branches, holes), depending on how the interval cover is positioned with respect to the critical values of the function. Its stability should also depend on this positioning. We propose [44] a theoretical framework that relates the structure of the Mapper to the one of the Reeb graph, making it possible to predict which features will be present and which will be absent in the Mapper given the function and the cover, and for each feature, to quantify its degree of instability. Using this framework, we can derive guarantees on the structure of the Mapper, on its stability, and on its convergence to the Reeb graph as the granularity of the cover I goes to zero.

7.2.7. *Persistence Theory: From Quiver Representations to Data Analysis*

Participant: Steve Oudot.

Persistence theory emerged in the early 2000s as a new theory in the area of applied and computational topology. This book [35] provides a broad and modern view of the subject, including its algebraic, topological, and algorithmic aspects. It also elaborates on applications in data analysis. The level of detail of the exposition has been set so as to keep a survey style, while providing sufficient insights into the proofs so the reader can understand the mechanisms at work.

7.3. Data Structures and Robust Geometric Computation

7.3.1. *A probabilistic approach to reducing the algebraic complexity of computing Delaunay triangulations*

Participant: Jean-Daniel Boissonnat.

In collaboration with Ramsay Dyer (Johann Bernoulli Institute, University of Groningen, Netherlands) and Arijit Ghosh (Max-Planck-Institut für Informatik, Saarbrücken, Germany).

Computing Delaunay triangulations in \mathbb{R}^d involves evaluating the so-called `in_sphere` predicate that determines if a point x lies inside, on or outside the sphere circumscribing $d + 1$ points p_0, \dots, p_d . This predicate reduces to evaluating the sign of a multivariate polynomial of degree $d + 2$ in the coordinates of the points x, p_0, \dots, p_d . Despite much progress on exact geometric computing, the fact that the degree of the polynomial increases with d makes the evaluation of the sign of such a polynomial problematic except in very low dimensions. In this paper, we propose a new approach that is based on the witness complex, a weak form of the Delaunay complex introduced by Carlsson and de Silva. The witness complex $\text{Wit}(L, W)$ is defined from two sets L and W in some metric space X : a finite set of points L on which the complex is built, and a set W of witnesses that serves as an approximation of X . A fundamental result of de Silva states that $\text{Wit}(L, W) = \text{Del}(L)$ if $W = X = \mathbb{R}^d$. In [25], [41], we give conditions on L that ensure that the witness complex and the Delaunay triangulation coincide when W is a finite set, and we introduce a new perturbation scheme to compute a perturbed set L' close to L such that $\text{Del}(L') = \text{Wit}(L', W)$. Our perturbation algorithm is a geometric application of the Moser-Tardos constructive proof of the Lovász local lemma. The only numerical operations we use are (squared) distance comparisons (i.e., predicates of degree 2). The time-complexity of the algorithm is sublinear in $|W|$. Interestingly, although the algorithm does not compute any measure of simplex quality, a lower bound on the thickness of the output simplices can be guaranteed.

7.3.2. Smoothed complexity of convex hulls

Participants: Marc Glisse, Rémy Thomasse.

In collaboration with O. Devillers (VEGAS Project-team) and X. Goaoc (Université Marne-la-Vallée)

We establish an upper bound on the smoothed complexity of convex hulls in \mathbb{R}^d under uniform Euclidean (ℓ^2) noise. Specifically, let $\{p_1^*, p_2^*, \dots, p_n^*\}$ be an arbitrary set of n points in the unit ball in \mathbb{R}^d and let $p_i = p_i^* + x_i$, where x_1, x_2, \dots, x_n are chosen independently from the unit ball of radius δ . We show that the expected complexity, measured as the number of faces of all dimensions, of the convex hull of $\{p_1, p_2, \dots, p_n\}$ is $O\left(n^{2-\frac{4}{d+1}}(1+1/\delta)^{d-1}\right)$; the magnitude δ of the noise may vary with n . For $d = 2$ this bound improves to $O\left(n^{\frac{2}{3}}(1+\delta^{-\frac{2}{3}})\right)$.

We also analyze the expected complexity of the convex hull of ℓ^2 and Gaussian perturbations of a nice sample of a sphere, giving a lower-bound for the smoothed complexity. We identify the different regimes in terms of the scale, as a function of n , and show that as the magnitude of the noise increases, that complexity varies monotonically for Gaussian noise but non-monotonically for ℓ^2 noise [31], [38].

7.3.3. Realization Spaces of Arrangements of Convex Bodies

Participant: Alfredo Hubard.

In collaboration with M. Dobbins (PosTech, South Korea) and A. Holmsen (KAIST, South Korea)

In [23], we introduce combinatorial types of arrangements of convex bodies, extending order types of point sets to arrangements of convex bodies, and study their realization spaces. Our main results witness a trade-off between the combinatorial complexity of the bodies and the topological complexity of their realization space. On one hand, we show that every combinatorial type can be realized by an arrangement of convex bodies and (under mild assumptions) its realization space is contractible. On the other hand, we prove a universality theorem that says that the restriction of the realization space to arrangements of convex polygons with a bounded number of vertices can have the homotopy type of any primary semialgebraic set.

7.3.4. Limits of order types

Participant: Alfredo Hubard.

In collaboration with X. Goaoc (Institut G. Monge), R. de Joannis de Verclos (CNRS-INPG), J-S. Sereni (LORIA), and J. Volec (ETH)

The notion of limits of dense graphs was invented, among other reasons, to attack problems in extremal graph theory. It is straightforward to define limits of order types in analogy with limits of graphs, and in [24] we examine how to adapt to this setting two approaches developed to study limits of dense graphs. We first consider flag algebras, which were used to open various questions on graphs to mechanical solving via semidefinite programming. We define flag algebras of order types, and use them to obtain, via the semidefinite method, new lower bounds on the density of 5- or 6-tuples in convex position in arbitrary point sets, as well as some inequalities expressing the difficulty of sampling order types uniformly. We next consider graphons, a representation of limits of dense graphs that enable their study by continuous probabilistic or analytic methods. We investigate how planar measures fare as a candidate analogue of graphons for limits of order types. We show that the map sending a measure to its associated limit is continuous and, if restricted to uniform measures on compact convex sets, a homeomorphism. We prove, however, that this map is not surjective. Finally, we examine a limit of order types similar to classical constructions in combinatorial geometry (Erdős-Szekeres, Horton...) and show that it cannot be represented by any somewhere regular measure; we analyze this example via an analogue of Sylvester's problem on the probability that k random points are in convex position.

GRACE Project-Team

7. New Results

7.1. Weight distribution of Algebraic-Geometry codes

V. Ducet worked on the weight distribution of geometric codes following a method initiated by Duursma. More precisely he implemented his method in magma and was able to compute the weight distribution of the geometric codes coming from two optimal curves of genus 2 and 3 over the finite fields of size 16 and 9 respectively. The aim is to compute the weight distribution of the Hermitian code over the finite field of size 16, for which computational improvements of the implementation are necessary.

7.2. Faster elliptic and hyperelliptic curve cryptography

B. Smith made several contributions to the development of faster arithmetic on elliptic curves and genus 2 Jacobians in 2015. First, an extended and more detailed treatment of his \mathbb{Q} -curve construction for endomorphism-accelerated elliptic curves (previously presented at ASIACRYPT 2013, and the basis of a successful implementation with C. Costello and H. Hisil presented at EUROCRYPT 2014) appeared in the Journal of Cryptology. A simplified approach to essential precomputations was published in the proceedings of AGCT-14. Finally, with C. Costello and P.-N. Chung, he gave a new, efficient, uniform, and constant-time scalar multiplication algorithm for genus 2 Jacobians exploiting fast Kummer surface arithmetic and features of differential addition chains.

7.3. Quantum factoring

Integer factorization via Shor's algorithm is a benchmark problem for general quantum computers, but surprisingly little work has been done on optimizing the algorithm for use as a serious factoring tool once large quantum computers are built (rather than as a proof of concept). In the meantime, given the limited size of contemporary quantum computers and the practical difficulties involved in building them, any optimizations to quantum factoring algorithms can lead to significant practical improvements. In a new interdisciplinary project with physicists F. Grosshans and T. Lawson, F. Morain and B. Smith have derived a simple new quantum factoring algorithm for cryptographic integers; its expected runtime is lower than Shor's factoring algorithm, and it should also be easier to implement in practice.

7.4. Cryptanalysis of code based cryptosystems by filtration attacks

The McEliece encryption scheme based on binary Goppa codes was one of the first public-key encryption schemes [35]. Its security rests on the difficulty of decoding an arbitrary code. The original proposal uses classical Goppa codes, and while it still remains unbroken, it requires a huge size of key. On the other hand, many derivative systems based on other families of algebraic codes have been subject to key recovery attacks. Up to now, key recovery attacks were based either on a variant of Sidelnikov and Shestakov's attack [36], where the first step involves the computation of minimum-weight codewords, or on the resolution of a system of polynomial equations using Gröbner bases.

In [3], A. Couvreur, P. Gaborit, V. Gauthier, A. Otmani and J.-P. Tillich introduced a new paradigm of attack called *filtration attacks*. The general principle decomposes in two steps:

1. **Distinguishing** the public code from a random one using the square code operation.
2. **Computing a filtration** of the public code using the distinguisher, and deriving from this filtration an efficient decoding algorithm for the public code.

This new style of attack allowed A. Couvreur, A. Otmani and J.-P. Tillich to break (in polynomial time) McEliece based on wild Goppa codes over quadratic extensions [7] and more recently to break the BBCRS cryptosystem [20]. A. Couvreur, Irene Márquez–Corbella, and R. Pellikaan broke McEliece based on algebraic geometry codes from curves of arbitrary genus [5], [6] by reconstructing optimal polynomial time decoding algorithms from the raw data of a generator matrix.

7.5. Quantum LDPC codes

Quantum codes are the analogous of error correcting codes for a quantum computer. A well known family of quantum codes are the CSS codes due to Calderbank, Shor and Steane can be represented by a pair of matrices (H_X, H_Z) such that $H_X H_Z^T = 0$. As in classical coding theory, if these matrices are sparse, then the code is said to be LDPC. An open problem in quantum coding theory is to get a family of quantum LDPC codes whose asymptotic minimum distance is in $\Omega(n^\alpha)$ for some $\alpha > 1/2$. No such family is known and actually, only few known families of quantum LDPC codes have a minimum distance tending to infinity.

In an article in preparation, Benjamin Audoux (I2M, Marseille) and A. Couvreur investigate a problem suggested by Bravyi and Hastings. They studied the behaviour of iterated tensor powers of CSS codes and prove in particular that such families always have a minimum distance tending to infinity. They propose also 3 families of LDPC codes whose minimum distance is in $\Omega(n^\beta)$ for all $\beta < 1/2$.

7.6. Discrete Logarithm computations in finite fields with the NFS algorithm

The best discrete logarithm record computations in prime fields and large characteristic finite fields are obtained with Number Field Sieve algorithm (NFS) at the moment. This algorithm is made of four steps:

1. polynomial selection;
2. relation collection (with a sieving technique);
3. linear algebra (computing the kernel of a huge matrix, of millions of rows and columns);
4. individual discrete logarithm computation.

The two more time consuming steps are the relation collection step and the linear algebra step. The polynomial selection is quite fast but is very important since it determines the complexity of the algorithm. Selecting better polynomials is a key to improve the overall running-time of the NFS algorithm. The final step: individual discrete logarithm, was though to be quite fast but F. Morain and A. Guillevic showed that it has an increasing complexity with respect to the extension degree of the finite field. A. Guillevic proposed a new method to reduce considerably the complexity, with at most a factor two speed-up in the exponent [22].

In 2015, F. Morain and A. Guillevic released with P. Gaudry and R. Barbulescu a major discrete logarithm record in a quadratic finite field $\text{GF}(p^2)$ of 180 decimal digits (dd), corresponding to 595 bits. This was presented at the international conference Eurocrypt [19].

7.6.1. DL Record computation in a quadratic finite field $\text{GF}(p^2)$

In order to compare the practical running time of discrete logarithm computation in prime fields and quadratic finite fields, F. Morain and A. Guillevic with P. Gaudry and R. Barbulescu launched a DL record in a 180dd finite field. The last DL record in a prime field was held by the CAMEL team of Nancy, in 2014, in a 180 dd prime field. The parameters chosen for the quadratic finite field are the following.

$$\begin{aligned}
 p &= 314159265358979323846264338327950288419716939937510582097494459230 \setminus \\
 &\quad 781640628620899877709223 \\
 \ell &= 392699081698724154807830422909937860524646174921888227621868074038 \setminus \\
 &\quad 47705078577612484713653 \\
 p-1 &= 6 \cdot h_0 \text{ with } h_0 \text{ a } 89 \text{ dd prime number} \\
 p+1 &= 8 \cdot \ell
 \end{aligned}$$

The discrete logarithm computation was made modulo ℓ , the largest prime factor of the multiplicative subgroup $GF(p^2)^*$, so that a DL computation with generic methods of complexity $O(\sqrt{\ell})$ was impracticable.

The two polynomials used in the NFS algorithm were chosen to be the following:

$$\begin{aligned} f &= x^4 + 1 \\ g &= 448225077249286433565160965828828303618362474 x^2 - 29606109908476368046927513730655796265782 \\ &\quad + 448225077249286433565160965828828303618362474 . \end{aligned}$$

We indeed designed a new polynomial selection method, that we called the Conjugation method. It is very well suited for quadratic and cubic finite fields $GF(p^2)$ and $GF(p^3)$ for the size range of the records.

We finally computed the discrete logarithm in basis $G = T + 2$ of the target $s = \lfloor (\pi(2^{298})/8) \rfloor t + \lfloor (\gamma \cdot 2^{298}) \rfloor$

$$\begin{aligned} \log_G s &\equiv 2762142436179128043003373492683066054037581738194144186101 \setminus \\ &\quad 9832278568318885392430499058012 \pmod{\ell}. \end{aligned}$$

The running time was very surprising: our record was much faster than the concurrent DL computation in a prime field of the same global size of 180dd, and even faster than the RSA modulus factorization of the same size.

Table 2. Comparison of running time for integer factorization (NFS-IF), discrete logarithm in prime field (NFS-DL(p)) and in quadratic field (NFS-DL(p²)) of same global size 180 dd.

Algorithm	relation collection	linear algebra	total
NFS-IF	5 years	5.5 months	5.5 years
NFS-DL(p)	50 years	80 years	130 years
NFS-DL(p ²)	157 days	18 days (GPU)	0.5 years

7.6.2. Individual discrete logarithm computation

A big difference between prime fields and finite fields of small extension such as $GF(p^3)$, $GF(p^4)$ and $GF(p^6)$ is the complexity of the final step of the NFS algorithm: computing the individual discrete logarithm of the target, given the large table of discrete logarithm of *small* elements. This table was obtained at the end of the linear algebra step. The target needs to be decomposed into small enough elements whose discrete logarithm is in the table, so that one can recompose the discrete logarithm of the target. This decomposition is quite fast for prime fields but we realized that it becomes more and more time consuming when the extension degree increase. A. Guillevic developed a new technique to improve considerably this step. The main idea is to use the structure of the finite field: the subfields. These improvements were presented at the Asiacrypt 2015 conference in Auckland, New Zealand and published in the proceedings [22].

7.7. Information sets of multiplicity codes

The codes we used in our PIR protocols, namely Reed-Muller and their generalization Multiplicity codes, are locally *correctable*: that means that local decoding allows to retrieve encoded symbols. In most applications, it is very desirable to retrieve *information* symbols. Another line of work in this topic was thus to find an encoding method for multiplicity codes so as to directly recover an information symbol from local correction, and not an encoded symbol. To do so we defined information sets for multiplicity codes, and design a systematic encoding based on this information set. This work was presented at ISIT'2015 in Hong-Kong in June [18].

7.8. Rank metric codes over infinite fields

Rank metric and Gabidulin codes over the rationals promise interesting applications to space-time coding. We have constructed optimal codes, similar to Gabidulin codes, in the case of infinite fields. We use algebraic extensions, and we have determined the condition on the considered extension to enable this construction. For example: we can design codes with complex coefficients, using number fields and Galois automorphisms. Then, in the rank metric setting, codewords can be seen as matrices. In this setting, a channel introduces errors (a matrix of small rank r added to the codeword) and erasures (s_r rows and s_c columns of the matrix are erased). We have developed an algorithm (adapted from the Welch–Berlekamp algorithm) to recover the right codeword in the presence of an error of rank weight up to $r + s_c + s_r \leq d - 1$, where d is the minimal distance of the code. As opposed to the finite field case, we are confronted by coefficient size growth. We solve this problem by computing modulo prime ideals. Using these codes we can completely bypass intermediate constructions using finite fields, which were the stumbling-block in classic constructions.

We also have used this framework to build rank-metric codes over the field of rational functions, using algebraic function fields with cyclic Galois group (Kummer and Artin extensions). These codes can be seen as a generator of infinitely many convolutional codes.

7.9. Hash function cryptanalysis

Cryptographic hash functions are versatile primitives that are used in many cryptographic protocols. The security of a hash function h is usually evaluated through two main notions: its preimage resistance (given a target t , the difficulty of finding a message m s.t. $h(m) = t$) and its collision resistance (the difficulty of finding two messages m, m' s.t. $h(m) = h(m')$).

A popular hash function is the SHA-1 algorithm. Although theoretical collision attacks were found in 2005, it is still being used in some applications, for instance as the hash function in some TLS certificates. Hence cryptanalysis of SHA-1 is still a major topic in cryptography.

In 2015, we improved the state-of-the-art on SHA-1 analysis in two ways:

- T. Espitau, P.-A. Fouque and P. Karpman improved the previous preimage attacks on SHA-1, reaching up to 62 rounds (out of 80), up from 57. The corresponding paper was published at CRYPTO 2015 [21].
- P. Karpman, T. Peyrin and M. Stevens developed collision attacks on the compression function of SHA-1 (i.e. freestart collisions). This exploits a model that is slightly more generous to the attacker in order to find explicit collisions on more rounds than what was previously possible. A first work resulted in freestart collisions for SHA-1 reduced to 76 steps; this attack takes less than a week to compute on a common GPU. The corresponding paper was published at CRYPTO 2015 [24]. This was later improved to attack the full compression function. Although the attack is more expensive it is still practical, taking less than two weeks on a 64 GPU cluster. The corresponding paper is currently under review for EUROCRYPT 2016 [32].

7.10. Block cipher design and analysis

Block ciphers are one of the most basic cryptographic primitives, yet block cipher analysis is still a major research topic. In recent years, the community also shifted focus to the more general setting of *authenticated encryption*, where one specifies an (set of) algorithm(s) providing both encryption and authentication for messages of arbitrary length. A major current event in that direction is the CAESAR academic competition, which aims to select a portfolio of good algorithms.

During this year, we helped to improve the state of the art in block cipher research in several ways:

- P. Karpman found a very efficient related-key attack on the CAESAR candidate Prøst-OTR. A related-key model is very generous to the attacker, but the attack in this case can be run instantaneously. The corresponding paper was published at ISC 2015 [23]

- B. Minaud, P. Derbez, P.-A. Fouque and P. Karpman developed a family of attacks that breaks all the remaining unbroken instances of the ASASA construction, that was presented at ASIACRYPT 2014. Using algebraic properties of the ciphers, for each type of instance, the attack allows to recover an algorithm equivalent to the secret key in near-practical time. This applies to a multivariate public-key scheme, a classical block cipher and small block ciphers used in white-box constructions. The corresponding paper was published at ASIACRYPT 2015 and was honoured as one of the three best papers of the conference [25].
- P. Karpman developed a compact 8-bit S-box with branch number three, which can be used as a basis to construct a lightweight block cipher particularly efficient on 8-bit microcontrollers. The corresponding paper is currently under review for FSE 2016.

MEXICO Project-Team

7. New Results

7.1. Highlights

Please note that three of our most important and novel results are given in the 'Highlights' section above.

7.2. Specifying and Verifying Concurrent C Programs with TLA+

Verifying software systems automatically from their source code rather than modelling them in a dedicated language gives more confidence in establishing their properties. In [37] we propose a formal specification and verification approach for concurrent C programs directly based on the semantics of C. We define a set of translation rules and implement it in a tool (C2TLA+) that automatically translates C code into a TLA+ specification. The TLC model checker can use this specification to generate a model, allowing to check the absence of runtime errors and dead code in the C program in a given configuration. In addition, we show how translated specifications interact with manually written ones to: check the C code against safety or liveness properties; provide concurrency primitives or model hardware that cannot be expressed in C; and use abstract versions of translated C functions to address the state explosion problem. All these verifications have been conducted on an industrial case study, which is a part of the microkernel of the PharOS real-time system.

7.3. Active Diagnosis with Observable Quiescence

Active diagnosis of a discrete-event system consists in controlling the system such that faults can be detected. In [27] we extend the framework of active diagnosis presented in [7] by introducing modalities for actions and states and a new capability for the controller, namely observing that the system is quiescent. We design a game-based construction for both the decision and the synthesis problems that is computationally optimal. Furthermore we prove that the size and the delay provided by the active diagnoser (when it exists) are almost optimal.

7.4. Test Case Generation for Concurrent Systems Using Event Structures

In [23] we deal with the test-case generation problem for concurrent systems that are specified by true-concurrency models such as Petri nets. We show that using true-concurrency models reduces both the size and the number of test cases needed for achieving certain coverage criteria. We present a test-case generation algorithm based on Petri net unfoldings and a SAT encoding for solving controllability problems in test cases. Finally, we evaluate our algorithm against traditional test-case generation methods under interleaving semantics.

7.5. State Space Reduction Strategie for Model Checking Concurrent C Programs

Model checking is an effective technique for uncovering subtle errors in concurrent systems. Unfortunately, the state space explosion is the main bottleneck in model checking tools. In [31] we propose a state space reduction technique for model checking concurrent programs written in C. The reduction technique consists in an analysis phase, which defines an approximate agglomeration predicate. This latter states whether a statement can be agglomerated or not. We implement this predicate using a syntactic analysis, as well as a semantic analysis based on abstract interpretation. We show the usefulness of using agglomeration technique to reduce the state space, as well as to generate an abstract TLA+ specification from a C program.

7.6. Simple Priced Timed Games Are Not That Simple

Priced timed games are two-player zero-sum games played on priced timed automata (whose locations and transitions are labeled by weights modeling the costs of spending time in a state and executing an action, respectively). The goals of the players are to minimise and maximise the cost to reach a target location, respectively. In [25] we consider priced timed games with one clock and arbitrary (positive and negative) weights and show that, for an important subclass of theirs (the so-called simple priced timed games), one can compute, in exponential time, the optimal values that the players can achieve, with their associated optimal strategies. As side results, we also show that one-clock priced timed games are determined and that we can use our result on simple priced timed games to solve the more general class of so-called reset-acyclic priced timed games (with arbitrary weights and one-clock).

7.7. A Hybrid-Dynamical Model for Passenger-flow in Transportation Systems

In a network with different transportation modes, or multimodal public transportation system (MPTS), modes are linked among one another not by resources or infrastructure elements—which are not shared, e.g., between different metro lines—but by the flow of passengers between them. Now, the movements of passengers are steered by the destinations that individual passengers have, and by which they can be grouped into trip profiles. To use the strength of fluid dynamics, introduce in [30] a multiphase hybrid Petri net model, in which the vehicle dynamics is rendered by individual tokens moving in an infrastructure net, while passenger quantities are given as vectors—whose components correspond to trip profiles—and evolve at stations according to fluid dynamics. This model is intended as a building block for obtaining supervisory control, via transport operator actions, to mitigate congestion.

7.8. An Algebraic View of Space/Belief and Extrusion/Utterance for Concurrency/Epistemic Logic

In [29] we enrich spatial constraint systems with operators to specify information and processes moving from a space to another. We shall refer to these new structures as spatial constraint systems with extrusion. We shall investigate the properties of this new family of constraint systems and illustrate their applications. From a computational point of view the new operators provide for process/information extrusion, a central concept in formalisms for mobile communication. From an epistemic point of view extrusion corresponds to a notion we shall call utterance; a piece of information that an agent communicates to others but that may be inconsistent with the agent's beliefs. Utterances can then be used to express instances of epistemic notions, which are common place in social media, such as hoaxes or intentional lies. Spatial constraint systems with extrusion can be seen as complete Heyting algebras equipped with maps to account for spatial and epistemic specifications.

7.9. Preserving Partial Order Runs in Parametric Time Petri Nets

Parameter synthesis for timed systems aims at deriving parameter valuations satisfying a given property. In [22] we target concurrent systems; it is well known that concurrency is a source of state-space explosion, and partial order techniques were defined to cope with this problem. Here we use partial order semantics for parametric time Petri nets as a way to significantly enhance the result of an existing synthesis algorithm. Given a reference parameter valuation, our approach synthesizes other valuations preserving, up to interleaving, the behavior of the reference parameter valuation. We show the applicability of our approach using acyclic asynchronous circuits.

7.10. Non-Atomic Transition Firing in Contextual Nets

The firing rule for Petri nets assumes instantaneous and simultaneous consumption and creation of tokens. In the context of ordinary Petri nets, this poses no particular problem because of the system's asynchronicity, even if token creation occurs later than token consumption in the firing. With read arcs, the situation changes, and several different choices of semantics are possible. The step semantics introduced by Janicki and Koutny

can be seen as imposing a two-phase firing scheme: first, the presence of the required tokens is checked, then consumption and production of tokens happens. Pursuing this approach further, we develop in [28] a more general framework based on explicitly splitting the phases of firing, allowing to synthesize coherent steps. This turns out to define a more general non-atomic semantics, which has important potential for safety as it allows to detect errors that were missed by the previous semantics. Then we study the characterization of partial-order processes feasible under one or the other semantics.

PARSIFAL Project-Team

7. New Results

7.1. The Checkers Proof Certifier

Participants: Tomer Libal, Giselle Reis, Hichem Chihani.

We presented a system description [29] of the Checkers proof certifier, which implements some of the theoretical ideas developed in the ProofCert project. This version of the system is capable of certifying a subset of the E-Prover superposition theorem prover. The system is mainly written in λ Prolog with a proof importing module written in Ocaml. The system is designed to allow modularity when designing the semantical translations of proof systems. For this capacity, the system supports, J. A. Robinson's resolution and the paramodulation technique of G. Robinson and L. Wos. On top of that, minimal support for some inference rules of the E-Prover was added.

7.2. Regular Patterns in Second-Order Unification

Participant: Tomer Libal.

We presented a paper [33] detailing a higher-order pre-unification procedure with improved termination over existing procedures. The classic higher-order unification procedure was presented by G. Huet in 1975 and is still used as the main unification procedure for higher-order automated theorem provers. This procedure does not terminate. In this project we have investigated the reasons for that and have shown that by choosing a specific (but complete) search strategy, an additional set of non-unifiable problems can be detected. As an example, we have shown that all unification problems generated by the Leo-III theorem prover when proving Cantor's theorem are decided by this procedure, in contrast to the classical unification procedure.

7.3. Static guarantees for message-passing computation

Participant: Stéphane Graham-Lengrand.

LCF [79] is a proof-search architecture, where search strategies are programmed via an API and successful proof-search runs are guaranteed correct, relying on the use of an abstract type theorem. We adapted the approach and defined principles for message-passing software architectures (where modules interact by exchanging messages), with the objective of guaranteeing message provenance and integrity. The principles rely on abstract types to *sign* messages at no run-time cost, and more generally rely on type-checking to provide static guarantees (i.e. at compile-time) that the messages produced by a trusted piece of code will not be altered or faked by an untrusted piece of code. We developed this primarily for safe theorem proving architectures, but the approach can be applied to other software architectures where modules with different levels of trust interact.

7.4. Proof-search with quantifiers and theories

Participant: Stéphane Graham-Lengrand.

We published our approach to proof-search on quantified problems in presence of one theory [22], where we identify the specifications required of the theory for the proof-search process to be sound and complete. Theories with unification procedures or quantifier elimination procedures satisfy our specifications, where *constraint streams* and *constraint projections* play a key role. Interestingly enough, Bjorner and Janota [52] independently achieved a similar result with *model projections*. Our theory-generic approach allows a clear formulation of what it could mean to combine several quantifier-handling theories, hopefully generalising what the Nelson-Oppen combination technique does in a quantifier-free context. We recently obtained two new results towards this:

- First, the cumbersome, stream-querying, and backtracking mechanisms that were required to implement [22] have been re-expressed in a more satisfying message-passing computational framework.
- Second, we re-expressed the standard quantifier-free combination techniques, mentioned above, as a concurrent message-passing interaction between different theory-specific procedures, and simplified their proofs of correctness. This led to the major redesign of Psyche, mentioned above.

7.5. Realizability semantics of abstract focusing, formalized

Participant: Stéphane Graham-Lengrand.

In [21] we presented a parametric system for abstract focusing, building on Zeilberger’s work [87], and parametrically capturing classical and intuitionistic focused systems. We presented its semantics, building on Munch-Maccagnoni’s work [80], in terms of *abstract realizability models* (which were independently identified by Krivine). The goal was to emphasize the similarities and differences between focusing and realizability, in the way they exploit the *polarities* of formulae. The system and its semantics led to a substantial formalisation in the proof assistant Coq.

7.6. The Meta-Theory of Bisimulation-Up-To

Participants: Kaustuv Chaudhuri, Matteo Cimini, Dale Miller.

The method of proof by bisimulation has proved to be a very successful technique for showing the equivalence of processes. Unfortunately, in process calculi with infinite transition systems, such as in calculi with a replication operator, finding a bisimulation requires exploring an infinite search space, which moreover often tends to have rather intricate and complex structure. One way to combat this complexity—i.e., reduce the size of candidate bisimulation sets—is to identify redundancies among their members and then to replace redundant classes by unique inhabitants. This yields families of *bisimulation-up-to* proof methods that are parametric over the redundancy relation. For instance, if we consider bisimilarity itself as the redundancy, then we obtain *bisimulation up to bisimilarity*; with this relation, the singleton set $\{(!a, !!a)\}$ is a candidate set for showing that the processes $!a$ and $!!a$ are bisimilar, for example, when the bisimulation set with redundancies is infinite.

Since *a priori* there is no restriction on such redundancy relations, a key theoretical question is when a bisimulation-up-to relation is *sound*, i.e., that it is contained in a bisimulation. In the literature there have been a number of techniques proposed for showing soundness, but they often require the use of complex reasoning about lattices of fixed points. In [19] (CPP’15) we show how to use the built-in coinduction facilities of the Abella theorem prover to produce comparatively lightweight proofs of the soundness of many common bisimulation-up-to techniques for CCS and the π -calculus. A key feature of our approach is that we can use the facilities already provided by the Abella system for reasoning about the binding constructs for the π -calculus.

7.7. Characterizing Independence in Type Theory

Participants: Kaustuv Chaudhuri, Yuting Wang.

In formal proof languages based on type theory, it is often the case that a theorem is proved for a certain kind of typing context, but needs to be used in a different context. For example, theorems about natural numbers may be proved in an empty typing context, since the type of natural numbers contains no higher-order features (i.e., natural numbers are closed), but we may need to use these properties of natural numbers when reasoning about λ -terms in De Bruijn notation, where the typing context is non-empty. In such a situation, it is useful to automatically transport the existing theorems to the new kinds of contexts, since we know that the theorem in question cannot depend on the properties of λ -terms. While this example is rather trivial, it becomes non-trivial when theorems are proved about higher-order data structures, which are commonly encountered when reasoning about syntax with binding constructs.

One way to achieve such reuse automatically is a technique called *subordination*, which is based on analyzing the constructors for a certain type and defining syntactic criteria under which certain normal terms of one type can have subterms of another type. Unfortunately, the classical definition of subordination lacks a proof-theoretic justification, and has surprising properties in third-order (and higher) signatures.

In [36] (TLCA'15), we propose a proof-theoretic characterization of a kind of dual to subordination, called *independence*, that characterizes when normal terms of one type *cannot* contain subterms of another type. This is achieved by means of proving an inductive *strengthening* lemma about the signatures in the two-level logic approach. We also show how to automatically prove such lemmas in certain commonly encountered situations in the theorem prover Abella.

7.8. Disproving Non-Theorems with Saturating Search

Participants: Taus Brock-Nannestad, Kaustuv Chaudhuri.

High-performance automated reasoning techniques such as resolution and the inverse method are well suited for proving *true* conjectures, but are ill-behaved for *false* conjectures. For example, for a simple theory of even numbers that states that 0 is even and that $n + 2$ is even whenever n is even, it is obviously the case that the conjecture “3 is even” is unprovable, but the algorithm would loop forever proving “0 is even”, “2 is even”, “4 is even”, etc. This behavior is observed even in the best saturation-based (i.e., forward-reasoning) theorem provers.

In [25] (TABLEAUX'15), we show how to finitely constrain the search space of saturation-based theorem provers by the use of *unsound* extensions of the goal query. These unsound extensions, when combined with forward subsumption, guarantee that only a finite number of consequences would ever be constructed based on any goal query, so the proof search procedure is guaranteed to terminate. If a proof is found among them that does not use the unsound extensions, then we can succeed with that proof. If no proof is found, then we can soundly assert that the original goal query was also unprovable, since even a weakened version of it was unprovable. The only other possibility is that a proof is found using the unsound extension; in this case, we use the particular instance of unsoundness to refine the original unsound goal to prevent it from being found again, while maintaining the invariant that the search space is finite, and rerun the search. Since first-order logic is undecidable, we may need to repeat the refinement procedure indefinitely, but for many kinds of domains, particularly those arising from typed signatures (such as the even numbers example above), we do eventually find a saturating approximation that guarantees that the conjecture has no proof.

This algorithm has been implemented as part of the Mætning theorem prover explained in the section on Software above. We plan to extend it in the future with various automatic refinement heuristics.

7.9. Encoding Bigraph Structure with Subexponentials

Participants: Kaustuv Chaudhuri, Giselle Reis.

Bigraphs were proposed by Robin Milner as a model of ubiquitous computing, which is computation that is aware both of *location* and of *connections*. As a formalism it subsumes many other process calculi such as CCS and the π -calculus. However, it has a number of problems *qua* syntax because it is based on graphs and a complicated theory of composition. The biggest of these problems is how to implement it in a formal reasoning system.

In recent years, many members (and ex-members) of the Parsifal team have been experimenting with a variant of linear logic that has not just a single pair but an arbitrary family of exponential connectives that are arranged in a pre-order. Each such pair of *subexponentials* may admit or reject the structural properties of weakening and contraction. One benefit of subexponentials is that it allows for *querying* the *absence* of certain kinds of exponential formulas without requiring all non-exponential formulas to be deleted as a consequence, which is the issue with ordinary linear logic.

In [28] (LPAR'15), we show how to represent the structure of bigraphs in terms of a simple theory of linear logic with subexponentials (SEL). We show that our representation is adequate, i.e., that it respects the composition and juxtaposition operations on bigraphs. Moreover, we show how one can ask queries about the nesting of places in the representation without modifying it, which gives us a technical means of encoding bigraph reactions as well. Some of the details for bigraph reactions remain to be worked out in future work.

7.10. Encoding Additive Connectives with Multiplicatives and Subexponentials

Participant: Kaustuv Chaudhuri.

In a recent workshop on Linearity [55], we have published the formal proof (that was obtained in 2009) that linear logic with three subexponentials in a certain lattice is undecidable. An extended version of this paper was submitted to a special issue on Linearity in Mathematical Structures in Computer Science and was accepted in November 2015.

The preprint of that extended paper [41] gives a direct embedding of propositional MALL (multiplicative and additive linear logic) using only multiplicative connectives and five subexponentials. This means that the additive connectives are, in fact, redundant when we have multiplicatives and subexponentials. Moreover, in the first-order case this encoding is polynomial and focally adequate, which means that MALL can be simulated at the highest fidelity – at the level of individual inference rules.

7.11. Computation in Focused Intuitionistic Logic

Participants: Taus Brock-Nannestad, Nicolas Guenot, Daniel Gustafsson.

Focusing is a proof-theoretical technique for eliminating unnecessary nondeterminism in proofs. Because it cuts down on nondeterminism, focusing is particularly useful for directing proof search. Focusing thus plays a key role in explaining the meaning and behaviour of logic programs.

Despite this success in clarifying the operational semantics of logic programming, focusing has not been as widely studied in the Curry-Howard style “proofs as programs” interpretation. Early results in this area established that λ -calculi associated with the focused calculi LJT and LJQ had evaluation strategies corresponding to call-by-name and call-by-value respectively. For the LJF calculus — which contains both LJT and LJQ as fragments — no such correspondence was known.

In [27] (PPDP'15) we show how a proof-term assignment to (a variant of) Liang and Miller’s focused sequent calculus LJF permits a uniform treatment of the call-by-value and call-by-name reduction strategies of the λ -calculus, as well as combinations of these strategies. Additionally, we show how to extract an abstract machine from LJF by considering machine states as certain configurations of instances of the cut rule. The aforementioned correspondence extends to this setting, and we show that well-known abstract machines for call-by-value and call-by-name are in fact exactly the abstract machines that one gets when considering certain fragments of LJF.

In the seminal work of Paul Blain Levy, the call-by-push-value language was introduced as a way of subsuming the call-by-value and call-by-name strategies of the λ -calculus. It was later conjectured that call-by-push-value was simply implementing a notion of focusing, and indeed this turns out to be the case, as we show in the aforementioned paper.

7.12. Focused Linear Logic and the λ -calculus

Participants: Taus Brock-Nannestad, Nicolas Guenot.

Linear Logic enjoys strong symmetries inherited from classical logic while providing a constructive framework comparable to intuitionistic logic. However, the computational interpretation of sequent calculus presentations of linear logic remains problematic, mostly because of the many rule permutations allowed in the sequent calculus.

In focused variants of Linear Logic, most of these rule permutations are eliminated by the focusing restriction — during focusing, a single formula is decomposed eagerly, and the focus is passed down to its subformulas. Conversely, during inversion, all invertible connectives are decomposed. Moreover, this decomposition is made fully deterministic by keeping the connectives in question in a list, and only decomposing the first connective of this list.

The end result of this is that a focused proof in Linear Logic almost always has one particular formula singled out as the one that will be decomposed. Thus, somewhat curiously, focused Linear Logic behaves much more like an intuitionistic sequent calculus (where at all times there is a single “special” formula on the right hand side of the sequent) than a classical calculus.

In [26] (MFPS’15), we study a term assignment for a focused version of Multiplicative Exponential Linear Logic (MELL), and show how the focusing technique gives rise to a calculus that straightforwardly embeds both a linear variant of the λ -calculus, and a sequent-based formulation of Parigot’s $\lambda\mu$ -calculus.

7.13. There is no complete linear term rewriting system for propositional logic

Participant: Lutz Straßburger.

Recently, we observed that the set of all sound linear inference rules in propositional logic is already coNP-complete [84]. This means that every boolean tautology can be written as a (left-and right-) linear rewrite rule. This raises the question of whether there is a rewriting system on linear terms of propositional logic that is sound and complete for the set of all such rewrite rules. We have shown (in a joint work with Anupam Das) that, as long as reduction steps are polynomial-time decidable, such a rewriting system does not exist unless coNP=NP. This is published in [20].

7.14. A (Bi)linear Implementation of Strong Call-by-Value

Participant: Beniamino Accattoli.

The elegant theory of the call-by-value λ -calculus relies on closed terms and weak evaluation (i.e., not under abstractions) and it is well-known that the number of call-by-value β -steps is a reasonable cost model. When turning to open terms or strong evaluation—that are used for instance in the implementation of Coq—the operational theory breaks, and the call-by-value λ -calculus has to be extended with some additional rewriting rules. In a joint work with Sacerdoti Coen [18], a proposal for open/strong call-by-value, called *fireball calculus*, is studied from the point of view of cost models and abstract machines. First, it is shown that open terms introduce a new malicious behavior, making the study of cost models non-trivial. Second, it is shown that the number of β -steps in the fireball calculus is a reasonable cost model. Third, a new abstract machine is introduced and its overhead is shown to be linear with respect to the number of β -steps and the size of the initial term, providing a surprisingly efficient implementation scheme.

7.15. Implementations of Strong Call-by-Name, Revisited

Participant: Beniamino Accattoli.

The literature about abstract machines for the strong evaluation (i.e., possibly under abstraction) of the ordinary (i.e., call-by-name) λ -calculus is scarce. Essentially, there is a single, old work: Crégut’s abstract machine [60] (1990), that is an extension of Krivine abstract machine to compute full normal forms. Crégut studies the correctness of the machine by means of an explicit substitutions calculus. In this joint work with Barenbaum and Mazza [17], Crégut’s work is revisited and simplified in the extreme. An alternative, simpler machine is introduced, the *Strong Milner abstract machine*. Its correctness is studied via *linear substitution calculus*, a new approach to explicit substitutions developed by Accattoli and Kesner that is much simpler than Crégut’s approach. Moreover, a complexity analysis of the machine is provided: its overhead is shown to be linear in the number of steps in the linear substitution calculus and in the size of the initial term.

7.16. Foundational Proof Certificates

We have continued to explore a number of new aspects of framework we call *Foundational Proof Certificates* (FPCs). Besides having defined and implemented prototype checkers for FPCs in classical and intuitionistic logic [37] we have also extended the proof theory underlying numerous modal logics so that FPCs can be applied to modal logics [35]. We have also extended the notion of FPC to work also in the model checking setting [31]. In both the modal logic and model checking domains, the key to getting FPCs to work is to have descriptions of *focused proof systems* available for those logics.

Given that FPCs are declarative and semantically simple structures, it has been possible to find numerous applications of them outside the problem of simply checking them. It was shown, for example, that FPCs can be used to help define the semantics of the output from traditional theorem provers [23]. We have also used FPCs as proof outlines in order to define high-level tactics to direct proof search [24].

7.17. Multi-level Delimited Control

There has been a great deal of interest in recent years to providing interesting functional programming primitives that are based on classical logic and not just intuitionistic logic. Unfortunately, the standard sequent calculus proof theory for classical logic is far too chaotic to provide such a foundation. We have recently proposed adding to classical (linear) logic an assortment of *subexponentials* and to provide a rigid structure for their placement within formulas. This new framework allows for sequent calculus proof theory to provide to the functional programming paradigm the feature often called *multi-level delimited control* [32]. The main result in that paper is also noteworthy in that it shows how to build certain complex synthetic connectives even though the standard approach (using focusing proof systems) cannot be used.

POSTALE Team

6. New Results

6.1. Parallel light speed labeling: the world's fastest connected component labeling for multicore processors

Participants: Lionel Lacassagne, Laurent Cabaret, Daniel Etiemble.

We have designed a parallel version of the Light Speed Labeling for shared-memory multicore processor. This algorithm outperforms the best algorithm by a factor $\times 10$. We are now working on the design of algorithms for GPU and manycore embedded processor and especially the TSAR architecture of LIP6 laboratory. More information is available at

- TSAR architecture: <https://www-soc.lip6.fr/trac/tsar>
- ALMOS operating system: <https://www-soc.lip6.fr/trac/almos>
- GIET-VM system: <https://www-soc.lip6.fr/trac/giet-vm>

The paper [20] introduces the parallel version of the Light Speed Labeling (LSL) and compares it with the parallel versions of the competitors. A benchmark shows that the parallel Light Speed Labeling is at least $\times 1.9$ faster than all the other algorithms for random images. This factor reach $\times 3.6$ for structured random images. More important, we show that thanks to its run-based processing (segments), LSL is intrinsically more efficient than all pixel-based algorithms.

6.2. Opening Polyhedral Compiler's Black Box

Participants: Lénaïc Bagnères, Oleksandr Zinenko, Stéphane Huot, Cédric Bastoul.

While compilers offer a fair trade-off between productivity and executable performance in single-threaded execution, their optimizations remain fragile when addressing compute-intensive code for parallel architectures with deep memory hierarchies. Moreover, these optimizations operate as black boxes, impenetrable for the user, leaving them with no alternative to time-consuming and error-prone manual optimization in cases where an imprecise cost model or a weak analysis resulted in a bad optimization decision. To address this issue, we propose a technique allowing to automatically translate an arbitrary polyhedral optimization, used internally by loop-level optimization frameworks of several modern compilers, into a sequence of comprehensible syntactic transformations as long as this optimization focuses on scheduling loop iterations. With our approach, we open the black box of the polyhedral frameworks enabling users to examine, refine, replay and even design complex optimizations semi-automatically in partnership with the compiler. [17]

6.3. Automating Resource Selection and Configuration in Inter-clouds through a Software Product Line Method

Participants: Alexandro Ferreira Leite, Vladimir Castro Alves, Genaina Nunes Rodrigues, Claude Tadonki, Christine Eisenbeis, Alba Cristina Alves de Melo.

Nowadays, cloud users face three important problems: (a) choosing one or more appropriate cloud provider(s) to run their application(s), (b) selecting appropriate cloud resources, which implies having enough information about the available resources, including their characteristics and constraints, and (c) configuring the cloud resources. These problems are mostly due to the wide range of resources. These resources usually have distinct dependencies, and they are offered at various clouds' layers. In this complex scenario, the users often have to handle cloud resources and their dependencies manually. This is an error-prone and time-consuming activity, even for skilled cloud users and system administrators. In this context, this paper proposes a software product line engineering (SPLE) method and a tool to deal with these issues. Our SPL-based engineering method enables a declarative and goal-oriented strategy. Furthermore, it allows resource selection and configuration in inter-cloud environments. In our proposal, the cloud users specify their applications and requirements, and our tool automatically selects and configures a suitable computing environment, taking into account temporal and functional dependencies. Experimental results on Amazon EC2 and Google Compute Engine (GCE) show that our approach enables unskilled users to have access to advanced inter-cloud computing configurations, without being concerned with the characteristics of each cloud. [18]

6.4. A Randomized LU-based Solver Using GPU and Intel Xeon Phi Accelerators

Participants: Marc Baboulin, Amal Khabou, Adrien Rémy de Zotti.

We present a fast hybrid solver for dense linear systems based on LU factorization. To achieve good performance, we avoid pivoting by using random butterfly transformations for which we developed efficient implementations on heterogeneous architectures. We used both Graphics Processing Units and Intel Xeon Phi as accelerators. The performance results show that the pre-processing due to randomization is negligible and that the solver outperforms the corresponding routines based on partial pivoting. [16]

6.5. Metaprogramming dense linear algebra solvers. Applications to multi and many-core architectures

Participants: Ian Masliah, Marc Baboulin, Joël Falcou.

The increasing complexity of new parallel architectures has widened the gap between adaptability and efficiency of the codes. As high performance numerical libraries tend to focus more on performance, we wish to address this issue using a C++ library called NT2. By analyzing the properties of the linear algebra domain that can be extracted from numerical libraries and combining them with architectural features, we developed a generic approach to solve dense linear systems on various architectures including CPU and GPU. We have then extended our work with an example of a least squares solver based on semi-normal equations in mixed precision that cannot be found in current libraries. For the automatically generated solvers, we report performance comparisons with state-of-the-art codes, and show that it is possible to obtain a generic code with a high-level interface (similar to MATLAB) which runs either on CPU or GPU without generating a significant overhead. [21] [23]

6.6. Using Random Butterfly Transformations in Parallel Schur Complement-Based Preconditioning

Participants: Marc Baboulin, Aygul Jamal, Masha Sosonkina.

We propose to use a randomization technique based on Random Butterfly Transformations (RBT) in the Algebraic Recursive Multilevel Solver (ARMS) to improve the preconditioning phase in the iterative solution of sparse linear systems. We integrated the RBT technique into the parallel version of ARMS (pARMS). The preliminary experimental results on some matrices from the Davis' collection show an improvement of the convergence and accuracy of the results when compared with existing implementations of the pARMS preconditioner. [15]

6.7. LU Preconditioning for Overdetermined Sparse Least Squares Problems

Participants: Gary Howell, Marc Baboulin.

We investigate how to use an LU factorization with the classical LSQR routine for solving overdetermined sparse least squares problems. Usually L is much better conditioned than A and iterating with L instead of A results in faster convergence. When a runtime test indicates that L is not sufficiently well-conditioned, a partial orthogonalization of L accelerates the convergence. Numerical experiments illustrate the good behavior of our algorithm in terms of storage and convergence. [19]

6.8. Dense Symmetric Indefinite Factorization on GPU Accelerated Architectures

Participants: Marc Baboulin, Jack Dongarra, Adrien Rémy de Zotti, Stanimire Tomov, Ichitaro Yamazaki.

We study the performance of dense symmetric indefinite factorizations (Bunch-Kaufman and Aasen's algorithms) on multicore CPUs with a Graphics Processing Unit (GPU). Though such algorithms are needed in many scientific and engineering simulations, obtaining high performance of the factorization on the GPU is difficult because the pivoting that is required to ensure the numerical stability of the factorization leads to frequent synchronizations and irregular data accesses. As a result, until recently, there has not been any implementation of these algorithms on hybrid CPU/GPU architectures. To improve their performance on the hybrid architecture, we explore different techniques to reduce the expensive communication and synchronization between the CPU and GPU, or on the GPU. We also study the performance of a symmetric indefinite factorization with no pivoting combined with the preprocessing technique based on Random Butterfly Transformations. Though such transformations only have probabilistic results on the numerical stability, they avoid the pivoting and obtain a great performance on the GPU. [14]

6.9. Computing least squares condition numbers on hybrid multicore/GPU systems

Participants: Marc Baboulin, Jack Dongarra, Rémi Lacroix.

We present an efficient computation for least squares conditioning or estimates of it. We propose performance results using new routines on top of the multicore-GPU library MAGMA. This set of routines is based on an efficient computation of the variance-covariance matrix for which, to our knowledge, there is no implementation in current public domain libraries LAPACK and ScaLAPACK. [22]

6.10. Towards a High-Performance Tensor Algebra Package for Accelerators

Participants: Marc Baboulin, Veselin Dobrev, Jack Dongarra, Christopher Earl, Joël Falcou, Azzam Haidar, Ian Karlin, Tzanio Kolev, Ian Masliah, Stanimire Tomov.

Numerous important applications, e.g., high-order FEM simulations, can be expressed through tensors. Examples are computation of FE matrices and SpMV products expressed as generalized tensor contractions. Contractions by the first index can often be represented as tensor index reordering plus gemm, which is a key factor to achieve high-performance. We present ongoing work on the design of a high-performance package in MAGMA for Tensor algebra that includes techniques to organize tensor contractions, data storage, and parametrization related to batched execution of large number of small tensor contractions. We apply auto-tuning and code generation techniques to provide an architecture-aware, user-friendly interface. [24]

SPECFUN Project-Team

6. New Results

6.1. Integration of rational functions

Periods of rational integrals are specific integrals, with respect to one or several variables, whose integrand is a rational function and whose domain of integration is closed. This particular class of integrals contains large families of functions naturally occurring in combinatorics and statistical physics, such as diagonals, constant terms and positive part of rational functions. Periods involving one parameter are classically known to satisfy *Picard-Fuchs equations*, a special type of linear differential equations with a very rich analytic and arithmetic structure. As for other special-function manipulations, handling periods through those differential equations is a good way to actually compute them, and this was the topic of Pierre Lairez' PhD thesis defended in 2014 [53] and awarded the "Ecole Polytechnique thesis prize" in 2015.

Computing multivariate integrals is one speciality of the team and our algorithms are known to treat much more general integrals than just periods of rational integrals. However, integration is still slow in practice when the number of variables goes increasing. By looking at periods of rational functions, the hope is to obtain relevant complexity bounds and faster algorithms.

The goal of reaching relevant theoretical complexity bounds had been reached in 2013 [31] but a practically fast algorithm was still missing. This year, we described a new algorithm which is efficient in practice [4], though its complexity is not known. This algorithm allows to compute quickly integrals that are too big to be computed with previous algorithms. As a challenging benchmark, we computed 210 integrals given by Batyrev and Kreuzer in their work on Calabi–Yau varieties. This achievement gave strong visibility to the paper and allowed a quick dissemination of the implementation, which is provided in Magma under a CeCILL B license. The algorithm is now used on a regular basis by several teams. We know of:

- Tom Coates' team (Dpt. of Mathematics, Imperial College, London, UK), which uses the software in their work about mirror symmetry and classification of Fano varieties;
- Duco van Straten (Institute of Mathematics, University of Mainz, Germany), who uses the software in his work in algebraic geometry;
- Gert Alkmvist (Dpt. of Mathematics, University of Lund, Sweden), who uses the software in his work of enumerating the Calabi–Yau differential equations.

6.2. Multiple binomial sums

Multiple binomial sums form a large class of multi-indexed sequences, closed under partial summation, which contains most of the sequences obtained by multiple summation of binomial coefficients and also all the sequences with algebraic generating function. We study in [14] the representation of the generating functions of binomial sums by integrals of rational functions. The outcome is twofold. Firstly, we show that a univariate sequence is a multiple binomial sum if and only if its generating function is the diagonal of a rational function. Secondly we propose algorithms that decide the equality of multiple binomial sums and that compute recurrence relations for them. In conjunction with geometric simplifications of the integral representations, this approach behaves well in practice. The process avoids the computation of certificates and the problem of accurate summation that afflicts discrete creative telescoping, both in theory and in practice.

6.3. Diagonals of rational functions and selected differential Galois groups

Diagonals of rational functions naturally occur in lattice statistical mechanics and enumerative combinatorics. In all the examples emerging from physics, the minimal linear differential operators annihilating these diagonals of rational functions have been shown to actually possess orthogonal or symplectic differential Galois groups. In order to understand the emergence of such orthogonal or symplectic groups, we exhaustively analyze in [1] three (constrained) sets of diagonals of rational functions, corresponding respectively to rational functions of three variables, four variables and six variables. The conclusion is that, even for these sets of examples which, at first sight, have no relation with physics, their differential Galois groups are always orthogonal or symplectic groups. We also discuss conditions on the rational functions such that the operators annihilating their diagonals do not correspond to orthogonal or symplectic differential Galois groups, but rather to generic special linear groups.

6.4. Algebraic Diagonals and Walks

The diagonal of a multivariate power series F is the univariate power series $\text{Diag}F$ generated by the diagonal terms of F . Diagonals form an important class of power series; they occur frequently in number theory, theoretical physics and enumerative combinatorics. In [7] we study algorithmic questions related to diagonals in the case where F is the Taylor expansion of a bivariate rational function. It is classical that in this case $\text{Diag}F$ is an algebraic function. We propose an algorithm that computes an annihilating polynomial for $\text{Diag}F$. Generically, it is its minimal polynomial and is obtained in time quasi-linear in its size. We show that this minimal polynomial has an exponential size with respect to the degree of the input rational function. We then address the related problem of enumerating directed lattice walks. The insight given by our study leads to a new method for expanding the generating power series of bridges, excursions and meanders. We show that their first N terms can be computed in quasi-linear complexity in N , without first computing a very large polynomial equation. An extended version of this work is presented in [13].

6.5. A human proof of the Gessel conjecture

Counting lattice paths obeying various geometric constraints is a classical topic in combinatorics and probability theory. Many recent works deal with the enumeration of 2-dimensional walks with prescribed steps confined to the positive quadrant. A notoriously difficult case concerns the so-called *Gessel walks*: they are planar walks confined to the positive quarter plane, that move by unit steps in any of the following directions: West, North-East, East and South-West. In 2001, Ira Gessel conjectured a closed-form expression for the number of such walks of a given length starting and ending at the origin. In 2008, Kauers, Koutschan and Zeilberger gave a computer-aided proof of this conjecture. The same year, Bostan and Kauers showed, using again computer algebra tools, that the trivariate generating function of Gessel walks is algebraic. We propose in [3] the first “human proofs” of these results. They are derived from a new expression for the generating function of Gessel walks in terms of special functions. This work has been published in the prestigious journal *Transactions of the AMS*.

6.6. Enumeration of 3-dimensional lattice walks confined to the positive octant

Small step walks in 2D are by now quite well understood, but almost everything remains to be done in higher dimensions. We explored in [2] the classification problem for 3-dimensional walks with unit steps confined to the positive octant. The first difficulty is their number: there are 11 074 225 cases (instead of 79 in dimension 2). In our work, we focused on the 35 548 that have at most six steps. We applied to them a combined approach, first experimental and then rigorous. Among the 35 548 cases, we first found 170 cases with a finite group; in the remaining cases, our experiments suggest that the group is infinite. We then rigorously proved D-finiteness of the generating series in all the 170 cases, with the exception of 19 intriguing step sets for which the nature of the generating function still remains unclear. In two challenging cases, no human proof is currently known, and we derived computer-algebra proofs, thus constituting the first proofs for those two step sets.

6.7. Efficient algorithms for rational first integrals

We presented in [29] fast algorithms for computing rational first integrals with degree bounded by N of a planar polynomial vector field of degree $d \leq N$. The main novelty is that such rational first integrals are obtained by computing via systems of linear equations instead of systems of quadratic equations. This leads to a probabilistic algorithm with arithmetic complexity $\tilde{O}(N^{2\omega})$ and to a deterministic algorithm for solving the problem in $\tilde{O}(d^2 N^{2\omega+1})$ arithmetic operations, where ω is the exponent of linear algebra. By comparison, the best previous algorithm uses at least $d^{\omega+1} N^{4\omega+4}$ arithmetic operations. Our new algorithms are moreover very efficient in practice.

6.8. Quasi-optimal computation of the p -curvature

The p -curvature of a system of linear differential equations in positive characteristic p is a matrix that measures to what extent the system is close to having a fundamental matrix of rational function solutions. This notion, originally introduced in the arithmetic theory of differential equations, has been recently used as an effective tool in computer algebra and in combinatorial applications. We have described in [6] a recent algorithm for computing the p -curvature, whose complexity is almost optimal with respect to the size of the output. The new algorithm performs remarkably well in practice. Its design relies on the existence of a well-suited ring, of so-called Hurwitz series, for which an analogue of the Cauchy–Lipschitz Theorem holds, and on a FFT-like method in which the “evaluation points” are Hurwitz series.

6.9. Axiomatic constraint systems for proof search modulo theories

Goal-directed proof search in first-order logic uses meta-variables to delay the choice of witnesses; substitutions for such variables are produced when closing proof-tree branches, using first-order unification or a theory-specific background reasoner. We have investigated a generalization of such mechanisms whereby theory-specific constraints are produced instead of substitutions. In order to design modular proof-search procedures over such mechanisms, we provide a sequent calculus with meta-variables, which manipulates such constraints abstractly. Proving soundness and completeness of the calculus leads to an acclimatization that identifies the conditions under which abstract constraints can be generated and propagated in the same way unifiers usually are. We then extract from our abstract framework a component interface and a specification for concrete implementations of background reasoners. This is a common work with Damien Rouhling (ENS Lyon), Stéphane Lengrand (CNRS, LIX) and Jean-Marc Notin (CNRS, LIX), based on the PhD contributions of Mahfuza Farooque (unaffiliated). It is described in [8].

6.10. DynaMoW: Dynamic Mathematics on the Web

The interactivity needed by our on-line encyclopedia DDMF is made possible by implementing it over our tool DynaMoW (<http://ddmf.msr-inria.inria.fr/DynaMoW/>). This Ocaml library simultaneously controls external symbolic calculations and web-page generation and was first developed from 2008 to 2011. With the evolution of Ocaml and web technologies, it became possible to hope for a more reactive and configurable tool, by using light-weight threads and websockets. A new design was elaborated this year by F. Chyzak and M. Guesdon, and DynaMoW was rewritten by the latter. Using this new DynaMoW will require a complete and potentially time-consuming port of DDMF. So we decided that experimenting with the port of a smaller DynaMoW-based application should be done to ascertain the new design of DynaMoW-based before going to scale with DDMF. To this end, we applied DynaMoW to another on-line encyclopedia of our’s, ECS. The code is now stabilizing, and will be released next year, after documentation is written.

6.11. ECS: Encyclopedia of Combinatorial Structures

The Encyclopedia of Combinatorial Structures (ECS, <http://algo.inria.fr/encyclopedia/>) originates as a project in Project-Team Algorithms, with a first release back in 1998. It is an on-line mathematical encyclopedia with an emphasis on sequences that arise in the context of decomposable combinatorial structures, with the possibility to search by the first terms in the sequence, keyword, generating function, or closed form. As such,

ECS ambitions to be seen as a young cousin of Sloane's famous Encyclopedia of Integer Sequences http://www.research.att.com/articles/featured_stories/2012_03/201203_OEIS.html?fbid=cibE46xiHwx. The latter lists more general types of sequences, and points to numerous entries in ECS for specific properties. With regard to our software development, ECS has served as a nice testbed for several evolutions of DynaMoW, in particular in 2009 and 2011. This year, F. Chyzak and M. Guesdon ported ECS to the language of the new DynaMoW. Public release is expected soon in 2016, and will please the many users waiting for this new release after the former website was discontinued for technical reasons.

6.12. Mathematical Components Library

We have released a new version of the Mathematical Components Library (<http://www.msr-inria.fr/projects/mathematical-components-2/>), including an updated version of the Ssreflect package (<http://ssr.msr-inria.inria.fr/>). A major refactoring of the archive now allows a more modular distribution, through several thematic packages, also available via the OPAM package manager. We have also opened our development repository and we mirror it on the GitHub platform, in order to better foster the community of users of the library.

TOCCATA Project-Team

7. New Results

7.1. Deductive Verification

- M. Clochard, J.-C. Filliâtre, and A. Paskevich proposed a novel method to prove the relative safety of operations over bounded integers in a large class of programs. Their approach consists of introducing dedicated abstract types for the bounded integers and restricting the set of allowed operations over these types in such a way that it is impossible to reach the bound during a realistic execution of the program: for example, it would take several hundred years to overflow a 64-bit integer. This technique is aimed at integer variables that serve essentially as counters or size measures. It can be used alongside the traditional methods of proving the absence of overflows for other integer values in the same program. The proposed approach is implemented in Why3 and was presented at VSTTE 2015 [26].
- J.-C. Filliâtre and M. Pereira proposed a new way to specify the behavior of a cursor data structure, with the objective of being able to verify both the implementation of a cursor and its use by client code. The approach is modular, which means that a program using a cursor can be verified independently of the way the cursor is implemented. An experimental evaluation has been conducted with Why3, with several implementations and client codes being verified. This work will be presented at JFLA 2016 [26].
- C. Fumex and C. Marché developed a new library for bit-vectors, in Why3 and SPARK [30]. This library is rich enough for the formal specification of functional behavior of programs that operate at the level of bits. It is also designed to exploit efficiently the support for bit-vectors built-in in some SMT solvers. This work is done in the context of the ProofInUse joint laboratory. The SPARK front-end of Why3, for the verification of Ada programs, is extended to exploit this new bit-vector theory. Several cases studies are conducted: efficient search for rightmost bit of a bit-vector, efficient computation of the number of bits set to 1, efficient solving of the n -queens problem. At the level of SPARK, a program inspired from some industrial code (originally developed in C par J. Gerlach, Fraunhofer FOKUS Institute, Germany and partially proved with Frama-C and Coq) is specified in SPARK and proved with automatic solvers only. The support for bit-vectors is already distributed with SPARK, and SPARK users already reported that several verification conditions, that couldn't be proved earlier, are now proved automatically.
- D. Hauzar and C. Marché worked on counterexample generation from failed proof attempts. They designed a new approach for generating potential counterexamples in the deductive verification setting, and implemented in Why3. When the logic goal generated for a given verification condition is not shown unsatisfiable by an SMT solvers, some solver can propose a model. By carefully reverting the transformation chain (from an input program through the VC generator and the various translation steps to solvers), this model is turned into a potential counterexample that the user can exploit to analyze why its original code is not proved. The approach is implemented in the chain from Ada programs through SPARK, Why3, and SMT solvers CVC4 and Z3. This implementation is robust enough to be distributed in the next release Pro 16 of SPARK. A research report on this subject will appear in January 2016.
- A. Charguéraud and F. Pottier (Inria Paris-Rocquencourt) obtained new results in the machine-checked verification of asymptotic complexity bounds, in addition to program correctness properties. Verifying the time usage of a program is very important, because otherwise a program might be proved to be functionally correct but may appear to run into an infinite loop for particular input data. More specifically, A. Charguéraud and F. Pottier started from the extension of CFML with *time credits* (encoding of time resources in Separation Logic), developed last year by A. Charguéraud, and

they used it to formally produce a machine-checked proof of the correctness and time complexity of a Union-Find data structure, implemented as an OCaml module. They thereby demonstrate that the approach scales up to difficult complexity analyses, and applies to actual executable code (as opposed to pseudo-code). This work was presented at ITP 2015 [24]. Furthermore, A. Charguéraud and F. Pottier co-advised the M2 internship of Armaël Guéneau, who extended the time credits approach so as to allow working conveniently with the big- O notation. He extended the CFML library and verified the time complexity of a binary random access list data structure due to Okasaki. This work has not been published yet.

- A. Charguéraud described a method for reasoning about mutable data structures that own their elements. In Separation Logic, representation predicates describe the ownership of a mutable data structure, by establishing a relationship between the entry point of the structure, the piece of heap over which this structure spans, and the logical model associated with the structure. When a data structure is polymorphic, such as in the case of a container, its representation predicate needs to be parameterized not just by the type of the items stored in the structure, but also by the representation predicates associated with these items. Such higher-order representation predicates can be used in particular to control whether containers should own their items. A. Charguéraud wrote a paper describing, through a collection of practical examples, solutions to the challenges associated with reasoning about accesses into data structures that own their elements. This paper will appear at CPP 2016 [23].

7.2. Automated Reasoning

- C. Dross, A. Paskevich, J. Kanig and S. Conchon published a journal paper [16] about integration of first-order axiomatizations with triggers as decision procedures in an SMT solver. This work extends a part of C. Dross PhD thesis [79]. A formal semantics of the notion of trigger is presented, with a general setting to show how a first-order axiomatization with triggers can be proved correct, complete, and terminating. An extended DPLL(T) algorithm can then integrate such an axiomatization with triggers, as a decision procedure for the theory it defines.

7.3. Certification of Languages, Tools and Systems

- M. Clochard and L. Gondelman developed a formalization of a simple compiler in *Why3*. It compiles a simple imperative language into assembler instructions for a stack machine. This case study was inspired by a similar example developed using Coq and interactive theorem proving. The aim is to improve significantly the degree of automation in the proofs. This is achieved by the formalization of a Hoare logic and a Weakest Precondition Calculus on assembly programs, so that the correctness of compilation is seen as a formal specification of the assembly instructions generated. This work was presented at the JFLA conference in 2015 [25].
- S. Boldo, C. Lelay, and G. Melquiond worked on the Coquelicot library, designed to be a user-friendly Coq library about real analysis. An easier way of writing formulas and theorem statements is achieved by relying on total functions in place of dependent types for limits, derivatives, integrals, power series, and so on. To help with the proof process, the library comes with a comprehensive set of theorems and some automation. We have exercised the library on several use cases: in an exam at university entry level, for the definitions and properties of Bessel functions, and for the solution of the one-dimensional wave equation. These results are published in the journal *Mathematics in Computer Science* [14].
- C. Lelay developed a new formalization of convergence with a focus on usability and genericity for the Coquelicot library. This formalization covers various parts of analysis: sequences, real functions, complex functions, vector functions, and so on. This work was presented at the 7th Coq Workshop [27].
- C. Paulin wrote a gentle introduction to the Calculus of Inductive Construction, the formalism on which the Coq proof assistant is based [28], discussing both theoretical and pragmatic aspects of the design.

7.4. Floating-Point and Numerical Programs

- É. Martin-Dorel and G. Melquiond worked on integrating the CoqInterval and CoqApprox libraries into a single package. The CoqApprox library is dedicated to computing verified Taylor models of univariate functions so as to compute approximation errors. The CoqInterval library reuses this work to automatically prove bounds on real-valued expressions. A large formalization effort took place during this work, so as to get rid of all the holes remaining in the formal proofs of CoqInterval. It was also the chance to perform a comparison between numerous decision procedures dedicated to proving nonlinear inequalities involving elementary functions. This work has been published in the *Journal of Automated Reasoning* [18].
- S. Boldo and G. Melquiond, with J.-H. Jourdan and X. Leroy (Gallium team, Inria Paris - Rocquencourt) extended the CompCert compiler to get the first formally verified C compiler that provably preserves the semantics of floating-point programs. This work, published in the *Journal of Automated Reasoning* [13], also covers the formalization of numerous algorithms of conversion between integers and floating-point numbers.
- S. Boldo worked on the fact that $a/\sqrt{a^2 + b^2}$ is always in the interval $[-1, 1]$ even when operations are done using floating-point arithmetic. This reduces to taking the square root of the square of a floating-point number as it is the worst case. Results in radix 2 (where $\sqrt{a^2} = |a|$) and other radices (where it might not hold) have been published at the 8th International Workshop on Numerical Software Verification [22].
- S. Boldo worked on programs computing the average of two floating-point numbers. As we want to take exceptional behaviors into account, we cannot use the naive formula $(x+y)/2$. Based on hints given by Sterbenz, she first wrote an accurate program and formally proved its properties. She also developed and formally proved a new algorithm that computes the correct rounding of the average of two floating-point numbers [21]. This was published at the 17th International Conference on Formal Engineering Methods.
- P. Roux formalized a theory of numerical analysis for bounding the round-off errors of a floating-point algorithm. This approach was applied to the formal verification of a program for checking that a matrix is semi-definite positive. The challenge here is that testing semi-definiteness involves algebraic number computations, yet it needs to be implemented using only approximate floating-point operations. This work has been published in the *Journal of Automated Reasoning* [19].
- C. Lelay and G. Melquiond worked on formalizing in Coq a numerical domain for the Verasco abstract interpreter built upon the CompCert verified compiler. This abstract domain is a relational domain based on affine forms (zonotopes). It is meant to help verifying floating-point programs and it is expected to perform faster (but less accurately) than a more generic domain based on polyhedrons.

7.5. Miscellaneous

- A. Charguéraud worked together with Umut Acar, Mike Rainey, and Filip Sieczkowski, as part of the ERC project *DeepSea*, on the development of efficient data structures and algorithms targeting modern, shared memory multicore architectures. A. Charguéraud was involved in two major results obtained this year.

The first result is the development of fast and robust parallel graph traversal algorithms based on depth-first-search. This algorithm leverages a new sequence data structure for representing the set of edges remaining to be visited. This sequence itself builds on prior work on bootstrapped chunked sequences [35]. In particular, the edge sequence structure uses a balanced split operation for partitioning the edges of a graph among the several processors involved in the computation. Compared with prior work, the new algorithm is designed and proved to be efficient not just for particular classes of graphs, but for all input graphs. This work has been published in the ACM/IEEE Conference on High Performance Computing (SC) [20].

Another result by A. Charguéraud and his co-authors is the development of a calculus for parallel computing on shared memory computers. Many languages for writing parallel programs have been developed. These languages offer several different abstractions for parallelism, such as fork-join, async-finish, futures, etc. While they may seem similar, these abstractions lead to different semantics, language design and implementation decisions. In this work, we consider the question of whether it would be possible to unify different approaches to parallelism. To this end, we propose a calculus, called *DAG-calculus* that can encode existing approaches to parallelism based on fork-join, async-finish, and futures paradigms and possibly others. We have shown that the approach is realistic by presenting an implementation in C++ and by performing an empirical evaluation. This work has been submitted for publication.

- A. Charguéraud developed a patch to the OCaml compiler for improving type error messages, in particular to make the language more accessible to beginners. The problem of improving type error messages in ML has received quite a bit of attention over the past two decades, and many different strategies have been considered. The challenge is not only to produce error messages that are both sufficiently concise and systematically useful to the programmer, but also to handle a full-blown programming language and to cope with large-sized programs efficiently. A. Charguéraud's novel approach consists of a slight modification to the traditional ML type inference algorithm implemented in OCaml that, by significantly reducing the left-to-right bias, produces error messages that are more helpful to the programmer. This work was published this year in the journal *Electronic Proceedings in Theoretical Computer Science* [15].

COMMANDS Project-Team

7. New Results

7.1. Optimal control of ordinary differential equations

7.1.1. Periodic optimal controls for the Purcell microswimmer

Participant: Pierre Martinon.

We investigate in [31] some geometric and numerical aspects related to optimal control problems for the so-called Purcell Three-link swimmer, in which the cost to minimize represents the energy consumed by the swimmer. More precisely, we focus on the periodic aspect of optimal trajectories and controls. Linearizing the control system along a reference extremal, we estimate the conjugate points, which play a crucial role for the second order optimality conditions. With techniques imported by the sub-Riemannian geometry, we also show that the nilpotent approximation of the system provides a model which is integrable, obtaining explicit expressions in terms of elliptic functions. This approximation allows to compute optimal periodic controls for small deformations of the body. Numerical simulations are presented using HAMPATH and BOCOP codes. A first paper was submitted in October 2015.

7.1.2. Study of optimal health insurance policies

Participant: Pierre Martinon.

In collaboration with the Economy department of Ecole Polytechnique, we analyze the design of an optimal medical insurance contract under ex post moral hazard, i.e., when illness severity cannot be observed by insurers and policyholders may exaggerate their health expenditures. This problem is reformulated in the optimal control framework, and we study the possible existence of deductibles or bunching phenomena in optimal contracts. A paper will be submitted in early 2016.

7.2. Optimal control of partial differential equations

7.2.1. Local minimization algorithms for dynamic programming equations

Participant: Axel Kröner.

The numerical realization of the dynamic programming principle for continuous-time optimal control leads to nonlinear Hamilton-Jacobi-Bellman equations which require the minimization of a nonlinear mapping over the set of admissible controls. This minimization is often performed by comparison over a finite number of elements of the control set. In this paper we demonstrate the importance of an accurate realization of these minimization problems and propose algorithms by which this can be achieved effectively. The considered class of equations includes nonsmooth control problems with l_1 -penalization which lead to sparse controls. See the reprint [28].

7.2.2. Suboptimal feedback control of PDEs by solving HJB equations on adaptive sparse grids

Participant: Axel Kröner.

An approach to solve finite time horizon sub-optimal feedback control problems for partial differential equations is proposed by solving dynamic programming equations on adaptive sparse grids. The approach is illustrated for the wave equation. A semi-discrete optimal control problem is introduced and the feedback control is derived from the corresponding value function. The value function can be characterized as the solution of an evolutionary Hamilton-Jacobi Bellman (HJB) equation which is defined over a state space whose dimension is equal to the dimension of the underlying semi-discrete system. Besides a low dimensional semi-discretization it is important to solve the HJB equation efficiently to address the curse of dimensionality. We propose to apply a semi-Lagrangian scheme using spatially adaptive sparse grids. Sparse grids allow the discretization of the value functions in (higher) space dimensions since the curse of dimensionality of full grid methods arises to a much smaller extent. For additional efficiency an adaptive grid refinement procedure is explored. We present several numerical examples studying the effect the parameters characterizing the sparse grid have on the accuracy of the value function and the optimal trajectory. See the report [27].

7.2.3. Numerical approximation of level set power mean curvature flow

Participant: Axel Kröner.

In this paper we investigate the numerical approximation of a variant of the mean curvature flow. We consider the evolution of hypersurfaces with normal speed given by H^k , $k \geq 1$, where H denotes the mean curvature. We use a level set formulation of this flow and discretize the regularized level set equation with finite elements. In a previous paper we proved an a priori estimate for the approximation error between the finite element solution and the solution of the original level set equation. We obtained an upper bound for this error which is polynomial in the discretization parameter and the reciprocal regularization parameter. The aim of the present paper is the numerical study of the behavior of the evolution and the numerical verification of certain convergence rates. We restrict the consideration to the case that the level set function depends on two variables, i.e. the moving hypersurfaces are curves. Furthermore, we confirm for specific initial curves and different values of k that the flow improves the isoperimetrical deficit. See the report [29].

7.3. Finance and stochastic control

7.3.1. Second order Pontryagin's principle for stochastic control problems

Participant: Frédéric Bonnans.

In this Hal reprint [25], we discuss stochastic optimal control problems whose volatility does not depend on the control, and which have finitely many equality and inequality constraints on the expected value of functions of the final state, as well as control constraints. The main result is a proof of necessity of some second order optimality conditions involving Pontryagin multipliers.

7.3.2. On the convergence of the Sakawa-Shindo algorithm in stochastic control

Participant: Frédéric Bonnans.

In the accepted paper [32], we analyze an algorithm for solving stochastic control problems, based on Pontryagin's maximum principle, due to Sakawa and Shindo in the deterministic case and extended to the stochastic setting by Mazliak. We assume that either the volatility is an affine function of the state, or the dynamics are linear. We obtain a monotone decrease of the cost functions as well as, in the convex case, the fact that the sequence of controls is minimizing, and converges to an optimal solution if it is bounded. In a specific case we interpret the algorithm as the gradient plus projection method and obtain a linear convergence rate to the solution.

7.3.3. Optimal multiple stopping problems

Participant: Frédéric Bonnans.

In the paper [13] we extend some results by Carmona and Touzi [8], who studied an optimal multiple stopping time problem in a market where the price process is continuous. We generalize their results when the price process is allowed to jump. Also, we generalize the problem associated to the valuation of swing options to the context of jump diffusion processes. We relate our problem to a sequence of ordinary stopping time problems. We characterize the value function of each ordinary stopping time problem as the unique viscosity solution of the associated Hamilton–Jacobi–Bellman variational inequality. In the paper [14] we deal with numerical solutions to an optimal multiple stopping problem. The corresponding dynamic programming (DP) equation is a variational inequality satisfied by the value function in the viscosity sense. The convergence of the numerical scheme is shown by viscosity arguments. An optimal quantization method is used for computing the conditional expectations arising in the DP equation. Numerical results are presented for the price of swing option and the behavior of the value function.

7.4. Electricity production

7.4.1. Equilibria over energy markets

Participant: Benjamin Heymann.

Motivated by electricity markets we introduce in this paper a general network market model, in which agents are located on the nodes of a graph, a traded good can travel from one place to another through edges considering quadratic losses. An independent operator has to match locally production and demand at the lowest expense. As argued in our previous paper “Cost-minimizing regulations for a wholesale electricity market” this setting is relevant to describe some real electricity markets, pricing behavior and market power coming from the fact that generators can bid above their true value. In a general setting of many distributed generator agents connected by a transmission network, bidding piece-wise linear cost functions, we propose a pricing optimal mechanism model to reduce market power. Our main results are the expression of the optimal mechanism design, two algorithms for the allocation problem and market power estimations. To deduce these nice properties, we intensively use convex analysis and some monotone behaviors of the set-valued maps involved. Furthermore, these algorithms make it possible to numerically compute a Nash equilibrium for the procurement auction, which is important to compare the optimal mechanism and the standard auction setting. Finally, we also show some interesting examples. In the continuation of this work, we introduce a class of bidding games for which we prove the existence of a Nash equilibrium. We give a sufficient condition for uniqueness, propose a numerical scheme to compute the extreme Nash Equilibria and show that the equilibrium strategies are convex for a subclass of games. We apply this framework to electricity auctions.

7.4.2. Energy management for a micro-grid

Participants: Frédéric Bonnans, Benjamin Heymann, Pierre Martinon, Olivier Tissot.

We study in [33] the energy management problem for a microgrid including a diesel generator and a photovoltaic plant with a battery storage system. The objective is to minimize the total operational cost over a certain timeframe, primarily the diesel consumption, while satisfying a prescribed power load. After reformulation, the decision variables can be reduced to the charging /discharging power for the battery system. We take into account the switching cost for the diesel generator, the non-convex objective, and the long-term aging of the batteries. We solve this problem using a continuous optimal control framework, with both a direct transcription method (time discretization) and a Dynamic Programming method (Hamilton Jacobi Bellman). This project is a collaboration between team COMMANDS (Inria Saclay, France) and Centro de Energia (Universidad de Chile, Chile). Ongoing works include more refined battery aging models, and modeling the stochastic nature of the photovoltaic power and power load.

7.5. Energy management in transport

7.5.1. Energy management for an hybrid vehicle

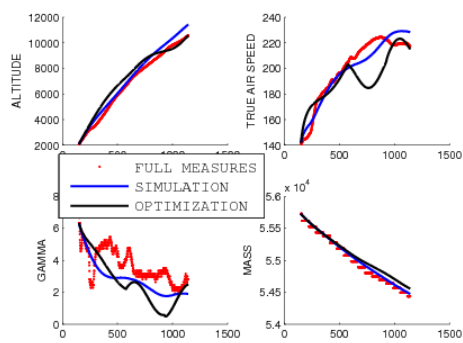
Participants: Florine Bleuse, Frédéric Bonnans, Pierre Martinon.

In the framework of the PhD thesis of F.Bleuse, ‘Optimal control and robustness for rechargeable hybrid vehicles’. The study is focused on the so-called parallel architecture, with both the thermal and electric engines able to move the vehicle. The main axis is to optimize the use of the thermal engine. We started to develop a methodology with two time scales for solving the problem of computing a feedback control.

7.5.2. Collaboration with the startup Safety Line

Participants: Frédéric Bonnans, Pierre Martinon, Olivier Tissot.

We pursue our collaboration with Safety Line, using more refined atmospheric models (including for instance predicted wind data). Future works include high performance optimization for the cruise phase as well as analyzing the validity of the parameter estimation performed with the data from the flight recorders.



 **SAFETY LINE**
take control of your safety



Figure 1. Parameter identification and trajectory optimization

DEFI Project-Team

6. New Results

6.1. Methods for inverse problems

6.1.1. Identifying defects in an unknown background using differential measurements

L. Audibert and H. Haddar

In the framework of the PhD thesis of Lorenzo Audibert we studied non destructive testing of concrete using ultrasonic waves, and more generally imaging in complex heterogeneous media. We assume that measurements are multistatic, which means that we record the scattered field on different points by using several sources. For this type of data we wish to build methods that are able to image the obstacle that created the scattered field. We use qualitative methods in this work, which only provide the support of the object independently from its physical property. The first part of this thesis consists of a theoretical analysis of the Linear Sampling Method. Such analysis is done in the framework of regularization theory, and our main contribution is to provide and analyze a regularization term that ensures good theoretical properties. Among those properties we were able to demonstrate that when the regularization parameter goes to zero, we actually construct a sequence of functions that strongly converges to the solution of the interior transmission problem. This behavior gives a central place to the interior transmission problem as it allows describing the asymptotic solution of our regularized problem. Using this characterization of our solution, we are able to give the optimal reconstruction we can get from our method. More importantly this description of the solution allows us to compare the solution coming from two different datasets. Based on the result of this comparison, we manage to produce an image of the connected component that contains the defect which appears between two measurement campaigns and this regardless of the medium. This method is well suited for the characteristics of the microstructure of concrete as shown on several numerical examples with realistic concrete-like microstructure. Finally, we extend our theoretical results to the case of limited aperture, anisotropic medium and elastic waves, which correspond to the real physics of the ultrasounds

6.1.2. Invisibility in scattering theory for small obstacles

L. Chesnel, X. Claeys and S.A. Nazarov

We are interested in a time harmonic acoustic problem in a waveguide containing flies. The flies are modelled by small sound soft obstacles. We explain how they should arrange to become invisible to an observer sending waves from $-\infty$ and measuring the resulting scattered field at the same position. We assume that the flies can control their position and/or their size. On the other hand, we show that any sound soft obstacle (non necessarily small) embedded in the waveguide always produces some non exponentially decaying scattered field at $+\infty$. As a consequence, the flies cannot be made completely invisible to an observer equipped with a measurement device located at $+\infty$.

6.1.3. New notion of regularization for Poisson data with an application to nanoparticle volume determination

F. Benvenuto, H. Haddar and B. Lantz

The aim of this work is to develop a fully automatic method for the reconstruction of the volume distribution of diluted polydisperse non-interacting nanoparticles with identical shapes from Small Angle X-ray Scattering measurements. The described method solves a maximum likelihood problem with a positivity constraint on the solution by means of an Expectation Maximization iterative scheme coupled with a robust stopping criterion. We prove that this is a regularization method according to an innovative notion of regularization specifically defined for inverse problems with Poisson data. Such a regularization, together with the positivity constraint results in high fidelity quantitative reconstructions of particle volume distributions making the method particularly effective in real applications. We test the performance of the method on synthetic data in the case of uni- and bi-modal particle volume distributions. Moreover, we show the reliability of the method on real data provided by a Xenocs device prototype.

6.1.4. A conformal mapping algorithm for the Bernoulli free boundary value problem

H. Haddar and R. Kress

We propose a new numerical method for the solution of Bernoulli's free boundary value problem for harmonic functions in a doubly connected domain D in R^2 where an unknown free boundary Γ_0 is determined by prescribed Cauchy data on Γ_0 in addition to a Dirichlet condition on the known boundary Γ_1 . Our main idea is to involve the conformal mapping method as proposed and analyzed by Akduman, Haddar and Kress for the solution of a related inverse boundary value problem. For this we interpret the free boundary Γ_0 as the unknown boundary in the inverse problem to construct Γ_0 from the Dirichlet condition on Γ_0 and Cauchy data on the known boundary Γ_1 . Our method for the Bernoulli problem iterates on the missing normal derivative on Γ_1 by alternating between the application of the conformal mapping method for the inverse problem and solving a mixed Dirichlet–Neumann boundary value problem in D . We present the mathematical foundations of our algorithm and prove a convergence result. Some numerical examples will serve as proof of concept of our approach.

6.1.5. Identification of small objects with near-field data in quasi-backscattering configurations

H. Haddar and M. Lakhali

We present a new sampling method for detecting targets (small inclusions or defects) immersed in a homogeneous medium in three-dimensional space, from measurements of acoustic scattered fields created by point source incident waves. We consider the harmonic regime and a data setting that corresponds with quasi-backscattering configuration: the data is collected by a set of receivers that are distributed on a segment centered at the source position and the device is swept along a path orthogonal to the receiver line. We assume that the aperture of the receivers is small compared with the distance to the targets. Considering the asymptotic form of the scattered field as the size of the targets goes to zero and the small aperture approximation, one is able to derive a special expression for the scattered field. In this expression a separation of the dependence of scattered field on the source location and the distance source-target is performed. This allows us to propose a sampling procedure that characterizes the targets location in terms of the range of a near-field operator constructed from available data. Our procedure is similar to the one proposed by Haddar-Rezac for far-field configurations. The reconstruction algorithm is based on the MUSIC (Multiple Signal Classification) algorithm.

6.2. Direct scattering problems

6.2.1. A numerical method to approximate black hole singularities in presence of metamaterials

L. Chesnel, A.-S. Bonnet-Ben Dhia, C. Carvalho and P. Ciarlet.

We investigate in a 2D setting the scattering of time-harmonic electromagnetic waves by a plasmonic device, represented as a non dissipative bounded and penetrable obstacle with a negative permittivity. Using the T-coercivity approach, we proved that the problem is well-posed in the classical frameworks if the negative permittivity does not lie in some critical interval whose definition depends on the shape of the device. When the latter has corners, for values inside the critical interval, unusual strong singularities for the electromagnetic field can appear. In that case, well-posedness is obtained by imposing a radiation condition at the corners to select the outgoing black-hole plasmonic wave, that is the one which carries energy towards the corners. We give a simple and systematic criterion to define what is the outgoing solution. We also propose an original numerical method based on the use of Perfectly Matched Layers at the corners. We emphasize that it is necessary to design an *ad hoc* technique because the field is too singular to be captured with standard finite element methods.

6.2.2. Boundary Integral Equations for the Transmission Eigenvalue Problem for Maxwell's Equations

Housseem Haddar, Shixu Meng and Fioralba Cakoni

We consider the transmission eigenvalue problem for Maxwell's equations corresponding to non-magnetic inhomogeneities with contrast in electric permittivity that changes sign inside its support. We formulate the transmission eigenvalue problem as an equivalent homogeneous system of boundary integral equation, and assuming that the contrast is constant near the boundary of the support of the inhomogeneity, we prove that the operator associated with this system is Fredholm of index zero and depends analytically on the wave number. Then we show the existence of wave numbers that are not transmission eigenvalues which by an application of the analytic Fredholm theory implies that the set of transmission eigenvalues is discrete with positive infinity as the only accumulation point.

6.2.3. A Volume integral method for solving scattering problems from locally perturbed periodic layers

Houssem Haddar and Thi Phong Nguyen

We investigate the scattering problem for the case of locally perturbed periodic layer in R^N ($N = 2, 3$). Using Floquet-Bloch transform in x_1 -direction we reformulate this scattering problem as an equivalent system of coupled volume integral equations. Using periodization in the x_2 -direction we apply a spectral method to discretize the problem and compute a numerical approximation of the solution. The convergence of this method is established and numerical validating results are conducted.

6.3. Shape and topology optimization

6.3.1. Deterministic approximation methods in shape optimization under random uncertainties

G. Allaire and C. Dapogny

This work is concerned with the treatment of uncertainties in shape optimization. We consider uncertainties in the loadings, the material properties, the geometry and the vibration frequency, both in the parametric and geometric optimization setting. We minimize objective functions which are mean values, variances or failure probabilities of standard cost functions under random uncertainties. By assuming that the uncertainties are small and generated by a finite number N of random variables, and using first- or second-order Taylor expansions, we propose a deterministic approach to optimize approximate objective functions. The computational cost is similar to that of a multiple load problems where the number of loads is N . We demonstrate the effectiveness of our approach on various parametric and geometric optimization problems in two space dimensions.

6.3.2. Molding direction constraints in structural optimization via a level-set method

G. Allaire, F. Jouve and G. Michailidis

In the framework of structural optimization via a level-set method, we develop an approach to handle the directional molding constraint for cast parts. A novel molding condition is formulated and a penalization method is used to enforce the constraint. A first advantage of our new approach is that it does not require to start from a feasible initialization, but it guarantees the convergence to a castable shape. A second advantage is that our approach can incorporate thickness constraints too. We do not address the optimization of the casting system, which is considered a priori defined. We show several 3d examples of compliance minimization in linearized elasticity under molding and minimal or maximal thickness constraints. We also compare our results with formulations already existing in the literature.

6.3.3. Identification of magnetic deposits in 2-D axisymmetric eddy current models via shape optimization

Zixian Jiang, Houssem Haddar, Armin Lechleiter and Mabrouka El-Guedri

The non-destructive control of steam generators is an essential task for the safe and failure-free operation of nuclear power plants. Due to magnetite particles in the cooling water of the plants, a frequent source for failures are magnetic deposits in the cooling loop of steam generators. From eddy current signals measured inside a U-tube in the steam generator, we propose and analyze a regularized shape optimization algorithm to identify magnetic deposits outside the U-tube with either known or unknown physical properties. Motivated by the cylindrical geometry of the U-tubes we assume an axisymmetric problem setting, reducing Maxwell's equations to a 2-D elliptic eddy current problem. The feasibility of the proposed algorithms is illustrated via numerical examples demonstrating in particular the stability of the method with respect to noise.

6.4. Asymptotic Analysis

6.4.1. *Ion transport through deformable porous media: derivation of the macroscopic equations using upscaling*

G. Allaire, O. Bernard, J.-F. Duf r che and A. Mikelic

We study the homogenization (or upscaling) of the transport of a multicomponent electrolyte in a dilute Newtonian solvent through a deformable porous medium. The pore scale interaction between the flow and the structure deformation is taken into account. After a careful adimensionalization process, we first consider so-called equilibrium solutions, in the absence of external forces, for which the velocity and diffusive fluxes vanish and the electrostatic potential is the solution of a Poisson-Boltzmann equation. When the motion is governed by a small static electric field and small hydrodynamic and elastic forces, we use O'Brien's argument to deduce a linearized model. Then we perform the homogenization of these linearized equations for a suitable choice of time scale. It turns out that the deformation of the porous medium is weakly coupled to the electrokinetics system in the sense that it does not influence electrokinetics although the latter one yields an osmotic pressure term in the mechanical equations. As a consequence, the effective tensor satisfies Onsager properties, namely is symmetric positive definite.

6.4.2. *On the asymptotic behaviour of the kernel of an adjoint convection-diffusion operator in a long cylinder*

G. Allaire and A. Piatnitski

This work studies the asymptotic behaviour of the principal eigenfunction of the adjoint Neumann problem for a convection diffusion operator defined in a long cylinder. The operator coefficients are 1-periodic in the longitudinal variable. Depending on the sign of the so-called longitudinal drift (a weighted average of the coefficients), we prove that this principal eigenfunction is equal to the product of a specified periodic function and of an exponential, up to the addition of fast decaying boundary layer terms.

6.4.3. *A comparison between two-scale asymptotic expansions and Bloch wave expansions for the homogenization of periodic structures*

G. Allaire, M. Briane and M. Vanninathan

In this work we make a comparison between the two-scale asymptotic expansion method for periodic homogenization and the so-called Bloch wave method. It is well-known that the homogenized tensor coincides with the Hessian matrix of the first Bloch eigenvalue when the Bloch parameter vanishes. In the context of the two-scale asymptotic expansion method, there is the notion of high order homogenized equation where the homogenized equation can be improved by adding small additional higher order differential terms. The next non-zero high order term is a fourth-order term, accounting for dispersion effects. Surprisingly, this homogenized fourth-order tensor is not equal to the fourth-order tensor arising in the Taylor expansion of the first Bloch eigenvalue, which is often called Burnett tensor. Here, we establish an exact relation between the homogenized fourth-order tensor and the Burnett fourth-order tensor. It was proved by Conca et al. that the Burnett fourth-order tensor has a sign. For the special case of a simple laminate we prove that the homogenized fourth-order tensor may change sign. In the elliptic case we explain the difference between the homogenized and Burnett fourth-order tensors by a difference in the source term which features an additional corrector term.

Finally, for the wave equation, the two fourth-order tensors coincide again, so dispersion is unambiguously defined, and only the source terms differ as in the elliptic case.

6.4.4. Influence of the geometry on plasmonic waves

L. Chesnel, X. Claeys and S.A. Nazarov

In the modeling of plasmonic technologies in time harmonic regime, one is led to study the eigenvalue problem $-\operatorname{div}(\sigma \nabla u) = \lambda u$ (P), where σ is a physical coefficient positive in some region Ω_+ and negative in some other region Ω_- . We highlight an unusual instability phenomenon for the source term problem associated with (P): for certain configurations, when the interface between Ω_+ and Ω_- presents a rounded corner, the solution may depend critically on the value of the rounding parameter. We explain this property studying the eigenvalue problem (P). We provide an asymptotic expansion of the eigenvalues and prove error estimates. We establish an oscillatory behaviour of the eigenvalues as the rounding parameter of the corner tends to zero. These theoretical results are illustrated by numerical experiments.

6.4.5. Effective boundary conditions for thin periodic coatings Participants

Mathieu Chamaillard, Houssein Haddar and Patrick Joly

We study the derivation of asymptotic model (generalized impedance boundary conditions) for periodic coating in 3-D configurations. The definition of periodicity for 3D surfaces cannot be done in an intrinsic way in general. We propose a definition based on the use of local parametrisations of the surface. This parametrization-dependent definition is somehow inspired from practical considerations in the manufacturing of periodic coatings. The asymptotic of the problem is constructed for the scalar problem and also for the electromagnetic problem. Approximate models of order 1 and 2 are derived for the scalar problem and are validated numerically. In the electromagnetic case, only conditions of order 1 are exhibited in the general case.

6.5. Diffusion MRI

Jing-Rebecca Li, Houssein Haddar, Simona Schiavi, Khieu Van Nguyen, Gabrielle Fournet

Diffusion Magnetic Resonance Imaging (DMRI) is a promising tool to obtain useful information on microscopic structure and has been extensively applied to biological tissues.

We obtained the following results.

- We derived using homogenization techniques a model of the time-dependent “apparent diffusion coefficient” (ADC) that is valid at a wide range of diffusion times. The ADC is a very important experimental quantity measured by diffusion MRI in biological tissues. This work resulted in one submitted article to a mathematical journal and we are preparing an article for a physics journal.
- We analyzed a dMRI model called the Karger model that is valid at long diffusion times. This resulted in one submitted article to a mathematical journal.
- We acquired dMRI data of the nerve cells of the *Aplysia Californica* at the high field brain MRI center Neurospin. This data is useful because the nerve cells are bigger than mammal neurons and so it is easier to obtain segmented geometrical information about these cells for model validation.
- We participated in the data analysis and numerical simulation of a MR imaging method to measure blood flow in micro-vessels in the brain. This resulted in a submitted article to a MRI journal.

DISCO Project-Team

7. New Results

7.1. Algebraic Analysis Approach to Linear Functional Systems

Participants: Alban Quadrat [Disco], Rosane Ushirobira [Non-A].

7.1.1. Artstein's transformation of linear time-delay systems

Artstein's classical results show that a linear first-order differential time-delay system with delays in the input is equivalent to a linear first-order differential system without delays thanks to an invertible transform which includes integral and delay operators. Within a constructive algebraic approach, we show how Artstein's reduction can be found again and generalized as a particular isomorphism problem between the finitely presented modules defined by the two above linear systems over the ring of integro-differential time-delay operators. Moreover, we show that Artstein's reduction can be obtained in an automatic way by means of symbolic computation, and thus can be implemented in computer algebra systems.

7.1.2. Algebraic analysis for the Ore extension ring of differential time-varying delay operators

No algebraic (polynomial) approach seems to exist for the study of linear differential time-delay systems in the case of a (sufficiently regular) time-varying delay. Based on the concept of skew polynomial rings developed by Ore in the 30s, we construct the ring of differential time-delay operators as an Ore extension and to analyze its properties. A characterization of classical algebraic properties of this ring, such as noetherianity, its homological and Krull dimensions and the existence of Gröbner bases, are given in terms of the time-varying delay function. The algebraic analysis approach to linear systems theory allows us to study linear differential time-varying delay systems (e.g. existence of autonomous elements, controllability, parametrizability, flatness, behavioral approach) through methods coming from module theory, homological algebra and constructive algebra.

7.2. New Techniques for Robust Control of Linear Infinite-Dimensional Systems

Participants: Yacine Bouzidi [Disco], Petteri Laakkonen [Univ. Tampere], Adrien Poteaux [Lille 1], Alban Quadrat [Disco], Arnaud Quadrat [SAGEM], Guillaume Rance [SAGEM], Fabrice Rouillier [Ouragan].

7.2.1. Computer algebra methods for testing the structural stability of multidimensional systems

We present new computer algebra based methods for testing the structural stability of n -D discrete linear systems (with $n \geq 2$). More precisely, starting from the usual stability conditions which resumes to deciding if an hypersurface has points in the unit polydisc, we show that the problem is equivalent to deciding if an algebraic set has real points and use state-of-the-art algorithms for this purpose. Our strategy has been implemented in Maple and its relevance demonstrated through numerous experimentations.

Moreover, we also consider the specific case of two-dimensional systems and focus on the practical efficiency aspect. For such systems, the problem of testing the stability is reduced to that of deciding if a bivariate algebraic system with finitely many solutions has real ones. Our first contribution is an algorithm that answers this question while achieving practical efficiency. Our second contribution concerns the stability of two dimensional systems with parameters. More precisely, given a two-dimensional system depending on a set of parameters, we present a new algorithm that computes regions of the parameters space in which the considered system is structurally stable.

7.2.2. Computer algebra methods for the stability analysis of differential systems with commensurate time-delays

Within the frequency-domain approach, the asymptotic stability of linear differential systems with commensurate delays is ensured by the condition that all the roots of the corresponding quasipolynomial have negative real parts. A classical approach for checking this condition consists in computing the set of critical zeros of the quasipolynomial, i.e., the roots (and the corresponding delays) of the quasipolynomial that lie on the imaginary axis, and then analyzing the variation of these roots with respect to the variation of the delay. Based on solving algebraic systems techniques, we propose a certified and efficient symbolic-numeric algorithm for computing the set of critical roots of a quasipolynomial. Moreover, using recent algorithmic results developed by the computer algebra community, we present an efficient algorithm for the computation of Puiseux series at a critical zero which allows us to finely analyze the stability of the system with respect to the variation of the delay

7.2.3. A fractional ideal approach to the robust regulation problem

We show how fractional ideal techniques developed in [8] can be used to obtain a general formulation of the internal model principle for stabilizable infinite-dimensional SISO plants which do not necessarily admit coprime factorization. This result is then used to obtain necessary and sufficient conditions for the robust regulation problem. In particular, we find again all the standard results obtained in the literature.

7.2.4. Robust control as an application to the homological perturbation lemma:

Within the lattice approach to transfer matrices developed in [8], we have recently shown how standard results on robust control can be obtained in a unified way and generalized when interpreted as a particular case of the so-called Homological Perturbation Lemma. This lemma plays a significant role in algebraic topology, homological algebra, algebraic and differential geometry, computer algebra Our results show that it is also central to robust control theory for infinite-dimensional linear systems.

7.2.5. A symbolic-numeric method for the parametric H_∞ loop-shaping design problem

We develop a symbolic-numeric method for solving the H_∞ loop-shaping design problem for a low order single-input single-output system with parameters. Due to the system parameters, no purely numerical algorithm can indeed solve the problem. Using Gröbner basis techniques and the rational univariate representation of zero-dimensional algebraic varieties, we first give a parametrization of all the solutions of the two algebraic Riccati equations associated with the H_∞ control problem. Then, using results on the spectral factorization problem, a certified symbolic-numeric algorithm is obtained for the computations of the positive definite solutions of these two algebraic Riccati equations. Finally, we present a certified symbolic-numeric algorithm which solves the H_∞ loop-shaping design problem for the above class of systems.

7.3. Improved algorithm for computing separating linear forms for bivariate systems

Participants: Yacine Bouzidi [Disco], Sylvain Lazard [Vegas], Guillaume Moroz [Vegas], Marc Pouget [Vegas], Fabrice Rouillier [Ouragan].

We present new algorithms for computing linear separating forms, RUR decompositions and isolating boxes of the solutions. We show that these three algorithms have worst-case bit complexity $\tilde{O}_B(d^6 + d^5\tau)$, where \tilde{O} refers to the complexity where polylogarithmic factors are omitted and O_B refers to the bit complexity. We also present probabilistic Las-Vegas variants of our two first algorithms, which have expected bit complexity $\tilde{O}_B(d^5 + d^4\tau)$. A key ingredient of our proofs of complexity is an amortized analysis of the triangular decomposition algorithm via subresultants, which is of independent interest.

7.4. Stable H_∞ Controller for Infinite-dimensional systems

The controllers, besides the stabilization, are often designed to achieve some performance and robustness objectives by minimizing H_∞ norm of some cost functions. The resulting controller may be stable or unstable. The unstable controllers, however, are more sensitive to sensor/actuator faults, or nonlinearities. It is not an easy task to design a stable controller for systems having infinitely many zeros and poles in the right-half-plane. By using the similar idea in [88], stable H_∞ controller design method will be presented for a certain class of infinite-dimensional plants. The plants may have infinitely many unstable zeros, however, it is assumed that these zeros are *uniformly separated*. Under some certain assumptions, first, a sufficient condition will be presented to construct a real unit function, which satisfies certain interpolation conditions at the right-half-plane zeros of the plant and some H_∞ norm constraints. Then, by utilizing this result, stable H_∞ controller design method are presented.

7.5. Multiplicity and Stable Varieties of Time-delay Systems: A Missing Link

Multiple spectral values in dynamical systems are often at the origin of complex behaviors as well as unstable solutions. In this work, an unexpected property of multiple spectral values is emphasized. It has been shown that the variety corresponding to such a multiple root defines a stable variety for the steady state. Under mild assumptions, for the reduced examples we show that such a multiple spectral value is nothing else than the spectral abscissa.

7.6. Delay effect in chemical reactions

Belousov-Zhabotinskii (BZ) reaction, which is a very complicated reaction, has been widely studied in bio-science and chemistry, since its dynamic behaviour is similar to real biological oscillators [89]. For certain type of chemical reactions, the use of the law of mass-action kinetics may lead to some simple models expressed by ordinary differential equations. The main feature of BZ reaction, oscillatory behaviour, has been represented by a simple mechanism and the model of this mechanism can be described by ordinary differential equations. However, delayed mass-actions kinetics lead to more accurate models by conserving the simplicity and a relative reduced number of parameters. In [83], [95], [96], [82], some delay-differential models are proposed for Belousov-Zhabotinskii (BZ) reaction with a fewer of concentrations compared to the models obtained by ordinary differential equations. However, in most of these works, the delay, which occurs due to the required time to provide sufficient energy, has not been taken into account. Recently, we consider a more realistic Belousov-Zhabotinskii model, which includes two independent delays. The novelty of the proposed model with respect to the existing ones in the literature can be summarized as follows; one of these delays is introduced to reproduce qualitatively the behavior of the model proposed by [84] with a less number of concentrations as in [95]. Second, the remaining delay appears naturally since the reactants do not react suddenly in the chemical reactions, i.e. the delay stems from the needed time for the occurrence of reaction, called “delayed concentration”.

7.7. H_∞ -stability analysis of neutral systems with commensurate delays

Participants: Catherine Bonnet, Le Ha Vy Nguyen.

We have analyzed [32] the H_∞ -stability of neutral systems with commensurate delays and multiple chains of poles asymptotic to a same set of points on the imaginary axis. First, by approximation, the location of poles of large modulus is determined. This analysis requires to consider several subclasses of systems where poles of high modulus exhibit various patterns. Second, we derive necessary and sufficient conditions for H_∞ -stability which are easy to check as expressed in terms of the degrees of the polynomials involved in the numerator and denominator of the transfer function.

7.8. H_∞ -Stabilization of neutral delay systems

Participants: Catherine Bonnet, Yutaka Yamamoto [Kyoto University].

We have considered two particular neutral delay systems with one delay having a chain of poles clustering the imaginary axis from left or right. For these systems the existence of coprime factorizations have been investigated. The extension to more general systems is still in progress.

7.9. Interval Observer of a new type

Participants: Frederic Mazenc [Disco], Emilia Fridman [Tel-Aviv University].

In [19], we addressed the fundamental problem of constructing for nonlinear systems observers that converge in finite time and, at the same time, provided with upper and lower bounds for the solutions when disturbances are present. This new technique of estimation relies on the use of past values of the output, as done to construct some already known observers which converge in finite time, and on a recent technical result pertaining to the theory of the monotone systems. The result applies to systems with additive disturbances and disturbances in the output. The nonlinear terms are not supposed to be globally Lipschitz, but it is requested that they depend only on the input and output variables. The fundamental advantage over classical interval observer techniques is that no information on the initial conditions of the solutions of the studied system are needed.

7.10. Trajectory based approach

Participants: Frederic Mazenc [Disco], Silviu Niculescu [Disco], Michael Malisoff [LSU].

In the work [22], we provided a new stability analysis technique, which is based on the study of the behaviors of the solutions over any interval $[t - T, T]$, where t represents the time and T is an appropriately chosen constant. Thus trajectory-based approach is completely new in the sense that it neither rely on Lyapunov functions nor on the small gain theorem. One of its most striking feature is that it applies to a broad number of systems (systems with delay, continuous/discrete systems, ODE coupled with difference equations).

In [30] and [55] we provided several significant applications of the main result of [22]. In [30] in two results, we use a Lyapunov function for a corresponding undelayed system to provide a new method to prove stability of linear continuous-time time-varying systems with bounded time-varying delays. We allow uncertainties in the coefficient matrices of the systems. Our main results use upper bounds on an integral average involving the delay. The results establish input-to-state stability with respect to disturbances. We also provided in [55] a novel reduction model approach that ensures global exponential stabilization of linear systems with a time-varying pointwise delay in the input, which allows the delay to be discontinuous and uncertain. Finally, we provided an alternative to the reduction model method, based on a different dynamic extension.

7.11. Positive Systems Approach

Participants: Frederic Mazenc [Disco], Michael Malisoff [LSU].

We presented new methods to prove stability of time-varying systems with delays by taking advantage of the theory of the positive and cooperative systems [24], [23]. We used linear time-varying Lyapunov functionals, operators with integral terms, and positive systems, and we provided robustness of the stability with respect to multiplicative uncertainty in the vector fields. We allowed cases where the delay may be unknown, and where the vector fields defining the systems are not necessarily bounded. The results apply to neutral time-varying systems but are very distinct from those of the paper [69].

7.12. Attitude dynamics, control and observation

Participants: Frederic Mazenc [Disco], Maruthi Akella [Univ of Texas], Divya Thakur [Univ of Texas], Sunpil Yang [Univ of Texas].

We addressed several problems pertaining to the control of fully actuated rigid-body attitude dynamics. The fundamental tool we used is the adaptation of the so called strictification approach to the features of the attitude dynamics (see [3] for an introduction to the 'strictification' paradigm). In particular

1) The contribution [31] output feedback stabilization of fully actuated rigid-body attitude dynamics in the presence of unknown point-wise time-delay in the input torque. Specifically, rate-gyros are unavailable here and only the attitude state represented by the unit quaternion is assumed to be measured. It is worth mentioning that the presence of unknown time-delay in the measured variables, imposes formidable technical challenges for the output-feedback attitude stabilization problem on hand. One of the central difficulties stems from the availability of only a weak Lyapunov-like function for the passivity based dynamic output feedback controller in the absence of delay. This obstacle is circumvented in this contribution by a novel process of partially strictifying the underlying weak Lyapunov-like function.

2) In [57], we considered stabilization of fully actuated rigid-body attitude dynamics in the absence of angular velocity measurements and presents new robustness results to bounded unknown external disturbance torques. In particular, it is assumed that only body orientation is measured in the form of a unit-quaternion signal. It is well known that the passivity properties of the dynamics allows design of velocity-free controllers using a first-order stable filter driven by measured states. When external disturbance torques are taken into account, however, the robustness properties of these passivity-based output feedback controllers cannot be readily analyzed because the Lyapunov-like function from which the controller is derived has a time-derivative that is only negative semidefinite, and therefore non-strict. This obstacle is circumvented through a new partial-strictification approach which ultimately allows the characterization of robustness properties for this closed-loop system.

3) In [18], we proposed a smooth angular velocity observer for the attitude tracking control of a rigid body in the absence of angular velocity measurements. The observer design ensures asymptotic convergence of angular velocity state estimation errors irrespective of the control torque or the initial attitude state of the spacecraft. Unlike existing rate observer formulations that attain estimation error convergence by imposing certain switching conditions or hybrid-logic, the proposed observer has a smooth structure that ensures continuity of all estimated states. Lyapunov strictification is again the key technical result making us to establish the results.

7.13. Introduction of artificial delays for control and observation

Participants: Frederic Mazenc [Disco], Silviu Niculescu [Disco], Michael Malisoff [LSU], Nikolaos Bekiaris-Liberis [Tech. Univ. of Crete].

It is well-known that, in some cases, control or observation problems for systems with or without inherent delay can be solved by artificially introducing delays. We obtained in this field of research two distinct new results.

1) In [56] and [54], we have considered a family of linear time-varying systems with an input, an output and delays in the input. We have shown that, under classical stabilizability and detectability assumptions, all the systems of this family can be exponentially stabilized through a time-varying feedback depending on past values of the output and the input and this without the use of observers or dynamic extensions. Hence, the simplicity of the design and the determination of the value of the solutions in finite time are the main features of the new approach.

2) In [54], we provided a new backstepping result for time-varying systems with input delays. The novelty of the contribution is in the bounds on the controls, and the facts that (i) one does not need to compute any Lie derivatives to apply our controls, (ii) the controls have no distributed terms, and (iii) no differentiability conditions on the available controls for the subsystems are needed. The result is obtained by the introduction of constant pointwise delay in the input. Thus this result is significantly different for backstepping results for systems with delay in the input as presented for instance in [70].

7.14. Continuous-discrete observers

Participants: Frederic Mazenc [Disco], Emilia Fridman [Tel-Aviv University], Michael Malisoff [LSU], Vincent Andrieu [LAGEP].

We solved several problems of observer design pertaining to the fundamental and difficult case where the measurements are available at discrete instants only.

1) We considered the problem of stabilizing a linear continuous-time system with discrete-time measurements and a sampled input with a pointwise constant delay [20]. In a first part, we designed a continuous-discrete observer which converges when the maximum time interval between two consecutive measurements is sufficiently small. In a second part, we constructed a dynamic output feedback by using a technique which is strongly reminiscent of the reduction model approach. It stabilizes the system when the maximal time between two consecutive sampling instants is sufficiently small. No limitation on the size of the delay is imposed and an ISS property with respect to additive disturbances is established.

2) The problem of designing continuous-discrete observers for a large class of continuous time nonlinear time-varying systems with discrete time measurements has been addressed in several contributions: [12], [17] and [53]. Some technical obstacle encountered in [12] were overcome in [17] by using the notion of cooperative systems, which led to results consisting in explicit expressions of the largest sampling interval under which the observers converge to the solutions of the original system

7.15. Sensor faults: detectability and distributed Fault Diagnosis. A positive invariance based approach.

Participants: Sorin Olaru [correspondent], Vasso Reppa [L2S], Abid Kodakkadan [L2S], Marios Polycarpou [University of Cyprus].

The paper [62] introduces the performance analysis of local monitoring modules of a distributed diagnosis scheme tailored to detect multiple sensor faults in a class of nonlinear systems. The local modules monitor the healthy operation of subsets of sensors (local sensor sets). Every module is designed to detect the occurrence of faults in the local sensor sets when some analytical redundancy relations (ARRs) are violated. The set of ARR is formulated using structured residuals and adaptive thresholds based on a nonlinear observer. In order to characterize the sensitivity of every monitoring module to local sensor faults, we obtain structural fault detectability conditions based on adaptive thresholds, and strong fault detectability conditions based on ultimate robust positively invariant sets. These conditions correspond to explicit relationships between the local sensor faults, the worst-case bounds on modeling uncertainties and the design parameters of the local monitoring module.

In a recent paper [44], we considered the abnormal functioning of sensors (measurement channels) deployed for monitoring and control of discrete linear time invariant systems affected by additive uncertainties. The main objective was to analyze the sensor fault detectability via a robust positive invariance based technique. The analysis relies on the categorization of detectable faults and leads to certain conditions for guaranteed nondetectability, guaranteed detectability and implicit detectability.

As a support to this line of research, in the paper [73] we presented a methodology for computing robust positively invariant sets for linear, discrete time-invariant systems that are affected by additive disturbances, with the particularity that these disturbances are subject to state-dependent bounds. The proposed methodology requires less restrictive assumptions compared to similar established techniques, while it provides the framework for determining the state-dependent (parameterized) ultimate bounds for several classes of disturbances. The added value of the proposed approach is illustrated by an optimization-based problem for detecting the mode of functioning of a switching system.

7.16. Fault tolerant control design for multi-sensor networked control systems

Participants: Sorin Olaru [correspondent], Nikola Stankovic [L2S], Silviu Niculescu [L2S], Florin Stoican [Univ. Politehnica, Bucharest].

In the paper [27], we consider a multi-sensor networked control configuration with linear plant which is affected by a bounded additive disturbance. Shared network is used for the communication between sensors and controller. It is assumed that the sensors are prone to abrupt faults, while the controller's input may be updated with a varying time-delay. In order to identify and isolate the sensor(s) providing faulty information, we equip the controller with a set-based detection and isolation routine. Furthermore, in the case when the network induces time-delays, control is performed based on the knowledge we have on the mathematical model of the plant. In the presence of model inaccuracies or disturbance, such a control action may not guarantee satisfying performance of the system. Therefore, a stabilising controller with delay compensation has been designed.

7.17. Constrained Control of Uncertain, Time-varying Linear Discrete-Time Systems Subject to Bounded Disturbances

Participants: Sorin Olaru [correspondent], Nam Nguyen [Technion, Israel], Per-Olof Gutman [Technion, Israel], Morten Hovd [NTNU, Trondheim, Norway].

In the paper [26], robust invariance for ellipsoidal sets with respect to uncertain and/or time-varying linear discrete-time systems with bounded additive disturbances is revisited. We provide an extension of an existing invariance condition and propose a novel robust interpolation based control design involving several local unconstrained robust optimal controls. It is shown that at each time instant a quadratic programming problem is solved on-line for the implementation. Proofs of recursive feasibility and input-to-state stability are provided to support the theoretical foundation.

7.18. Explicit robust constrained control for linear systems : analysis, implementation and design based on optimization

Participants: Sorin Olaru [correspondent], Ngoc Anh Nguyen [L2S], Pedro Rodriguez Ayerbe [L2S], Martin Gulán [STUBA, Bratislava].

Piecewise affine (PWA) feedback control laws is relevant for the control of constrained systems, hybrid systems; equally for the approximation of nonlinear control. However, they are associated with serious implementation issues. Motivated from the interest in this class of particular controllers, the thesis of Ngoc Anh Nguyen is mostly related to their analysis and design. The first part of this thesis aims to compute the robustness and fragility margins for a given PWA control law and a linear discrete-time system. More precisely, the robustness margin was defined as the set of linear time-varying systems such that the given PWA control law keeps the trajectories inside a given feasible set. On a different perspective, the fragility margin contains all the admissible variations of the control law coefficients such that the positive invariance of the given feasible set is still guaranteed. The second part of this thesis focuses on inverse optimality problem for the class of PWA controllers. Namely, the goal is to construct an optimization problem whose optimal solution is equivalent to the given PWA function. The methodology is based on convex lifting [33]: an auxiliary 1-dimensional variable which enhances the convexity characterization into recovered optimization problem. Accordingly, if the given PWA function is continuous [35], the optimal solution to this reconstructed optimization problem will be shown to be unique. Otherwise, if the continuity of this given PWA function is not fulfilled, this function will be shown to be one optimal solution to the recovered problem [36]. In view of applications in linear model predictive control (MPC), it was shown that any continuous PWA control law can be obtained by a linear MPC problem with the prediction horizon at most equal to 2 prediction steps [65]. Aside from the theoretical meaning, this result can also be of help to facilitate implementation of PWA control laws by avoiding storing state space partition [66]. Another utility of convex liftings [34] will be shown in the last part of this thesis to be a control Lyapunov function. Accordingly, this convex lifting will be deployed in the so-called robust control design based on convex liftings for linear system affected by bounded additive disturbances and polytopic uncertainties [60]. Both implicit and explicit controllers can be obtained. This method can also guarantee the recursive feasibility and robust stability. However, this control Lyapunov function is only defined over the maximal λ -contractive set for a given subunitary λ which is known to be smaller than the maximal controllable

set. Therefore, an extension of the above method to the N-steps controllable set will be presented. This method is based on a cascade of convex liftings where an auxiliary variable will be used to emulate a Lyapunov function. Namely, this variable will be shown to be non-negative, to strictly decrease for N first steps and to stay at 0 afterwards. Accordingly, robust stability is sought.

7.19. Predictive Control for multi-agent (multi-vehicle) systems

Participants: Sorin Olaru [correspondent], Ionela Prodan [LCIS], Minh Tri Nguyen [L2S], Cristina Stoica [L2S], Silviu Niculescu [L2S], Fernando Fontes [FEUP U. Porto], Joao Sousa [FEUP U. Porto], Fernando Lobo Pereira [FEUP U. Porto], Alexandra Grancharova [U. Sofia, Bulgaria].

We continued a mature line of research on the tracking problems for multi-agent systems. In [75] we presented a series of developments on predictive control for path following via a priori generated trajectory for autonomous aerial vehicles. The strategy partitions itself into offline and runtime procedures with the assumed goal of moving the computationally expensive part into the offline phase and of leaving only tracking decisions to the runtime. First, it will be recalled that differential flatness represents a well-suited tool for generating feasible reference trajectory. Next, an optimization-based control problem which minimizes the tracking error for the nonholonomic system is formulated and further enhanced via path following mechanisms. Finally, possible changes of the selection of sampling times along the path and their impact on the predictive control formulation will be discussed in detail.

On a relatively different framework, in [71] we investigate multiple agents evolving in the same environment with the objective of preservation of a predefined formation. This formation aims to reinforce the safety of the global system and further lighten the supervision task. One of the major issues for this objective is the task assignment problem, which can be formulated in terms of an optimization problem by employing set-theoretic methods. In real time the agents will be steered into the defined formation via task (re)allocation and classical feedback mechanisms. The task assignment calculation is often performed in an offline design stage, without considering the possible variation of the number of agents in the global system. These changes (i.e., including/excluding an agent from a formation) can be regarded as a typical fault, due to some serious damages on the components or due to the operator decision. In this context, our work proposes a new algorithm for the dynamical task assignment formulation of multi-agent systems in view of real-time optimization by including fault detection and isolation capabilities. This algorithm allows to detect whether there is a fault in the global multi-agent system, to isolate the faulty agent and to integrate a recovered/healthy agent. The proposed methods will be illustrated by means of a numerical example with connections to multi-vehicle systems.

7.20. Invariant sets for time-delay systems

Participants: Sorin Olaru [correspondent], Mohammed Laraba [L2S], Silviu Niculescu [L2S].

The characterization of invariant sets for dynamical systems affected by time-delays is a long standing research topic in our group and this year new results have been obtained [52], [51] towards construction of invariant sets in the original state space (also called D-invariant sets) by exploiting the forward mappings. As novelties, the present paper contains a sufficient condition for the existence of ellipsoidal D-contractive sets for dDDEs, and a necessary and sufficient condition for the existence of D-invariant sets in relation to time-varying dDDE stability. Another contribution is the clarification of the relationship between convexity (convex hull operation) and D-invariance. In short, this is shown that the convex hull of two D-invariant sets is not D-invariant but the convex hull of a non-convex D-invariant set is D-invariant.

7.21. Biochemical system modeled through a PDE

Participants: Frédéric Mazenc, Abdou Dramé [City Univ. of New York], Peter Wolenski [LSU].

We studied a model of chemostat relying on a Partial Differential Equation. More precisely, we studied in [13] the stability of periodic solutions of distributed parameters biochemical system with periodic input $S_{in}(t)$ (which represent the substrate input concentration). We established that if the function $S_{in}(t)$ is periodic then the system has a periodic solution that possess the robust stability property called input to state stability and this when sufficiently small perturbations are acting on $S_{in}(t)$.

7.22. Modelling of cell dynamics in Acute Myeloid Leukemia

Participants: Catherine Bonnet, Jean Clairambault [BANG project-team], François Delhommeau [INSERM Paris (Team18 of UMR 872) Cordeliers Research Centre and St. Antoine Hospital, Paris], Walid Djema, Emilia Fridman [Tel-Aviv University], Pierre Hirsch [INSERM Paris (Team18 of UMR 872) Cordeliers Research Centre and St. Antoine Hospital, Paris], Frédéric Mazenc.

Modelling of Acute Myeloid Leukemia started a few years ago. Starting from a PDE model of hematopoiesis given in [77], we have derived several models of healthy or cancer cell dynamics in hematopoiesis which according to some conditions admit one or two equilibrium points. Often taking profit of the positivity of the system we have derived this year several sufficient (or necessary and sufficient) conditions which ensure stability properties ranging from local asymptotic stability to global exponential stability and obtained, when appropriate, an estimation of a subset of the basin of attraction [14], [40].

7.23. Observability analysis of AC electric machines

Participants: Mohamad Koteich [CentraleSupélec, L2S, Renault], Guillaume Sandou [correspondent], Gilles Duc [CentraleSupélec, L2S], Abdelmalek Maloum [Renault].

High-performance control of electric drives requires an accurate knowledge of the rotor position and/or speed. These mechanical variables are traditionally measured using sensors, which increases the cost and reduces both the robustness and the reliability of the system. This emphasizes the importance of electric drives control without shaft sensors, often referred to as sensorless control : it consists of replacing sensors with a state observer algorithm, that estimates the desired mechanical variables from currents and voltages sensing and based on the system's model. Nevertheless, before designing a state observer, the observability of the system should be examined, that is, it should be checked whether the states to be estimated can be reconstructed, unambiguously, from the input/output signals of the system.

This work addresses the modeling and the observability analysis of electric drives in the view of mechanical sensors removal. Firstly, electrical machines models are elaborated, and it is shown that a unified modeling of alternating current machines is feasible, for the purpose of designing unified control and estimation strategies. The observability of the machines' models is next studied in the view of sensorless control. The local instantaneous observability theory is applied, which enables us to formulate physically insightful analytic conditions that can be easily interpreted and tested in real time. The validity of the observability conditions is confirmed by numerical simulations and experimental data, using an extended Kalman observer. This work contributes to novel outlooks on the sensorless alternating current drives and to a deeper understanding of its properties, in order to develop higher performance estimation techniques in the critical operating conditions (mainly at standstill and/or zerostator-frequency). The concepts introduced throughout this work, such as the equivalent flux and the observability vector, with the obtained results, open new horizons in a domain that seems to become mature enough [48], [76], [46], [49], [45], [15], [16], [91], [92], [47].

7.24. Optimization of Line of Sight controller based on high-level optronic criterion

Participants: Sophie Frasnedo [CentraleSupélec, L2S, Sagem], Guillaume Sandou [correspondent], Gilles Duc [CentraleSupélec, L2S], Philippe Feyel [Sagem], Cédric Chapuis [Sagem].

A method to tune the parameters of the controller of an inertially stabilized platform is proposed. This platform carries an electro-optical system. The image quality is obviously influenced by the movements of the platform: the Line of Sight (LoS) of the imager has to remain fixed in an inertial frame. The more the LoS controller manages to counter the movement of the platform, the better the image quality will be. The motion Modulation Transfer Function (motion MTF) measures the amount of blur brought into the image by the motion of the platform. It represents the contrast over spatial frequencies. Up to now, it has mostly been used as a validation tool for controllers already tuned from derived low level and conservative considerations. The proposed methodology aims to tune LoS controllers using directly the motion MTF as a criterion in the design procedure [42].

7.25. Optimization of controller using bayesian optimization

Participants: Sophie Frasnedo [CentraleSupélec, L2S, Sagem], Julien Bect [CentraleSupélec, L2S], Gilles Duc [CentraleSupélec, L2S], Guillaume Sandou [correspondent], Philippe Feyel [Sagem], Cédric Chapuis [Sagem].

A method to globally optimize the parameters of the controller of an inertially stabilized platform is presented. This platform carries an electro-optical system. The quality of the produced image is obviously influenced by the capacity of the controller to compensate for the unwanted motion of the platform. The motion Modulation Transfer Function (motion MTF) measures the amount of blur brought into the image by those parasite movements. The controller is tuned by minimizing a criterion which includes the motion MTF. However, evaluating this criterion is time-consuming. Using an optimization method that needs numerous evaluations of the criterion is not compatible with industrial constraints. Bayesian optimization methods consist in combining prior information about the criterion and previous evaluation results in order to choose efficiently new evaluation points and reach the global minimizer within a reasonable time. In this paper, a Bayesian approach is used to optimize the motion MTF-based criterion. The results are compared with a local optimization of the same MTF-based criterion, initialized with an acceptable initial point. Similar performances are achieved by the proposed methodology, without requiring an initialization point [41].

7.26. Particle Swarm Optimization based Approach for Model Predictive Control Tuning

Participants: Mohamed Lotfi Derouiche [CentraleSupélec, L2S, Ecole Nationale d'Ingénieur de Tunis], Guillaume Sandou [correspondent], Soufiene Bouallegue [Ecole Nationale d'Ingénieur de Tunis], Joseph Haggège [Ecole Nationale d'Ingénieur de Tunis].

In this work, a new Model Predictive Controller (MPC) parameters tuning strategy is proposed using a perturbed Particle Swarm Optimization (pPSO) approach. This original LabVIEW implementation of this metaheuristic algorithm is firstly validated on some test functions in order to show its efficiency and validity. The optimization results are compared with the standard PSO as well as a LabVIEW implemented Genetic Algorithm (GA) approaches. The parameters tuning problem, i.e. the weighting factors on the output error and input increments of the MPC algorithm, is after that formulated and systematically resolved, using the proposed LabVIEW pPSO algorithm. The case of a Magnetic Levitation (MAGLEV) system is investigated to illustrate the robustness and superiority of the proposed pPSO-based tuning MPC approach. All obtained simulation results, as well as the statistical analysis tests, are compared and discussed in order to improve the effectiveness of the proposed pPSO-based MPC tuning methodology.

7.27. Traffic rescheduling for CBTC train system running in a mixed traffic

Participants: Juliette Pochet [CentraleSupélec, L2S, SNCF], Guillaume Sandou [correspondent], Sylvain Baro [SNCF].

Railway companies need to achieve higher capacities on existing infrastructures such as high density suburban mainlines. Communication based train control (CBTC) systems have been widely deployed on dedicated subway lines. However, deployment on shared rail infrastructure, where CBTC and non CBTC trains run, leads to a mixed positioning and controlling system with different precision levels and restrictions. New performance and complexity issues are to arise. In this work, a method for traffic rescheduling, adapted to a CBTC system running in a mixed traffic, is introduced. A genetic algorithm solves the problem to optimize the cost function. It determines the dwell times and running times of CBTC-equipped trains, taking into account the non-equipped trains planning and fixed-block localization. In addition, reordering can be allowed by modifying the problem constraints. The work is supported by a new simulation tool developed by SNCF and adapted to mixed traffic study. The approach is illustrated with a case study based on a part of an East/West line in Paris region network, proving the ability of the method to find good feasible solutions when delays occur in the traffic.

7.28. Combined Feedback Linearization and MPC for Wind Turbine Power Tracking

Participants: Nicolo Gionfra [CentraleSupélec, L2S], Guillaume Sandou [correspondent], Houria Siguerdjane [CentraleSupélec, L2S], Damien Faille [EDF], Philippe Loevenbruck [EDF].

The problem of controlling a variable-speed-variable-pitch wind turbine in non conventional operating points is addressed. We aim to provide a control architecture for a general active power tracking problem for the turbine's entire operating envelope. The presented control enables to cope with system non linearities while handling state and input constraints, and avoiding singular points. Simulations are carried out based on the CART turbine parameters. Comparative results show that the proposed controller outperforms the classic PI regulator.

GECO Project-Team

6. New Results

6.1. New results: geometric control

Let us list some new results in sub-Riemannian geometry and hypoelliptic diffusion obtained by GECO's members.

- In [12] and [20] we study the sub-Finsler geometry as a time-optimal control problem. In particular, we consider non-smooth and non-strictly convex sub-Finsler structures associated with the Heisenberg, Grushin, and Martinet distributions. Motivated by problems in geometric group theory, we characterize extremal curves, discuss their optimality, and calculate the metric spheres, proving their Euclidean rectifiability.
- In [18] we compare different notions of curvature on contact sub-Riemannian manifolds. In particular we introduce canonical curvatures as the coefficients of the sub-Riemannian Jacobi equation. The main result is that all these coefficients are encoded in the asymptotic expansion of the horizontal derivatives of the sub-Riemannian distance. We explicitly compute their expressions in terms of the standard tensors of contact geometry. As an application of these results, we obtain a sub-Riemannian version of the Bonnet-Myers theorem that applies to any contact manifold.
- In sub-Riemannian geometry the coefficients of the Jacobi equation define curvature-like invariants. We show in [21] that these coefficients can be interpreted as the curvature of a canonical Ehresmann connection associated to the metric, first introduced by Zelenko and Li. We show why this connection is naturally nonlinear, and we discuss some of its properties.
- On a sub-Riemannian manifold we define in [22] two type of Laplacians. The macroscopic Laplacian, as the divergence of the horizontal gradient, once a volume is fixed, and the microscopic Laplacian, as the operator associated with a geodesic random walk. We consider a general class of random walks, where all sub-Riemannian geodesics are taken in account. This operator depends only on the choice of a complement to the sub-Riemannian distribution. We address the problem of equivalence of the two operators. This problem is interesting since, on equiregular sub-Riemannian manifolds, there is always an intrinsic volume (e.g. Popp's one) but not a canonical choice of complement. The result depends heavily on the type of structure under investigation: we describe the relationship between the two approaches in the case of contact structures, Carnot groups, quasi-contact structures.
- In [2] we study 3D almost-Riemannian manifolds, that is, generalized Riemannian manifolds defined locally by 3 vector fields that play the role of an orthonormal frame, but could become collinear on some singular set. Almost-Riemannian manifolds were deeply studied in dimension 2. In this paper we start the study of the 3D case which appear to be reacher with respect to the 2D case, due to the presence of abnormal extremals which define a field of directions on the singular set. We study the type of singularities of the metric that could appear generically, we construct local normal forms and we study abnormal extremals. We then study the nilpotent approximation and the structure of the corresponding small spheres. We finally give some preliminary results about heat diffusion on such manifolds.

New results on motion planning are the following.

- In [7] (written while D. Prandi was PhD student in the team) we study the complexity of the motion planning problem for control-affine systems. Such complexities are already defined and rather well-understood in the particular case of nonholonomic (or sub-Riemannian) systems. Our aim is to generalize these notions and results to systems with a drift. Accordingly, we present various definitions of complexity, as functions of the curve that is approximated, and of the precision of the approximation. Due to the lack of time-rescaling invariance of these systems, we consider geometric

and parametrized curves separately. Then, we give some asymptotic estimates for these quantities. As a byproduct, we are able to treat the long-time local controllability problem, giving quantitative estimates on the cost of stabilizing the system near a non-equilibrium point of the drift.

- In [11] and [1] we propose new conditions guaranteeing that the trajectories of a mechanical control system can track any curve on the configuration manifold. We focus on systems that can be represented as forced affine connection control systems and we generalize the sufficient conditions for tracking known in the literature. The new results are proved by a combination of averaging procedures by highly oscillating controls and the notion of kinematic reduction.
- In
- [17] we introduce the concept of Developmental Partial Differential Equation (DPDE), which consists of a Partial Differential Equation (PDE) on a time-varying manifold with complete coupling between the PDE and the manifold's evolution. In other words, the manifold's evolution depends on the solution to the PDE, and vice versa the differential operator of the PDE depends on the manifold's geometry. DPDE is used to study a diffusion equation with source on a growing surface whose growth depends on the intensity of the diffused quantity. The surface may, for instance, represent the membrane of an egg chamber and the diffused quantity a protein activating a signaling pathway leading to growth. Our main objective is to show controllability of the surface shape using a fixed source with variable intensity for the diffusion. More specifically, we look for a control driving a symmetric manifold shape to any other symmetric shape in a given time interval. For the diffusion we take directly the Laplace-Beltrami operator of the surface, while the surface growth is assumed to be equal to the value of the diffused quantity. We introduce a theoretical framework, provide approximate controllability and show numerical results. Future applications include a specific model for the oogenesis of *Drosophila melanogaster*.

6.2. New results: quantum control

New results have been obtained for the control of the bilinear Schrödinger equation.

- In [4] we study the so-called spin-boson system, namely a two-level system in interaction with a distinguished mode of a quantized bosonic field. We give a brief description of the controlled Rabi and Jaynes–Cummings models and we discuss their appearance in the mathematics and physics literature. We then study the controllability of the Rabi model when the control is an external field acting on the bosonic part. Applying geometric control techniques to the Galerkin approximation and using perturbation theory to guarantee non-resonance of the spectrum of the drift operator, we prove approximate controllability of the system, for almost every value of the interaction parameter.
- The main result in [9] is the approximate controllability of a bilinear Schrödinger equation modeling a system of two ions trapped in a cavity. A new spectral decoupling technique is introduced, which allows to analyze the controllability of the infinite-dimensional system through finite-dimensional considerations. The controllability of a simplified version of the model has been obtained in [16].
- In [13] and [3] we study the controllability of a closed control-affine quantum system driven by two or more external fields. We provide a sufficient condition for controllability in terms of existence of conical intersections between eigenvalues of the Hamiltonian in dependence of the controls seen as parameters. Such spectral condition is structurally stable in the case of three controls or in the case of two controls when the Hamiltonian is real. The spectral condition appears naturally in the adiabatic control framework and yields approximate controllability in the infinite-dimensional case. In the finite-dimensional case it implies that the system is Lie-bracket generating when lifted to the group of unitary transformations, and in particular that it is exactly controllable. Hence, Lie algebraic conditions are deduced from purely spectral properties. We conclude the analysis by proving that approximate and exact controllability are equivalent properties for general finite-dimensional quantum systems.

In [26], written with the members of the European project QUAINT, state-of-the-art quantum control techniques are reviewed and put into perspective by a consortium uniting expertise in optimal control theory and applications to spectroscopy, imaging, quantum dynamics of closed and open systems. Key challenges are addressed and a roadmap to future developments is sketched.

6.3. New results: neurophysiology

In [27] we present a new version of the image inpainting algorithm that GECO developed in the recent years. This new version is called the Averaging and Hypoelliptic Evolution (AHE) algorithm, and is based upon a semi-discrete variation of the Citti–Petitot–Sarti model of the primary visual cortex V1. In particular, we focus on reconstructing highly corrupted images (i.e. where more than the 80% of the image is missing).

6.4. New results: switched systems

- In [5] we consider a continuous-time linear switched system on \mathbb{R}^n associated with a compact convex set of matrices. When it is irreducible and its largest Lyapunov exponent is zero there always exists a Barabanov norm associated with the system. We look at two types of issues: (a) properties of Barabanov norms such as uniqueness up to homogeneity and strict convexity; (b) asymptotic behaviour of the extremal solutions of the linear switched system. Regarding Issue (a), we provide partial answers and propose four related open problems. As for Issue (b), we establish, when $n = 3$, a Poincaré–Bendixson theorem under a regularity assumption on the set of matrices. We then revisit a noteworthy result of N.E. Barabanov describing the asymptotic behaviour of linear switched system on \mathbb{R}^3 associated with a pair of Hurwitz matrices $\{A, A + bc^T\}$.
- Motivated by an open problem posed by J.P. Hespanha, in [23] we extend the notion of Barabanov norm and extremal trajectory to classes of switching signals that are not closed under concatenation. We use these tools to prove that the finiteness of the L_2 -gain is equivalent, for a large set of switched linear control systems, to the condition that the generalized spectral radius associated with any minimal realization of the original switched system is smaller than one.
- In [14] in the totally observed case and in [23] in the general case, we answer an open problem posed by J.P. Hespanha in 2003. We first extend the notion of Barabanov norm and extremal trajectory to classes of switching signals that are not closed under concatenation. We use these tools to prove that the finiteness of the L_2 -gain is equivalent, for a large set of switched linear control systems, to the condition that the generalized spectral radius associated with any minimal realization of the original switched system is smaller than one.
- In [24] we address the stability of non-autonomous difference equations by providing a suitable representation of the solution at time t in terms of the initial condition and time-dependent matrix coefficients. This enables us to characterize the asymptotic behavior of solutions in terms of that of such coefficients. As a consequence, we obtain necessary and sufficient stability criteria for non-autonomous linear difference equations. In the case of difference equations with arbitrary switching, we obtain a generalization of the well-known criterion for autonomous systems due to Hale and Silkowski, which, as the latter, is delay-independent. These results are applied to transport and wave propagation on networks. In particular, we show that the wave equation on a network with arbitrarily switching damping at external vertices is exponentially stable if and only if the network is a tree and the damping is bounded away from zero at all external vertices but one.
- For linear systems in continuous time with random switching, we characterize in [25] the Lyapunov exponents using the Multiplicative Ergodic Theorem for an associated system in discrete time. An application to control systems shows that here a controllability condition implies that arbitrary exponential decay rates for almost sure stabilization can be obtained.

A result related to switched system is the one obtained in [6] and [15]: we study the stability of linear time-varying delay differential equations where the delay enters as a switching parameter. In [6] we give a collection of converse Lyapunov–Krasovskii theorems for uncertain retarded differential equations. We show that the

existence of a weakly-degenerate Lyapunov–Krasovskii functional is a necessary and sufficient condition for the global exponential stability of linear retarded functional differential equations. In [15] the fundamental question that we consider is the following: assuming that every individual (constant-delay) subsystem is exponentially stable, can we characterize the cases when the system is not exponentially stable? This is nothing else than the so-called Markus-Yamabe instability and we give new conditions ensuring it.

Maxplus Team

7. New Results

7.1. Théorie spectrale max-plus et géométrie métrique/Max-plus spectral theory and metric geometry

7.1.1. Introduction

Participants: Marianne Akian, Stéphane Gaubert, Cormac Walsh.

Étant donné un noyau $a : S \times S \rightarrow \mathbb{R} \cup \{-\infty\}$, on peut lui associer le problème spectral max-plus

$$\sup_{y \in S} a(x, y) + u(y) = \lambda + u(x), \quad \forall x \in S, \quad (1)$$

dans lequel on cherche le vecteur propre $u : S \rightarrow \mathbb{R} \cup \{-\infty\}$ et la valeur propre correspondante $\lambda \in \mathbb{R} \cup \{-\infty\}$. Comme nous l'avons rappelé dans les §3.2 et 3.3, le problème spectral (9) intervient en contrôle ergodique: l'ensemble S est l'espace des états, et l'application $a(x, y)$ fournit le gain associé à la transition $x \rightarrow y$. Le cas où S est fini est classique, l'on a alors un résultat précis de représentation de l'espace propre, à l'aide d'un certain graphe, dit graphe critique. Des résultats existent également lorsque S est compact et que le noyau vérifie certaines propriétés de régularité.

Dans [51], nous avons considéré le cas où S est non compact. Lorsque $\lambda = 0$, l'espace propre est analogue à l'espace des fonctions harmoniques défini en théorie (classique ou probabiliste) du potentiel. En introduisant l'analogue max-plus de la frontière de Martin, nous avons obtenu un analogue de la formule de représentation de Poisson des fonctions harmoniques : toute solution u de (9) peut être représentée sous la forme :

$$u = \sup_{w \in \mathcal{M}_m} w + \mu_u(w), \quad (2)$$

où $\mathcal{M}_m \subset (\mathbb{R} \cup \{-\infty\})^S$ est l'analogue max-plus de la frontière de Martin minimale (l'ensemble des fonctions harmoniques extrémales normalisées), et où μ_u joue le rôle de la mesure spectrale. Nous avons montré aussi que les éléments de l'espace de Martin minimal peuvent être caractérisés comme les limites de "quasi-géodésiques". La frontière de Martin max-plus généralise dans une certaine mesure la frontière d'un espace métrique construite à partir des horo-fonctions (fonctions de Busemann généralisées), ou horo-frontière. Ces résultats inspirent les travaux des sections suivantes, qui portent sur des cas remarquables d'espaces métriques (§7.1.2) ou sur des applications en théorie des jeux (§7.2.2).

English version

Let the kernel $a : S \times S \rightarrow \mathbb{R} \cup \{-\infty\}$ be given. One may associate the max-plus spectral equation (9), where the eigenvector $u : S \rightarrow \mathbb{R} \cup \{-\infty\}$ and the eigenvalue $\lambda \in \mathbb{R} \cup \{-\infty\}$ are unknown. As we recalled in §3.2 and refmonotone, this spectral problem arises in ergodic optimal control: the set S is the *state space*, and the map $a(x, y)$ is the *transition reward*. The case when S is finite is classical, a precise spectral theorem is known, with a characterisation of the eigenspace in terms of a critical graph. Some results have been shown when S is compact, assuming that the kernel a satisfies some regularity properties.

In [51], we considered the case where S is non-compact. When $\lambda = 0$, the eigenspace is analogous to the set of harmonic functions defined in classical or probabilistic potential theory. By introducing a max-plus analogue of the classical Martin boundary, we obtained an analogue of the Poisson representation of harmonic functions, showing that any solution u of (9) may be represented as in (10) where $\mathcal{M}_m \subset (\mathbb{R} \cup \{-\infty\})^S$ is a max-plus analogue of the minimal Martin boundary (the set of normalised extremal harmonic functions), and μ_u plays the role of the spectral measure. We also showed that the elements of the minimal Martin boundary can be characterised as limits of certain “almost-geodesics”. The max-plus Martin boundary generalises to some extent the boundary of metric spaces defined in terms of horofunctions (generalised Busemann functions), or horoboundary. These results have inspired the work of the next sections, which deal either with interesting examples of metric spaces (§7.1.2) or with applications to zero-sum games (§7.2.2).

7.1.2. *Isométries de la géométrie de Hilbert/Isometries of the Hilbert geometry*

Participants: Cormac Walsh, Bas Lemmens [Kent University, UK].

Dans nos travaux précédents, nous avons étudié la géométrie de Hilbert (d’un ensemble convexe) en dimension finie, en particulier son horo-frontière et son groupe des isométries. Le chapitre de livre [167] donne une vue d’ensemble de ces travaux. Le cas de la dimension infinie est aussi intéressant, et a été utilisé depuis de nombreuses années en analyse non linéaire. Malgré cela, la géométrie de ces espaces est très peu connue en dimension infinie.

On s’intéresse par exemple au problème suivant. En dimension finie, il est connu que la géométrie de Hilbert est isométrique à un espace normé si et seulement si le convexe est un simplexe. On a montré [38] plus généralement que la géométrie de Hilbert est isométrique à un espace de Banach si et seulement si le convexe est le cône des fonctions positives continues sur un espace topologique compact. Pour cela, on a étudié l’horo-frontière en dimension infinie.

On continue à travailler sur ce sujet avec Bas Lemmens de l’Université de Kent.

English version

Previously, we have been studying the Hilbert geometry in finite dimensions, especially its horofunction boundary and isometry group. The book chapter [167] contains a survey of this work. However, the infinite dimensional case is also interesting, and has been used as a tool for many years in non-linear analysis. Despite this, very little is known about the geometry of these spaces when the dimension is infinite.

An example of a problem in which we are interested is the following. In finite dimension it is known that a Hilbert geometry is isometric to a normed space if and only if it is a simplex. We have shown [38] that, more generally, a Hilbert geometry is isometric to a Banach space if and only if it is the cross-section of a positive cone, that is, the cone of positive continuous functions on some compact topological space. To solve this problem we found it useful to study the horofunction boundary in the infinite-dimensional case.

We are continuing to study similar problems in relation to this topic in collaboration with Bas Lemmens of the University of Kent.

7.1.3. *Croissance des boules dans la géométrie de Hilbert/Volume growth in the Hilbert geometry*

Participants: Cormac Walsh, Constantin Vernicos [Université Montpellier 2].

Avec Constantin Vernicos de l’Université Montpellier 2, nous étudions la croissance du volume de la boule d’une géométrie de Hilbert (d’un ensemble convexe) en fonction du rayon. En particulier, nous étudions l’entropie volumique:

$$\lim_{r \rightarrow \infty} \frac{\log \text{Vol } B(x, r)}{r}, \quad (3)$$

où $B(x, r)$ désigne la boule de centre x et de rayon r , et Vol est une notion de volume particulière, telle que celle définie par Holmes–Thompson ou celle de Busemann. L’entropie ne dépend pas du choix particulier de x , ni de celui du volume. Il est connu que pour l’espace hyperbolique, ou toute géométrie de Hilbert dont la frontière est C^2 et de courbure strictement positive, l’entropie est égale à $n - 1$ lorsque la dimension de l’espace est n , et il a été prouvé récemment que ceci correspond aussi à l’entropie maximale d’une géométrie de Hilbert en dimension n .

Constantin Vernicos a montré que, en dimension 2 et 3, l’entropie volumique d’une géométrie d’Hilbert sur une convexe est égale à l’*approximabilité* de la convexe, ce qui est le taux de croissance exponentielle du nombre de sommets nécessaire pour approximer la convexe par un polytope à ϵ près, quand ϵ diminue.

Ceci motive l’étude de la croissance du volume dans le cas de polytopes. Dans ce cas, la croissance est polynomiale de degré n , plutôt qu’exponentielle, et il est important de comprendre le lien entre le coefficient dominant du polynôme exprimant le volume et la complexité du polytope. Nous avons obtenu une formule pour ce coefficient, laquelle dépend de la structure combinatoire du polytope. Cette formule suggère de définir une nouvelle notion de *approximabilité* en utilisant une quantité combinatoire différente que le nombre de sommets, et d’étudier la relation entre cette *approximabilité* et l’entropie volumique. On pourrait supposer que les deux quantités sont égales, ce qui impliquerait en particulier que l’entropie volumique d’une convexe est égale à celle de son dual.

English version

In a collaboration with Constantin Vernicos of Université Montpellier 2, we are investigating how the volume of a ball in a Hilbert geometry grows as its radius increases. Specifically, we are studying the volume entropy (11) where $B(x, r)$ is the ball with center x and radius r , and Vol denotes some notion of volume, for example, the Holmes–Thompson or Busemann definitions. Note that the entropy does not depend on the particular choice of x , nor on the choice of the volume. It is known that the hyperbolic space, or indeed any Hilbert geometry with a C^2 -smooth boundary of strictly positive curvature, has entropy $n-1$, where n is the dimension, and it has recently been proved that this is the maximal entropy possible for Hilbert geometries of the given dimension.

Constantin Vernicos has shown that, in dimension 2 and 3, the volume entropy of a Hilbert geometry on a convex body is equal to the *approximability* of the body, that is, the exponential rate of growth of the number of vertices needed to approximate the body by a polytope within ϵ , as ϵ decreases.

This motivates studying the volume growth in the polytopal case. Here the growth is polynomial rather than exponential, of degree n , and it is important to know how the constant on front of the highest term depends on the complexity of the polytope. We have a formula for this constant in terms of the combinatorial structure of the polytope. This formula suggests defining a new notion of *approximability* using a different combinatorial quantity from the number of vertices, and studying the relationship between this *approximability* and the volume entropy. One might conjecture that the two quantities are equal, which would imply in particular that the volume entropy of a convex body is equal to that of its dual.

7.1.4. Consensus non-commutatif et contraction d’opérateurs de Kraus/Noncommutative consensus and contraction of Kraus maps

Participants: Stéphane Gaubert, Zheng Qu.

Dans le travail [16], on s’est intéressé à la vitesse de convergence vers l’équilibre d’une itération de la forme $x^{k+1} = T(x^k)$, $x^k \in X$, où T est une application linéaire préservant un cône dans un espace de Banach X , telle que $T(e) = e$, pour un certain vecteur e dans l’intérieur du cône. On s’intéresse aussi à l’itération dans l’espace dual, $y^{k+1} = T^*(y^k)$, $y^k \in X^*$, lorsque $\langle y^0, e \rangle = 1$.

Le cas classique est celui où $T(x) = Px$ est un opérateur de Markov. L’itération primale traduit alors la convergence vers le “consensus”, et l’itération duale traduit la convergence de la distribution de probabilité en temps k vers l’état stationnaire. Dans ce cas, le taux de contraction (en un coup) $\kappa(P)$ d’une itération primale, pour la semi-norme de Hilbert $\|z\|_H := \max_i z_i - \min_j z_j$, ainsi que le taux de contraction d’une itération duale, pour la métrique en variation totale, coïncident et sont caractérisés par une formule due à Doeblin et Dobrushin (coefficient d’ergodicité),

$$\kappa(P) := 1 - \min_{i,j} \sum_{s=1}^n \min(P_{is}, P_{js}).$$

On a donné ici une généralisation de cette formule au cas d'opérateurs abstraits, qui s'applique en particulier aux opérateurs de Kraus qui interviennent en information quantique. Ces derniers opèrent sur l'espace des matrices symétriques, et sont de la forme

$$T(x) = \sum_k a_k x a_k^* \quad \text{avec} \quad \sum_k a_k a_k^* = I .$$

Dans [114], nous avons étudié des questions de complexité pour les applications de Kraus, montrant en particulier qu'il est NP-dur de vérifier qu'une application de Kraus envoie le cône dans son intérieur.

English version

In [16], we studied the speed of convergence to equilibrium of an iteration of the form $x^{k+1} = T(x^k)$, $x^k \in X$, where T is a linear map preserving a cone in a Banach space X , such that $T(e) = e$, for some vector e in the interior of the cone. We also considered the iteration in the dual space X^* , $y^{k+1} = T^*(y^k)$, $y^k \in X^*$, where $\langle y^0, e \rangle = 1$.

The classical application arises when $T(x) = Px$ is a Markov operator. Then, the primal iteration represents the dynamics of consensus, whereas the dual iteration represents the evolution of the probability distribution as a function of time. Then, the (one-shot) contraction rate $\kappa(P)$ of the primal iteration, with respect to Hilbert's seminorm $\|z\|_H := \max_i z_i - \min_j z_j$, and the contraction rate of the dual iteration, with respect to the total variation metric, coincide, and are characterized by a formula of Doeblin and Dobrushin (ergodicity coefficient),

$$\kappa(P) := 1 - \min_{i,j} \sum_{s=1}^n \min(P_{is}, P_{js}).$$

We gave here a generalization of this formula to an abstract operators on a cone. This covers in particular the Kraus maps arising in quantum information theory. The latter maps act on the space of symmetric matrices. They can be written as

$$T(x) = \sum_k a_k x a_k^* \quad \text{with} \quad \sum_k a_k a_k^* = I .$$

In [114], we studied complexity issues related to Kraus maps, and showed in particular that checking whether a Kraus map sends the cone to its interior is NP-hard.

7.2. Algèbre linéaire max-plus, convexité tropicale et jeux à somme nulle/Max-plus linear algebra, tropical convexity and zero-sum games

7.2.1. Polyèdres tropicaux/Tropical polyhedra

Participants: Xavier Allamigeon, Stéphane Gaubert, Eric Goubault [CEA], Ricardo Katz [Conicet, Argentine].

On étudie les analogues max-plus ou tropicaux des ensembles convexes. Ceux-ci sont utiles en particulier pour représenter de manière effective les ensembles d'états accessibles de systèmes à événements discrets [9], ils sont aussi apparus récemment en géométrie tropicale, dans toute une série de travaux à la suite de Sturmfels et Develin [97]. Les polyèdres max-plus peuvent aussi être vus comme des limites de déformations de polyèdres classiques, sur lesquels ils donnent un éclairage de nature combinatoire. Toutes ces motivations ont inspiré la recherche d'analogues des résultats fondamentaux d'analyse convexe classique: séparation, projection, points extrémaux, à la suite en particulier de [8].

Dans un travail de X. Allamigeon, S. Gaubert, et E. Goubault [62], [63], on a mis en évidence un critère combinatoire pour la caractérisation des sommets des polyèdres tropicalement convexes. Celui-ci s'exprime à l'aide d'hypergraphes orientés, et de leurs composantes fortement connexes. Ce critère possède la propriété d'être vérifiable en un temps presque linéaire en la taille de l'hypergraphe.

On en déduit un analogue tropical de la méthode de la double description [63] (méthode très utilisée sur les polyèdres classiques, et due à Motzkin *et al.* [148]). Cet algorithme permet de calculer les sommets d'un polyèdre défini de façon externe (intersection de demi-espaces ou d'hyperplans tropicaux). Grâce au critère combinatoire précédent, l'algorithme améliore de plusieurs ordres de grandeur les techniques connues jusqu'alors. Ceci est confirmé par de nombreuses expérimentations. Ce travail est motivé par des applications à l'analyse statique [61] et aux systèmes à événements discrets [102], dans lesquelles la manipulation de tels polyèdres est le goulot d'étranglement.

Il est connu qu'un polyèdre tropical peut être représenté comme l'enveloppe convexe d'un ensemble minimal de points et rayons, donnés par ses sommets et ses rayons extrêmes [112]. Dans un travail réalisé par X. Allamigeon et R. Katz [64], et effectué en partie lors de visites de R. Katz à Inria, on étudie la question duale de la caractérisation des représentations minimales par demi-espaces. On montre qu'un polyèdre tropical possède *essentiellement* une unique représentation minimale par demi-espaces, lorsque leurs apex appartiennent au polyèdre. On montre que les apex de ces demi-espaces non-redondants correspondent à certains sommets du complexe tropical introduit par Develin et Sturmfels [97]. On introduit également un critère combinatoire pour l'élimination de demi-espaces redondants à l'aide d'hypergraphes orientés.

Dans un travail de X. Allamigeon et R. Katz [65], nous étudions la tropicalisation des représentations par demi-espaces des polyèdres convexes sur le corps des séries de Puiseux. Nous démontrons ainsi une conjecture de Develin et Yu [98]. Celle-ci assure qu'étant donné un polytope tropical pur, il existe un polytope *relevé* sur les séries de Puiseux, dont les demi-espaces associés aux faces se "tropicalisent" en une représentation par demi-espaces du polytope tropical initial.

Des applications de ces travaux à l'algorithmique, concernant en particulier les jeux répétés, sont discutées dans la Section 7.4.1 .

English version

We study the max-plus or tropical analogues of convex sets. These have been used in particular to represent effectively the accessible sets of certain discrete event systems [9]. They also appeared in tropical geometry, following the work of Sturmfels and Develin [97]. Max-plus polyhedra can be thought of as limits of deformations of classical polyhedra, on which they give a combinatorial insight. These motivations have inspired the investigation of analogues of basic results of classical convex analysis: separation, projection, representation by extreme points, following [8].

In a work of X. Allamigeon, S. Gaubert, and E. Goubault [63], we introduce a combinatorial criterion for the characterization of the vertices of tropically convex polyhedra. It is expressed in terms of directed hypergraphs and their strongly connected components. This criterion can be verified in almost linear time in the size of the hypergraph.

This allows to develop a tropical analogue of the double description method [63] (this method is widely used for classical convex polyhedra, and is due to Motzkin *et al.* [148]). This algorithm is able to determine all the vertices of a polyhedron defined externally (intersection of tropical half-spaces of hyperplanes). Thanks to the combinatorial criterion mentioned above, the algorithm improves the existing methods by several orders of magnitude. This is confirmed by several experiments. This is motivated by applications to static analysis [61] and discrete event systems [102], in which computing such polyhedra turns out to be the bottleneck.

It is well-known that a tropical polyhedron can be represented as the convex hull of a minimal set of points and rays, provided by its vertices and extreme rays [112]. In a work of X. Allamigeon and R. Katz [64], partly done during visits of R. Katz at Inria, the dual problem of characterizing the minimal representations by half-spaces is studied. We show that a tropical polyhedron admits *essentially* a unique minimal external representation by half-spaces, provided that their apices belong to the polyhedron. We prove that the apices of

these half-spaces correspond to certain vertices of the tropical complex introduced by Develin and Sturmfels [97]. We also establish a combinatorial criterion allowing to eliminate redundant half-spaces using directed hypergraphs.

In a work of X. Allamigeon and R. Katz [35], we study the tropicalization of the representation by half-spaces of convex polyhedra over the field of Puiseux series. In particular, we prove a conjecture of Develin and Yu [98]. It states that, given a pure tropical polytope, there exists a lifting polytope over Puiseux series, such that the facet-defining half-spaces are “tropicalized” into a representation by half-spaces of the initial polytope.

Some algorithmic applications of this work concerning in particular mean payoff games, will be discussed in Section 7.4.1 .

7.2.2. *Points fixes d’applications monotones homogènes et jeux à somme nulle/Fixed points of order preserving homogeneous maps and zero-sum games*

Participants: Marianne Akian, Stéphane Gaubert, Antoine Hochart.

Pour les jeux répétés à somme nulle, un problème de base est de savoir si le paiement moyen par unité de temps est indépendant de l’état initial. Ici, on définit le paiement moyen directement au moyen de l’opérateur de Shapley (ou de la programmation dynamique) du jeu, lequel préserve l’ordre et commute avec l’addition d’une constante. Dans le cas particulier des jeux à zero joueur, i.e. de chaînes de Markov avec fonctionnelle additive, la solution du problème ci-dessus est fournie par le théorème ergodique. Dans [11], [21], on généralise ce résultat au cas des jeux répétés à espace d’états fini. Cette généralisation est basée sur l’étude de la sous-classe d’opérateurs de Shapley *sans-paiement* (le paiement a lieu seulement le dernier jour), lesquels commutent avec la multiplication par une constante positive. L’intérêt de cette sous-classe est qu’elle inclue la fonction de récession d’un opérateur de Shapley, lorsqu’elle existe. Nous montrons que le paiement moyen est indépendant de l’état initial pour toutes les perturbations des paiements instantanés dépendantes de l’état si, et seulement si, une condition d’ergodicité est vérifiée. Cette dernière est caractérisée par l’unicité, à constante additive près, du point fixe de la fonction de récession de l’opérateur de Shapley, ou, dans le cas particulier des jeux stochastiques à nombre fini d’actions et information parfaite, par une condition d’accessibilité dans un hypergraphe orienté, entre deux sous-ensembles conjugués d’états. On montre aussi que l’ergodicité d’un jeu ne dépend que de la probabilité de transition et qu’elle peut être vérifiée en temps polynomial lorsque le nombre d’états est fixé. Dans [26], on généralise la condition d’accessibilité dans un hypergraphe orienté au cas de jeux avec espaces d’actions arbitraires.

English version

A basic question for zero-sum repeated games consists in determining whether the mean payoff per time unit is independent of the initial state. Here the mean payoff is defined in terms of the Shapley operator (dynamic programming operator) of the game, which is an order preserving map commuting with the addition of a constant. In the special case of “zero-player” games, i.e., of Markov chains equipped with additive functionals, the answer to the above question is provided by the mean ergodic theorem. In [11], [21], we generalize this result to repeated games with a finite state space. This generalization is based on the study of the subclass of *payment-free* Shapley operators (the payment only occurs when the game stops), which are commuting with the multiplication by a positive constant, and which include the recession function of any Shapley operator, when it exists. We show that the mean payoff is independent of the initial state for all state-dependent perturbations of the rewards if and only if an ergodicity condition is satisfied. The latter is characterized by the uniqueness modulo additive constants of the fixed point of the recession function of the Shapley operator, or, in the special case of stochastic games with finite action spaces and perfect information, by a reachability condition involving conjugate subsets of states in directed hypergraphs. We show that the ergodicity condition for games only depends on the support of the transition probability and that it can be checked in polynomial time when the number of states is fixed. In [26], we generalize the above reachability condition to the case of games with arbitrary actions spaces,

7.2.3. *Puissances extérieures tropicales de matrices/Tropical compound matrix identities*

Participants: Marianne Akian, Stéphane Gaubert, Adi Niv.

English version

In [43], [45], we proved some identities on matrices using a weak and strong transfer principles. In the present work, we prove identities on compound matrices in extended tropical semirings. Such identities include analogues to properties of conjugate matrices, powers of matrices and $\text{adj}(A) \det(A)^{-1}$, all of which have implications on the eigenvalues of the corresponding matrices. A tropical Sylvester-Franke identity is provided as well. Even though part of these identities hold over any commutative ring, they cannot be adjusted to semirings with symmetry using the existing weak and strong transfer principles. By reducing these identities to definite matrices, we introduce a transfer principle for formal series, allowing us to infer tropical identities from classical ones. We provide the proofs both via this wider principle and by means of graph theory arguments.

7.2.4. Matrices totalement positives tropicales/Tropical totally positive matrices

Participants: Stéphane Gaubert, Adi Niv.

English version

We investigate totally positive and totally non-negative matrices over the tropical symmetrized semiring. We show these matrices are diagonally dominant, and are determined by their 2×2 minors. We provide the role of the classical double echelon forms in the tropical setting, and find a so called “staircase” form to finite matrices. We establish the connection to the classical sets of totally positive and totally non-negative matrices through the valuation on the field of Puiseux series. In particular, we find the connection to elementary matrix factorization, positivity of eigenvalues and planar networks.

7.2.5. Algèbre supertropicale/Supertropical algebra

Participant: Adi Niv.

English version

Several properties of matrices over the tropical algebra are studied using the supertropical algebra introduced in [126].

The only invertible matrices in tropical algebra are diagonal matrices, permutation matrices and their products. However, the pseudo-inverse A^∇ , defined as $\frac{1}{\det(A)} \text{adj}(A)$, with $\det(A)$ being the tropical permanent, inherits some classical algebraic properties and has some surprising new ones. In [19], defining B and B' to be tropically similar if $B' = A^\nabla B A$, we examine the characteristic (max-)polynomials of tropically similar matrices as well as those of pseudo-inverses. Other miscellaneous results include a new proof of the identity for $\det(AB)$ and a connection to stabilization of the powers of definite matrices.

In a joint work with Louis Rowen (Bar Ilan Univ.) [37], we study the pathology that causes tropical eigenspaces of distinct supertropical eigenvalues of a non-singular matrix A , to be dependent. We show that in lower dimensions the eigenvectors of distinct eigenvalues are independent, as desired. The index set that differentiates between subsequent essential monomials of the characteristic polynomial, yields an eigenvalue λ , and corresponds to the columns of the eigenmatrix $A + \lambda I$ from which the eigenvectors are taken. We ascertain the cause for failure in higher dimensions, and prove that independence of the eigenvectors is recovered in case the “difference criterion” holds, defined in terms of disjoint differences between index sets of subsequent coefficients. We conclude by considering the eigenvectors of the matrix $A^\nabla := \frac{1}{\det(A)} \text{adj}(A)$ and the connection of the independence question to generalized eigenvectors.

In a joint work with Zur Izhakian (University of Aberdeen) and Louis Rowen (Bar Ilan Univ.) [36], extending earlier work on supertropical adjoints and applying symmetrization, we provide a symmetrized supertropical version SL_n of the special linear group, which we partition into submonoids, based on “quasi-identity” matrices, and we display maximal sub-semigroups of SL_n . We also study the monoid generated by SL_n . Several illustrative examples are given of unexpected behavior. We describe the action of elementary matrices on SL_n , which enables one to connect different matrices in SL_n , but in a weaker sense than the classical situation.

7.3. Algèbre max-plus, déformations et asymptotiques /Max-plus algebra, deformations and asymptotic analysis

7.3.1. Introduction

Comme indiqué dans le §3.7 , l'algèbre max-plus est la limite d'une déformation de l'algèbre classique, ou plutôt du semi-corps des réels positifs. Elle peut aussi fournir des estimations de ces déformations, puisque

$$\max(a, b) \leq \epsilon \log(e^{a/\epsilon} + e^{b/\epsilon}) \leq \epsilon \log(2) + \max(a, b) . \quad (4)$$

L'utilisation de ces propriétés a déjà conduit dans le passé aux travaux sur les perturbations de valeurs propres [42], [41], [40], ou sur les grandes déviations [1], [47]. Dans les travaux qui suivent, nous exploitons ces propriétés dans des contextes reliés ou similaires à ceux de nos travaux précédents.

English version

As detailed in §3.7 , max-plus algebra is the limit of a deformation of classical algebra, or more precisely of the semi-field of usual real positive numbers. It can also give estimations for these deformations using for instance (12) . By using these properties, we already obtained some works on singular perturbations of matrix eigenvalues [42], [41], [40], or on large deviations [1], [47]. In the works described below, we are exploiting again these properties in contexts that are related or similar to those of our earlier works.

7.3.2. Méthodes tropicales de localisation de valeurs propres de matrices/Tropical methods for the localisation of matrix eigenvalues

Participants: Marianne Akian, Stéphane Gaubert, Andrea Marchesini.

Dans un travail avec Meisam Sharify [50], on a comparé les modules des valeurs propres d'un polynôme matriciel au moyen des racines tropicales du polynôme obtenu en appliquant une norme donnée aux coefficients. En particulier, on a obtenu des inégalités de type majorisation qui généralisent les bornes obtenues par Polya et Ostrowski dans le cas de polynômes scalaires.

Une partie de la thèse d'Andrea Marchesini, présentée dans [49], montre des inégalités de type majorisation entre les modules des valeurs propres d'une matrice et les valeurs propres tropicales de la matrice de ses modules. En particulier, les majorations généralisent l'inégalité de Friedland [108] concernant le rayon spectral.

Nous avons aussi amélioré et généralisé ces inégalités [27], en appliquant différents changements de variables diagonaux à la matrice complexe initiale, lesquels sont construits à partir des variables duales du problème d'affectation optimale paramétrique construit à partir d'une matrice tropicale associée à la matrice complexe. En particulier, lorsqu'on les applique à une matrice companion par blocs, ces inégalités sont similaires à celles de [50].

English version

In a work with Meisam Sharify [50], we compared the moduli of the eigenvalues of a matrix polynomial to the tropical roots of a polynomial obtained by applying a norm to the coefficients of the original matrix polynomial. In particular, we obtained majorization type inequalities which generalize the bounds of Polya and Ostrowski available for scalar polynomials.

One part of the thesis of Andrea Marchesini, presented in [49], shows majorization type inequalities between the moduli of the eigenvalues of a complex matrix and the tropical eigenvalues of the matrix obtained by applying the modulus entrywise. In particular, the upper bounds generalize the inequality of Friedland [108] concerning the spectral radius. The above inequalities were obtained by using the permanent and tropical analogues of the exterior power of a matrix and by showing (combinatorially) properties of their eigenvalues similar to the ones of usual exterior powers.

We also improved and generalized these inequalities, see [27], by applying to the original complex matrix, different diagonal scalings constructed from the dual variables of the parametric optimal assignment constructed from an associated tropical matrix. In particular, when applied to a block companion matrix, our inequalities are similar to the ones in [50].

7.3.3. Méthodes tropicales pour le calcul numérique de valeurs propres de matrices/Tropical methods for the numerical computation of matrix eigenvalues

Participants: Marianne Akian, Stéphane Gaubert, Andrea Marchesini.

Un des buts de la thèse d'Andrea Marchesini était d'utiliser les résultats de localisation de valeurs propres tels que ceux obtenus ci-dessus pour améliorer la précision des algorithmes de calcul numérique de valeurs propres de matrices ou de polynômes matriciels, en particulier en construisant des changements d'échelle exploitant les calculs tropicaux, à effectuer préalablement à l'appel d'algorithmes classiques comme QZ. Le "changement d'échelle tropical" introduit par Stéphane Gaubert et Meisam Sharify [115] dans le cas de polynôme matriciels quadratiques consiste en un changement de variable multiplicatif de la variable scalaire du polynôme matriciel. Dans la deuxième partie de la thèse d'Andrea Marchesini, en collaboration avec Françoise Tisseur de l'Université de Manchester [22], on considère un changement de variables diagonal du polynôme matriciel construit à partir des variables duales du problème d'affectation optimale paramétrique construit dans l'esprit de [40], [34]. On montre l'intérêt de ces changements d'échelle en terme de conditionnement des valeurs propres, et la supériorité du changement de variables diagonal par rapport au changement d'échelle tropical.

English version

One of goals of the PhD thesis of Andrea Marchesini was to use results on the localisation of eigenvalues like the above ones, to improve the accuracy of the numerical computation of the eigenvalues of a complex matrix or matrix polynomial, in particular by applying scaling methods using tropical techniques, which may be used before calling usual algorithms as QZ. The "tropical scaling" introduced by Stéphane Gaubert and Meisam Sharify [115] in the case of quadratic matrix polynomials consists in a multiplicative scaling of the scalar variable of the matrix polynomial. In the second part of the PhD thesis of Andrea Marchesini, corresponding to a work with Françoise Tisseur from Manchester University [22], we consider a diagonal scaling of the matrix polynomial constructed from the dual variables of the parametric optimal assignment constructed in the same spirit as in [40], [34]. We show the interest of these scaling methods on the eigenvalue conditioning, and the superiority of the diagonal scaling with respect to the tropical scaling.

7.3.4. Tropicalisation du chemin central, et application à la courbure/Tropicalization of the central path and application to the curvature

Participants: Xavier Allamigeon, Pascal Benchimol, Stéphane Gaubert, Michael Joswig [TU Berlin].

En optimisation, une classe importante d'algorithmes, dits *de points intérieurs*, consiste à suivre une courbe appelée *chemin central* jusqu'à atteindre la solution optimale. Le chemin central d'un programme linéaire $LP(A, b, c) \equiv \min\{c \cdot x \mid Ax \leq b, x \geq 0\}$ est défini comme l'ensemble des solutions optimales (x^μ, w^μ) des problèmes à barrière logarithmique:

$$\begin{aligned} \text{minimiser} \quad & c \cdot x - \mu \left(\sum_{j=1}^n \log x_j + \sum_{i=1}^m \log w_i \right) \\ \text{sous les contraintes} \quad & Ax + w = b, \quad x > 0, \quad w > 0 \end{aligned}$$

Les performances d'un algorithme de point intérieur sont intimement liées à la forme du chemin central. En particulier, la courbure mesure de combien un chemin diffère d'une ligne droite. Intuitivement, un chemin central à forte courbure devrait être plus difficile à approximer par des segments de droites, ce qui suggère davantage d'itérations des algorithmes de points intérieurs. La courbure totale du chemin central a été étudiée par Dedieu, Malajovich et Shub [94] à travers le théorème de Bezout dans le cas multihomogène, et par De

Loera, Sturmfels and Vinzant [93] à l'aide de la théorie des matroïdes. Ces deux travaux fournissent une borne supérieure en $O(n)$ sur la courbure totale moyenne sur l'ensemble des régions formées par l'arrangement d'hyperplans en dimension n . Le cube de Klee-Minty redondant de [100] et le "serpent" de [99] sont des instances qui montrent que la courbure totale peut être de l'ordre de $\Omega(m)$ pour un polytope défini par m inégalités.

Dans un travail de X. Allamigeon, P. Benchimol, S. Gaubert, and M. Joswig, nous avons étudié la tropicalisation du chemin central. Le *chemin central tropical* est défini comme la limite logarithmique des chemins centraux d'une famille paramétrique de programmes linéaires $\text{LP}(A(t), b(t), c(t))$, où les entrées $A_{ij}(t)$, $b_i(t)$ and $c_j(t)$ sont des fonctions définissables dans une structure o-minimale appelée *corps de Hardy*.

Une première contribution a été de fournir une caractérisation entièrement géométrique du chemin central tropical. Nous avons montré que le centre analytique est donné par le plus grand élément de l'ensemble des points tropicaux admissibles. De plus, tout point du chemin central tropical coïncide avec le plus grand élément de l'ensemble admissible tropical intersecté avec un ensemble de sous-niveau de la fonction de coût tropicale.

Grâce à cette caractérisation, nous avons réfuté l'analogue continu de la conjecture de Hirsch proposé par Deza, Terlaky et Zinchenko. Ainsi, nous avons construit une famille de programmes linéaires définis par $3r + 4$ inégalités in dimension $2r + 2$, où le chemin central a une courbure totale en $\Omega(2^r)$. Cette famille est obtenue en relevant des programmes linéaires tropicaux qui proviennent d'une construction de Bezem, Nieuwenhuis et Rodríguez-Carbonell [73]. Afin d'estimer la courbure dans ce contre-exemple, nous introduisons une notion d'angle combinatoire, qui est de nature tropicale. Cela nous permet de définir un analogue combinatoire de la courbure totale qui fournit un minorant de la courbure totale classique.

Ces résultats sont rassemblés dans le document [60].

English version

In optimization, path-following interior point methods are driven to an optimal solution along a trajectory called the central path. The *central path* of a linear program $\text{LP}(A, b, c) \equiv \min\{c \cdot x \mid Ax \leq b, x \geq 0\}$ is defined as the set of the optimal solutions (x^μ, w^μ) of the barrier problems:

$$\begin{aligned} & \text{minimize} && c \cdot x - \mu \left(\sum_{j=1}^n \log x_j + \sum_{i=1}^m \log w_i \right) \\ & \text{subject to} && Ax + w = b, \quad x > 0, \quad w > 0 \end{aligned}$$

The performance of an interior point method is tightly linked to the shape of its central path. In particular, the curvature measures how far a path differs from a straight line. Intuitively, a central path with high curvature should be harder to approximate with line segments, and thus this suggests more iterations of the interior point methods. The total curvature of the central path has been studied by Dedieu, Malajovich and Shub [94] via the multihomogeneous Bézout Theorem and by De Loera, Sturmfels and Vinzant [93] using matroid theory. These two papers provide an upper bound of $O(n)$ on the total curvature averaged over all regions of an arrangement of hyperplanes in dimension n . The redundant Klee-Minty cube of [100] and the "snake" in [99] are instances which show that the total curvature can be in $\Omega(m)$ for a polytope described by m inequalities. By analogy with the classical Hirsch conjecture, Deza, Terlaky and Zichenko [99] conjectured that $O(m)$ is also an upper bound for the total curvature.

In a work of X. Allamigeon, P. Benchimol, S. Gaubert, and M. Joswig, we have studied the tropicalization of the central path. The *tropical central path* is defined as the logarithmic limit of the central paths of a parametric family of linear programs $\text{LP}(A(t), b(t), c(t))$, where the entries $A_{ij}(t)$, $b_i(t)$ and $c_j(t)$ are definable functions in an o-minimal structure called the *Hardy field*.

A first contribution is to provide a purely geometric characterization of the tropical central path. We have shown that the tropical analytic center is the greatest element of the tropical feasible set. Moreover, any point of the tropical central path is the greatest element of the tropical feasible set intersected with a sublevel set of the tropical objective function.

Thanks to this characterization, we disprove the continuous analog of the Hirsch conjecture proposed by Deza, Terlaky and Zinchenko, by constructing a family of linear programs with $3r + 4$ inequalities in dimension $2r + 2$ where the central path has a total curvature in $\Omega(2^r)$. This family is gotten by lifting tropical linear programs which come from a construction of Bezem, Nieuwenhuis and Rodríguez-Carbonell [73]. In order to estimate the curvature in this counter example, we introduce a notion of a combinatorial angle, which is tropical in nature. This allows us to define a combinatorial analogue of the total curvature which provides a lower bound for the classical total curvature.

These results are gathered in the preprint [60].

7.3.5. *Etude des ensembles semi-algébriques tropicaux/Tropicalization of semi-algebraic sets*

Participants: Xavier Allamigeon, Stéphane Gaubert, Mateusz Skomra.

Suite à son stage de M2, Mateusz Skomra a débuté en octobre 2015 une thèse sous la direction de Xavier Allamigeon et Stéphane Gaubert portant sur la tropicalisation des ensembles semi-algébriques. Cette thèse est financée par une bourse de la région Ile-de-France. La thèse de Mateusz vise en particulier à étudier l'analogie tropical de la programmation sur le cône des matrices positives, dans le but d'obtenir de nouveaux algorithmes ou résultats de complexité pour des sous-classes de problèmes.

English version

Following his M2 internship, Mateusz Skomra has started in October 2015 a PhD thesis under the supervision of Xavier Allamigeon and Stéphane Gaubert, on the tropicalization of semi-algebraic sets. This thesis is funded by a grant from Ile-de-France. One goal is to study the tropical analogue of semidefinite programming, in order to define new algorithms or to establish new complexity results for some subclasses of problems.

7.4. Algorithmes/Algorithms

7.4.1. *Algorithmique des polyèdres tropicaux/Algorithmics of tropical polyhedra*

Participants: Xavier Allamigeon, Pascal Benchimol, Stéphane Gaubert, Michael Joswig [TU Berlin].

Dans un travail de X. Allamigeon, P. Benchimol, S. Gaubert et M. Joswig [13], nous avons défini un analogue tropical de l'algorithme du simplexe qui permet de résoudre les problèmes de *programmation linéaire tropicale*, i.e.

$$\begin{aligned} & \text{minimiser} && \max_{1 \leq j \leq n} c_j + x_j \\ & \text{sous les contraintes} && \max(\max_{1 \leq j \leq n} (a_{ij}^+ + x_j), b_i^+) \geq \max(\max_{1 \leq j \leq n} (a_{ij}^- + x_j), b_i^-), \quad i = 1, \dots, m \\ & && x \in (\mathbb{R} \cup \{-\infty\})^n \end{aligned} \quad (5)$$

où les entrées du programme a_{ij}^\pm, b_i^\pm, c_j sont à valeur dans $\mathbb{R} \cup \{-\infty\}$. Ces problèmes sont intimement liés à la résolution de jeux répétés à somme nulle, puisque résoudre un jeu à paiement moyen déterministe est équivalent à déterminer si un problème de programmation linéaire admet un point réalisable [44].

Comme son homologue usuel, le simplexe tropical pivote entre des points de base (tropicaux), jusqu'à atteindre l'optimum du programme linéaire. La différence fondamentale avec l'algorithme du simplexe classique est que le pivotage est réalisé de manière purement combinatoire, en s'appuyant sur des descriptions locales du polyèdre tropical défini par les contraintes à l'aide d'(hyper)graphes orientés. Ceci nous a permis de prouver que *l'étape de pivotage (incluant le calcul des coûts réduits) a la même complexité en temps que dans l'algorithme classique, i.e. $O(n(m+n))$* . Ceci est d'autant plus inattendu que la structure des arêtes tropicales entre deux points de base sont géométriquement plus complexes (elles sont constituées de plusieurs segments de droite, jusqu'à n).

Le simplexe tropical a la propriété d'être fortement corrélé avec l'algorithme du simplexe classique. Grâce au principe de Tarski, le simplexe usuel peut être transposé tel quel sur des programmes linéaires dont les coefficients en entrée sont non plus des réels, mais sur le corps $\mathbb{R}\{\{t\}\}$ des séries de Puiseux généralisées en une certaine indéterminée t , *i.e.* des objets de la forme :

$$c_{\alpha_1} t^{\alpha_1} + c_{\alpha_2} t^{\alpha_2} + \dots \quad (6)$$

où les α_i sont des réels, les coefficients c_{α_i} sont des réels non-nuls, et où la séquence des $\alpha_1, \alpha_2, \dots$ est strictement croissante et soit finie, soit non-bornée. L'opposé du plus petit exposant de la série, $-\alpha_1$, est appelé *valuation* de la série. Un programme linéaire tropical est dit *relevé* en un problème linéaire sur $\mathbb{R}\{\{t\}\}$, si la valuation des coefficients en entrée de ce dernier sont égaux aux coefficients du problème tropical. Dans nos travaux, nous avons établi la correspondance suivante entre le simplexe usuel et le simplexe tropical : *pour tout programme linéaire tropical générique, l'algorithme du simplexe tropical trace l'image par la valuation du chemin sur l'algorithme du simplexe usuel sur n'importe quel relèvement du programme tropical dans $\mathbb{R}\{\{t\}\}$.*

Les résultats présentés ci-dessus sont rassemblés dans l'article [13]. Ils ont fait l'objet de plusieurs présentations en conférence [54], [55] [59].

Ces résultats ouvrent la possibilité de relier la complexité du l'algorithme du simplexe usuel avec celles des jeux déterministes. Pour ces derniers, on sait seulement que leur résolution est dans la classe de complexité $NP \cap coNP$, et on ignore s'il existe un algorithme de complexité polynomiale. De façon similaire, on ne sait pas caractériser de façon précise la complexité de l'algorithme du simplexe usuel. Celle-ci dépend fortement de la règle de pivotage utilisée, et il existe des problèmes sur lesquelles de nombreuses règles de pivotage ont une complexité exponentielle. L'existence d'une règle de pivotage qui permettrait au simplexe de terminer en temps polynomial sur n'importe quelle instance est encore aujourd'hui une question ouverte.

Dans un deuxième travail, nous avons relié les deux problèmes ouverts précédents, grâce à l'algorithme du simplexe tropical. Nous avons en effet exhibé une classe de règles de pivotage, dites *combinatoires*, et avons montré qu'elles satisfont la propriété suivante : *s'il existe une règle de pivotage combinatoire qui permet de résoudre tout problème de programmation linéaire usuel en temps polynomial, alors on peut résoudre les jeux à paiement moyen en temps (fortement) polynomial.* Le terme *combinatoire* fait référence au fait que la règle est définie en fonction du signe des mineurs de la matrice des coefficients du problème linéaire. Ce résultat est décrit dans l'article [56], et a été présenté dans plusieurs conférences [57], [58].

Enfin, dans un travail de X. Allamigeon, P. Benchimol et S. Gaubert [53], nous avons étendu les résultats aux règles de pivotage *semi-algébriques*, classe incluant la règle dite du *shadow-vertex*. Celle-ci est connue pour avoir fourni plusieurs bornes de complexité moyenne et lisse sur l'algorithme du simplexe. Nous avons donc tropicalisé l'algorithme du simplexe shadow-vertex, et nous avons montré que cet algorithme permet de résoudre les jeux à paiement moyen en temps polynomial en moyenne.

English version

In an ongoing work of X. Allamigeon, P. Benchimol, S. Gaubert and M. Joswig, we introduced a tropical analogue of the simplex algorithm, allowing one to solve problems of *tropical linear programming*, which are of the form (13), where the coefficients of the program, a_{ij}^\pm, b_i^\pm, c_j take their values in the max-plus semiring $\mathbb{R} \cup \{-\infty\}$. These problems are closely related to mean payoff games, as solving a game of this kind is equivalent to determine whether a tropical linear program admits a feasible point [44].

Like the classical simplex algorithm, the tropical simplex algorithm performs pivoting operations between basis points, until it reaches the optimum. The main discrepancy with the classical algorithm is that the pivoting is now a purely combinatorial operation, which is performed by using a local description of the polyhedron by a directed hypergraph. This allowed us to show that *a tropical pivoting step (including computing reduced costs) has the same complexity as in the classical simplex algorithm, i.e. $O(n(m+n))$.* This is all the more

surprising as the tropical edge between two given points has a geometrically more complex structure in the tropical case (it is constituted of up to n ordinary line segments).

The tropical simplex algorithm turns out to be closely related to the classical one. Thanks to Tarski's principle, the latter is also valid for linear programs over the field $\mathbb{R}\{\{t\}\}$ of generalized Puiseux series in an indeterminate t . These series are of the form (14), where the α_i are real numbers, the coefficients c_{α_i} are non-zero reals, and the sequence $\alpha_1, \alpha_2, \dots$ is strictly increasing and either finite or unbounded. The opposite of the smallest exponent of the series, $-\alpha_1$, is called *valuation*. A tropical linear program is said to be *lifted* to a linear program over $\mathbb{R}\{\{t\}\}$ if the valuation of the coefficients of the latter are sent to the coefficients of the former by the valuation. We showed the following relation between the classical simplex algorithm and its tropical analogue: *for all generic tropical linear program, the tropical simplex algorithm computes the image by the valuation of the path of the classical simplex algorithm, applied to any lift in $\mathbb{R}\{\{t\}\}$ of the original program.*

These results are gathered in the article [13]. They have been presented in several conferences [54], [55] [59].

They allow one to relate the complexity of the classical simplex algorithm with the complexity of mean payoff games. The latter is unsettled, these games are known to be in the class $\text{NP} \cap \text{coNP}$ but it is not known whether they can be solved in polynomial time. Basic complexity issues regarding the classical simplex algorithm are also unsettled: its execution time depends on the pivoting rule, and many pivoting rules have been shown to have exponential worst case behaviors. The existence of a pivoting rule leading the simplex to terminate in polynomial time is still an open question. In a second work, we related these two open questions, via the tropical simplex algorithm. We identified a class of pivoting rules, which are said to be *combinatorial*, and show that they have the following property: *if there is a combinatorial pivoting rule allowing one to solve every classical linear programming problem in polynomial time, then, mean payoff games can be solved in (strongly) polynomial time.* By *combinatorial*, we mean that the rule depends only of the coefficients of the system through the signs of minors of the coefficients matrix. This result is given in the article [56]. It has been presented to the conferences [57], [58].

Finally, in a work of X. Allamigeon, P. Benchimol and S. Gaubert [53], we extended the latter results to *semi-algebraic* pivoting rules, which include the so-called *shadow-vertex* rule. This rule has been exploited in the literature to establish several average-case and smooth complexity bounds on the simplex algorithm. We tropicalized the shadow-vertex simplex algorithm, and showed that it solves mean payoff games in polynomial time on average.

7.4.2. *Approximation max-plus de fonctions valeurs et équations de Riccati généralisées/Max-plus approximation of value functions and generalized Riccati equations*

Participants: Stéphane Gaubert, Zheng Qu, Srinivas Sridharan.

Le travail de thèse de Zheng Qu, supervisée par S. Gaubert et S. Tang, a porté sur le développement de méthodes tropicales en programmation dynamique approchée [154]. Celle-ci permettent d'atténuer la malédiction de la dimension, pour certaines classes de problèmes de contrôle optimal.

Un développement de ce travail est paru dans [17], où il est montré qu'une classe de relaxations convexes introduites par Sridharan et al. pour traiter numériquement un problème de contrôle quantique sont en fait exactes (pas de saut de relaxation).

English version

The PhD work of Zheng Qu, supervised by S. Gaubert and S. Tang, dealt with the développement of tropical methods in approximate dynamic programming [154]. These allow one to attenuate the curse of dimensionality for certain optimal control problems.

A development of this work appeared in [17]. It is shown there that a class of convex relaxations introduced Sridharan et al. to solve numerically some quantum control problem is exact.

7.4.3. Approximation probabiliste d'équations d'Hamilton-Jacobi-Bellman et itération sur les politiques

Participants: Marianne Akian, Eric Fodjo.

La thèse d'Eric Fodjo traite de problèmes de contrôle stochastique (de diffusions) issus en particulier de problèmes de gestion de portefeuille avec coûts de transaction. La programmation dynamique conduit à une équation aux dérivées partielles d'Hamilton-Jacobi-Bellman, sur un espace de dimension au moins égale au nombre d'actifs risqués. La malédiction de la dimension ne permet pas de traiter numériquement ces équations en dimension grande (supérieure à 5). On se propose d'aborder ces problèmes avec des méthodes numériques associant itération sur les politiques, discrétisations probabilistes, et discrétisations max-plus, afin d'essayer de monter plus en dimension. Une autre piste est de remplacer l'itération sur les politiques par une approximation par des problèmes avec commutations optimales.

Nous considérons actuellement des équations d'Hamilton-Jacobi-Bellman fortement non-linéaires associées à des problèmes de contrôle de diffusions faisant intervenir un contrôle discret (prenant un nombre fini de valeurs) et éventuellement un contrôle continu. On construit un algorithme numérique probabiliste de faible complexité, en combinant les propriétés de distributivité idempotente obtenues par McEneaney, Kaise et Han [128], [145] pour le même type d'équations et la méthode numérique probabiliste proposée par Fahim, Touzi et Warin [104] pour résoudre des équations d'Hamilton-Jacobi-Bellman fortement non-linéaires, lorsque la volatilité ne varie pas trop.

English version

The PhD thesis of Eric Fodjo concerns stochastic control problems obtained in particular in the modelisation of portfolio selection with transaction costs. The dynamic programming method leads to a Hamilton-Jacobi-Bellman partial differential equation, on a space with a dimension at least equal to the number of risky assets. Curse of dimensionality does not allow one to solve numerically these equations for a large dimension (greater to 5). We propose to tackle these problems with numerical methods combining policy iterations, probabilistic discretisations, max-plus discretisations, in order to increase the possible dimension. Another solution is to replace policy iterations by an approximation with optimal switching problems.

Our current work concerns fully nonlinear Hamilton-Jacobi-Bellman equations associated to diffusion control problems with finite horizon involving a finite set-valued (or switching) control and possibly a continuum-valued control. We construct a lower complexity probabilistic numerical algorithm by combining the idempotent expansion properties obtained by McEneaney, Kaise and Han [128], [145] for solving such problems with a numerical probabilistic method such as the one proposed by Fahim, Touzi and Warin [104] for solving some fully nonlinear parabolic partial differential equations, when the volatility does not oscillate too much.

7.5. Applications

7.5.1. Introduction

Nous présentons maintenant plusieurs travaux de nature appliquée, touchant à des domaines variés, dans lesquels nous exploitons certaines des techniques mathématiques présentées précédemment, et particulièrement celles qui relèvent de la théorie de Perron-Frobenius non-linéaire et de la convexité tropicale. Ces applications utilisent aussi des techniques d'algèbre linéaire ou d'optimisation convexe.

English version

In this section, we describe several applied works in which we use some of the theoretical tools developed by the team, including non-linear Perron-Frobenius theory and tropical convexity. Some of these applications also make an intensive use of linear algebraic and convex programming methods.

7.5.2. Preuve formelle d'inégalités non-linéaires/Formal proofs of non-linear inequalities

Participants: Xavier Allamigeon, Stéphane Gaubert, Victor Magron, Benjamin Werner [LIX].

La thèse de Victor Magron [140], dirigée par Benjamin Werner, codirigée par Stéphane Gaubert et Xavier Allamigeon, a porté sur la certification de bornes inférieures de fonctions multivariées à valeurs réelles, définies par des expressions semi-algébriques ou transcendentes, et sur la preuve de validité de celles-ci au moyen de certificats dans l'assistant de preuves Coq.

L'un des développements de ce travail est paru dans [18],

English version

The PhD work of Victor Magron [140], supervised by Benjamin Werner, and cosupervised by Stéphane Gaubert and Xavier Allamigeon, dealt with the certification of lower bounds for multivariate functions, defined by semi-algebraic or transcendental expressions, and their correctness proof through certificates checked in the Coq proof assistant. A development of this work appeared in [18]

7.5.3. Géométrie de l'ordre de Loewner et application au calcul d'invariants quadratiques en analyse statique de programme/Geometry of the Loewner order and application to the synthesis of quadratic invariants in static analysis of program

Participants: Xavier Allamigeon, Stéphane Gaubert, Éric Goubault [LIX], Sylvie Putot [LIX], Nikolas Stott.

Nous introduisons un nouveau domaine abstrait numérique reposant sur les ellipsoïdes pour la vérification formelle de systèmes linéaires switchés. La nouveauté de ce domaine ne réside pas dans l'utilisation des ellipsoïdes en tant qu'abstraction, mais dans le fait que nous dépassons deux difficultés clés qui ont jusqu'à maintenant limité l'utilisation des ellipsoïdes en interprétation abstraite. La première difficulté est que l'ensemble des ellipsoïdes ne constitue pas un treillis. Par conséquent, il n'y a pas a priori de choix canonique pour l'abstraction de l'union de deux ensembles, ce qui rend l'analyse moins prévisible (elle dépend du choix de "bonnes" bornes supérieures). La seconde difficulté est que les travaux récents utilisant les ellipsoïdes reposent sur des méthodes à base d'inégalités linéaires matricielles (LMI). Ces dernières sont efficaces sur des exemples de taille modérée, mais elles sont limitées par la complexité des algorithmes de points intérieurs. Ceux-ci ne passent pas aussi bien à l'échelle dans le cas des LMI que dans le cas de la programmation linéaire ou du second ordre.

Plus précisément, nous réduisons la question de l'abstraction de l'union de deux ensembles par une ellipsoïde à la sélection d'une borne inférieure de deux matrices positives pour l'ordre de Löwner. Nous montrons qu'il existe une unique procédure de sélection ayant la propriété d'être invariante par transformation linéaire des variables de programmes. Nous montrons que la borne inférieure ainsi sélectionnée peut être calculée en $O(n^3)$ opérations arithmétiques. Nous établissons aussi que cette borne inférieure coïncide avec l'ellipsoïde de volume minimal, si bien que, de façon surprenante, deux approches distinctes et naturelles mènent à la même sélection. Par ailleurs, nous montrons qu'un invariant ellipsoïdal peut être calculé de manière efficace. Notre algorithme est fondé sur une généralisation non-linéaire de l'algorithme power, utilisé habituellement pour calculer la plus grande valeur propre d'une matrice. Nous illustrons notre approche en l'appliquant à des exemples de systèmes linéaires switchés. Nous montrons que l'algorithme power conduit à des gains en temps de calcul très importants par rapport aux méthodes de type LMI, au prix d'une perte de précision limitée.

Ce travail est décrit dans l'article [29], qui a reçu le prix du meilleur article à la conférence EMSOFT 2015.

English version

We introduce a new numerical abstract domain based on ellipsoids designed for the formal verification of switched linear systems. The novelty of this domain does not consist in the use of ellipsoids as abstractions, but rather in the fact that we overcome two key difficulties which so far have limited the use of ellipsoids in abstract interpretation. The first issue is that the ordered set of ellipsoids does not constitute a lattice. This implies that there is a priori no canonical choice of the abstraction of the union of two sets, making the analysis less predictable as it relies on the selection of good upper bounds. The second issue is that most recent works using on ellipsoids rely on LMI methods. The latter are efficient on moderate size examples but they are inherently limited by the complexity of interior point algorithms, which, in the case of matrix inequality problems, do not scale as well as for linear programming or second order cone programming problems.

In more details, we reduce the question of abstracting by an ellipsoid the union of two sets to the selection of a minimal upper bound of two positive semidefinite matrices with respect to the Löwner order. We show that there is a unique selection procedure which has the property of being invariant with respect to linear transformations of the program variables. We show that the minimal upper bound can be computed with the same cost as performing a Cholesky decomposition, i.e., in $O(n^3)$ arithmetic operations. We also establish that it coincides with the minimal volume ellipsoid, so that, surprisingly, two distinct natural approaches lead to the same choice of selection. Moreover, we show that an invariant ellipsoid can be computed by a scalable algorithm. This is based on a non-linear generalization of the power algorithm which is classically used to compute the dominant eigenvalue of a matrix. We illustrate our approach by applying it to examples of switched linear systems. We show that the power iteration leads to important speedups by comparison with LMI based methods, at the price of a limited loss of precision.

This work is described in the article [29], and won the best paper award at the conference EMSOFT 2015.

7.5.4. *Optimisation de l'affectation temps réel des moyens de secours des pompiers/Optimization of the real time assignment of firemen vehicles*

Participants: Marianne Akian, Xavier Allamigeon, Vianney Boeuf, Stéphane Gaubert, Stéphane Raclot [BSPP].

La thèse de Vianney Boeuf est effectuée en partenariat avec la Brigade de sapeurs pompiers de Paris (BSPP). Elle est motivée par l'optimisation des moyens de secours, en incluant les questions de dimensionnement et d'affectation temps réel des moyens. On s'intéresse en particulier à l'affectation des engins et véhicules de secours, éventuellement empruntés à différentes casernes. Ce travail intervient en complément du travail de l'équipe au sein du projet ANR Democrite, qui porte sur l'évaluation du risque en milieu urbain.

English version

The PhD work of Vianney is carried out with the Brigade of Paris Firemen (BSPP). It is motivated by the issue of optimization of emergency resources, including the real time dynamic assignment of engines or emergency vehicles. This work is carried out in complement to the ANR project Democrite, dealing with risk evaluation in urban environment.

7.5.5. *Analyse de performance d'un centre de réception des appels d'urgence fondée sur les systèmes polynomiaux tropicaux/Performance evaluation of an emergency call center based on tropical polynomial systems*

Participants: Xavier Allamigeon, Vianney Boeuf, Stéphane Gaubert, Stéphane Raclot [BSPP], Régis Reboul [PP].

Ce travail a pris sa source dans un problème d'évaluation de performance et de dimensionnement, posé par Régis Reboul (Préfecture de Police), portant sur l'analyse de l'évolution projetée de la procédure de traitement des appels d'urgence (17-18-112). Ce travail a aussi bénéficié de l'appui du LtL Stéphane Raclot (BSPP).

Il a amené à la contribution suivante. Nous introduisons une méthode algébrique qui permet d'analyser les performances de systèmes mettant en jeu des priorités et modélisés par des réseaux de Petri. Cette méthode s'applique à la classe de réseaux de Petri dans lesquels les places peuvent être partitionnées en deux catégories : le routage dans certaines places est sujet à des règles de priorité, tandis que le routage dans les autres places est à choix libre.

Nous montrons que les variables compteurs, qui déterminent le nombre de tirage des transitions comme une fonction du temps, sont les solutions d'un système dynamique linéaire par morceaux. Par ailleurs, nous établissons que dans le modèle fluide, les régimes stationnaires sont précisément les solutions d'un ensemble d'équations linéaires par morceaux et lexicographiques, qui constituent un système polynomial sur un semi-corps tropical (min-plus) de germes.

En substance, ce résultat montre que calculer les régimes stationnaires se réduit à la résolution d'un système polynomial tropical. Ceci est l'un des problèmes de base en géométrie tropicale. Cette interprétation fournit des informations sur la nature des solutions ainsi que des algorithmes. En particulier, l'approche tropicale permet de déterminer les différentes phases de congestion du système.

Nous appliquons ensuite cette approche à un cas d'étude relié au projet actuel de la Préfecture de Police de Paris et la Brigade de sapeurs-pompiers de Paris sur la mise en place d'une nouvelle organisation de réception des appels 17/18/112 sur Paris et sa petite couronne. Nous introduisons pour cela un modèle simplifié d'organisation de réception des appels, et nous nous concentrons sur l'analyse d'une fonctionnalité clé de cette organisation : la procédure de réception des appels à deux niveaux. Les opérateurs de niveau 1 reçoivent les appels, les qualifient en fonction de leur urgence, gèrent les appels non-urgents, et transfèrent les appels urgents à des opérateurs de niveau 2 spécialisés et qui complètent l'instruction. Nous résolvons le système d'équations polynomiales tropicales correspondant, et obtenons un calcul explicite des différents phases de congestion en fonction du rapport entre les nombres d'opérateurs de niveau 2 et 1. Nos résultats analytiques sont obtenus pour le modèle fluide. Cependant, ils sont confirmés par des simulations dans lesquelles la sémantique initiale du réseau de Petri est utilisée.

Ce travail a fait l'objet de la publication [28] à la conférence FORMATS 2015.

English version

This work arose from a question raised by Régis Rebol (Préfecture de Police), regarding the analysis of the projected evolution of the treatment of emergency calls (17-18-112). This work also benefited from the help of LtL Stéphane Raclot (BSPP).

It led to the following contribution. We introduce an algebraic approach which allows to analyze the performance of systems involving priorities and modeled by timed Petri nets. Our results apply to the class of Petri nets in which the places can be partitioned in two categories: the routing in certain places is subject to priority rules, whereas the routing at the other places is free choice.

We show that the counter variables, which determine the number of firings of the different transitions as a function of time, are the solutions of a piecewise linear dynamical system. Moreover, we establish that in the fluid model, the stationary regimes are precisely the solutions of a set of lexicographic piecewise linear equations, which constitutes a polynomial system over a tropical (min-plus) semifield of germs.

In essence, this result shows that computing stationary regimes reduces to solving tropical polynomial systems. Solving tropical polynomial systems is one of the most basic problems of tropical geometry. The latter provides insights on the nature of solutions, as well as algorithmic tools. In particular, the tropical approach allows one to determine the different congestion phases of the system.

We apply this approach to a case study relative to the current project led by Préfecture de Police de Paris (PP), involving the Brigade de sapeurs-pompiers de Paris (BSPP), of a new organization to handle emergency calls to Police (number 17), Firemen (number 18), and untyped emergency calls (number 112), in the Paris area. To this purpose, we introduce a simplified model of emergency call center, and we concentrate on the analysis of an essential feature of the organization: the two level emergency procedure. Operators at level 1 initially receive the calls, qualify their urgency, handle the non urgent ones, and transfer the urgent cases to specialized level 2 operators who complete the instruction. We solve the associated system of tropical polynomial equations and arrive at an explicit computation of the different congestion phases, depending on the ratio of the numbers of operators of level 2 and 1. Our analytical results are obtained only for the approximate fluid model. However, they are confirmed by simulations in which the original semantics of the Petri nets (with integer firings) is respected.

This work has been published in the proceedings of the conference FORMATS 2015 [28].

7.5.6. Tarification du tarif des données dans les réseaux mobiles/Smart Data Pricing

Participants: Marianne Akian, Mustapha Bouhtou [Orange Labs], Jean-Bernard Eytard.

Le travail de PhD de Jean-Bernard Eytard, qui a démarré en Octobre, concerne l'optimisation de la tarification des données dans les réseaux mobiles.

English version

The PhD work of Jean-Bernard Eytard, which started in October, concerns the optimal pricing of data traffic in mobile networks.

POEMS Project-Team

6. New Results

6.1. Wave propagation in non classical media

6.1.1. Modal analysis of electromagnetic dispersive media

Participants: Anne-Sophie Bonnet-Ben Dhia, Christophe Hazard.

Except in vacuum, the velocity of electromagnetic waves generally depends on the frequency. This dispersion plays in particular a vital role in situations where the effective index takes values below unity or negative, which happens with metamaterials or plasmonic devices. However, most of the studies in this domain are considering only the time-harmonic regime, forgetting dispersion, which leads to apparent paradoxes. We have elaborated a project, in collaboration with the Institut Fresnel in Marseille. Our objective is to gather physical and mathematical points of view to explore a frequency-to-time approach for dispersive media. This approach is based on a general technique which allows to hide dispersion in an augmented formulation of Maxwell's equations. Using this tool, our aim is first to carry the spectral analysis of dispersive systems, take advantage of this analysis to predict the time-dependent behaviour of dispersive systems, then design adapted numerical methods for their simulation and finally confirm predictions by real experiments. To begin with, during the internship of Bilal Yezza, a toy problem has been studied, where the presence of accumulation points in the spectrum is due to the dispersion. This project has been submitted to the ANR for the second year and has already led to preliminary common works and discussions, in particular during the workshop *Leaky days* organized by Christophe Hazard in Palaiseau in June 2015.

6.1.2. Perfectly Matched Layers in plasmas and metamaterials

Participants: Eliane Bécache, Patrick Joly, Maryna Kachanovska, Valentin Vinoles.

We work on the stability of Generalized Perfectly Matched Layers (GPMLs) in dispersive media for which classical PMLs are in general unstable. These new PMLs involve, in addition to the absorption parameter $\sigma \geq 0$, a real valued rational function of the frequency $\psi(\omega)$. We first worked on isotropic media and derived, using Fourier analysis methods, a necessary and sufficient condition on the function $\psi(\omega)$ for the stability of the PML model. This result has been presented in several conferences and used to design new stable PMLs for negative index metamaterials and uniaxial anisotropic plasmas (even though this last model is anisotropic, the anisotropy has a structure that permits a special decomposition of vector fields that give a new equivalent model adapted for our GPMLs).

We are currently working on the generalization of this analysis to a class of anisotropic dispersive models using a different approach based on Laplace transform in time.

However, this theory does not apply to more general cold plasma models that we wish to treat. Finding good PMLs in this case still remains a challenging open question. Several attempts, such as radial PMLs (which we discussed about with our visitor Martin Halla from TU Wien), have failed.

6.2. Wave propagation in heterogeneous media

6.2.1. Homogenization of layered media

Participant: Jean-François Mercier.

Metamaterials have revived interest in the theory of homogenization techniques because some standard techniques, based on the Ross Nicholson-Weir method, can lead to unphysical effective parameters, since depending on the incident wave. In collaboration with Agnès Maurel and Abdelkader Ourir from the Langevin Institut and Simon Felix from the LAUM, we have proposed more suitable homogenization methods to describe wave propagation in artificial environments, by considering homogenization of sliced media. When the medium is structured at a sub-wavelength scale, it can be described as a simpler equivalent medium, homogeneous and anisotropic, with a tensor mass density and an effective modulus of elasticity. We considered two cases:

- for a propagating incident wave, we obtained the diffusion properties of the medium and we have shown that the effective medium correctly captures the acoustic properties of the real medium.

- however, in the real problem, evanescent waves are generated and if one of them is resonant, the properties of transmission and reflection of the incident wave are changed: this happens for the electromagnetic waves (Wood anomalies, "spoof plasmon"). To capture these resonance effects, we have considered evanescent incident waves. We then showed that the homogenization predicts the dispersion curves of the resonant waves: in the homogenized problem, they correspond to guided waves by the anisotropic layer.

6.2.2. *High order transmission conditions between homogeneous and homogenized periodic half-spaces*

Participants: Sonia Fliss, Valentin Vinales.

This work is a part of the PhD of Valentin Vinales, and is done in collaboration with Xavier Claeys (LJLL, Paris VI). It is motivated by the fact that classical homogenization theory poorly takes into account interfaces, which is particularly unfortunate when considering negative materials, because important phenomena arise precisely at their surface (plasmonic waves for instance). To overcome this limitation, we want to construct high order transmission conditions. Using matched asymptotics, we have treated the case of a plane interface between a homogeneous and a homogenized periodic half space. The analysis is based on an original combination of Floquet-Bloch transform and a periodic version of Kondratiev techniques. The obtained conditions involve Laplace- Beltrami operators at the interface and require to solve *cell problems* in infinite strips. The numerical computations are based on specific transparent conditions for periodic media. The error analysis and the numerical study are on-going works.

6.2.3. *Scattering by small heterogeneities*

Participants: Patrick Joly, Simon Marmorat.

Simon Marmorat has defended his thesis, done in collaboration with the CEA-LIST and with Xavier Claeys (LJLL, Paris VI). The goal was to develop an efficient numerical approach to simulate the propagation of waves in concrete, which is modelled as a smooth background medium, with many small embedded heterogeneities. To do so, one has proposed two reduced models relying on the asymptotic analysis of the problem with respect to the (small) size of the heterogeneities. The first model looks like a fictitious domain method in which the analysis of the near field (closed to the heterogeneities) is exploited. The second one is a method of auxiliary sources, based on the analysis of the far field (far from the heterogeneities). Rigorous error estimates have been established. From the numerical point of view, some points, related to the Galerkin enrichment of standard finite element methods, still need to be completed.

6.2.4. *Effective boundary conditions for strongly heterogeneous thin layers*

Participants: Mathieu Chamaillard, Patrick Joly.

This topic is the object of the PhD of Mathieu Chamaillard, done in collaboration with Housseem Haddar (Inria, Defi). We are interested in the construction of effective boundary conditions for the diffraction of waves by an obstacle covered with a thin coating whose physical characteristics vary "periodically". The width of the coating and the period are both proportional to the same small parameter δ .

The results obtained previously on scalar propagation models have been extended to 3D Maxwell's equations resulting in the construction of an effective condition of the form $E \times n = \delta ik \mathcal{Z}_\Gamma (n \times (H \times n))$ where the impedance operator \mathcal{Z}_Γ , a second order tangential differential operator along Γ , depends on the geometry of the obstacle and of the material properties of the coating. The analysis, which is much more involved than in the scalar case (in particular in what concerns the stability analysis), provides error estimates in $O(\delta^2)$.

The thesis will be defended in the end of January 2016.

6.3. Spectral theory and modal approaches for waveguides

6.3.1. Guided modes in ladder-like open periodic waveguides

Participants: Sonia Fliss, Patrick Joly, Khac Long Nguyen, Elizaveta Vasilevskaya.

The general objective is the study of localized modes in locally perturbed periodic media and of guided modes in periodic media with a lineic perturbation. We investigate the existence theory of such modes as well as their numerical computations.

The problem, that is investigated in the framework of the PhD thesis of E. Vasilevskaya, in collaboration with Bérandère Delourme (Paris 13 University), is the case where the propagation medium is a thin structure whose limit is a periodic graph. We exhibit situations where the introduction of a line defect into the geometry of the domain leads to the appearance of guided modes. From the theoretical point of view, the problem is studied by asymptotic analysis methods, the small parameter being the thickness of the domain, so that when the thickness of the structure is small enough, the domain approaches a graph. The spectral theory of the underlying limit operator defined in the graph plays a key role in the analysis. For 2D configurations, we have shown that for sufficiently thin structures, it suffices to reduce the width of one rung to make appear guided modes. Moreover, using matched asymptotic expansions, we have constructed asymptotic expansions at any order of the corresponding eigenvalues and guided modes. For 3D configurations, the spectral theory of the underlying limit operator was already studied. In a further step, one can expect, again by asymptotic analysis, to get corresponding existence results for the original problem, at least for sufficiently thin structures.

From a numerical point of view, the modes can be computed using non linear eigenvalue problems and specific transparent boundary conditions for periodic media. During his post-doc, Khac Long Nguyen has implemented an exact method based on Dirichlet-to-Neumann operators to compute localized modes in 2D locally perturbed periodic media or guided modes in 3D periodic media with a lineic perturbation. This was already done few years ago for waveguides configurations but here the construction of the transparent boundary conditions are much more involved.

6.3.2. Reduced graph models for networks of thin co-axial electromagnetic cables

Participants: Geoffrey Beck, Patrick Joly.

This work is the object of the PhD of Geoffrey Beck and is done in collaboration with Sébastien Imperiale (Inria, MEDISIM). The general context is the non destructive testing by reflectometry of electric networks of co-axial cables with heterogeneous cross section and lossy materials, which was the subject of the ANR project SODDA. We consider electromagnetic wave propagation in a network of thin coaxial cables (made of a dielectric material which surrounds a metallic inner-wire). The goal is to reduce 3D Maxwell's equations to a 1D like model. During the past two years, we derived and justified generalized telegraphers model for a single cable. This year, we incorporated in our model the losses due to the skin effect induced by the non perfectly conducting nature of the metallic wire. Finally using the method of matched asymptotics, we have derived and justified improved Kirchhoff conditions.

6.3.3. Multimodal methods for the propagation of acoustic and electromagnetic waves

Participant: Jean-François Mercier.

In collaboration with Agnès Maurel from the Langevin Institut and Simon Felix from the LAUM, we have developed fast multimodal methods to describe the acoustic propagation in rigid waveguides or in periodic arrays. An incident wave is scattered by penetrable inclusions or by the succession of different penetrable media separated by interfaces of any shape. The difficulties are: to take into account the modes coupling and to get modes naturally decoupled at the entrance and at the exit of the computational domain. A weak formulation of the problem provides a modal formulation taking exactly into account the matching conditions at the interfaces. A consequence is that the obtained convergence is the best convergence expected, given the regularity of the solution. After the study of isotropic cases, we have generalized this approach to the case of anisotropic media, the difficulty being to take into account a tensor in the propagation equation.

6.3.4. Plasmonic waveguides

Participants: Anne-Sophie Bonnet-Ben Dhia, Camille Carvalho, Patrick Ciarlet.

This work, which is a part of the PhD of Camille Carvalho, is done in collaboration with Lucas Chesnel (Inria, Defi). A plasmonic waveguide is a cylindrical structure consisting of metal and dielectric parts. In a certain frequency range, the metal can be seen as a lossless material with a negative dielectric permittivity. The study of the modes of a plasmonic waveguide is then presented as an eigenvalue problem with a sign-change of coefficients in the main part of the operator. Depending on the values of the contrast of permittivities at the metal / dielectric interface, different situations may occur. In the "good" case, the problem is self-adjoint with compact resolvent and admits two sequences of eigenvalues tending to $+$ and $-\infty$. But when the interface presents corners, for a particular contrast range, the problem is neither self-adjoint nor with compact resolvent. In this case, Kondratiev's theory of singularities allows to build extensions of the operator, with compact resolvent. Finally, we show that the eigenvalues for one of these extensions can be computed by combining finite elements and Perfectly Matched Layers at the corners. The paradox is that a specific treatment has to be done to capture the corners singularities, even to compute regular eigenmodes.

6.4. Inverse problems

6.4.1. Quasi-Reversibility method and exterior approach for evolution problems

Participant: Laurent Bourgeois.

This work is a collaboration with Jérémie Dardé from Toulouse University. We address some linear ill-posed problems involving the heat or the wave equation, in particular the heat/wave equation with lateral Cauchy data. We have introduced several kinds of variational mixed formulations of quasi-reversibility which enable us to solve these ill-posed problems by using classical Lagrange finite elements. We have also designed a new approach called the "exterior approach" to solve inverse obstacle problems with initial condition and lateral Cauchy data for the heat/wave equation. It is based on a combination of an elementary level set method and the quasi-reversibility methods we have just mentioned. Some numerical experiments have proved the feasibility of our strategy to identify obstacles from lateral Cauchy data for the heat equation in 2D and for the wave equation in 1D. Our objective is now to focus on the wave equation in 2D. Firstly we wish to obtain a minimal value of the final time in order to ensure uniqueness of the obstacle from the lateral Cauchy data. Secondly we want to test our exterior approach numerically. We expect better results than with the heat equation.

6.4.2. Higher-order expansion of misfit functional for defect identification in elastic solids

Participants: Marc Bonnet, Rémi Cornaggia.

This work, done in the context of the PhD of Rémi Cornaggia, concerns the identification of scatterers of moderate size, modelled as elastic inhomogeneities embedded in an homogeneous elastic background medium, by time-harmonic elastodynamic measurements. Least-squares functionals, commonly used for defect identification, are expanded in powers of the small characteristic radius a of a trial inhomogeneity. This entails the expansion of the elastodynamic scattering problem, which is needed only on the support of the trial inhomogeneity and is established by means of a Lippmann-Schwinger volume integral equation. This approach generalizes, to higher orders in a , the well-known concept of topological derivative. Such expansion,

whose derivation and evaluation are facilitated by using an adjoint state, provides a basis for the quantitative estimation of flaws whereby a region of interest may be exhaustively probed at reasonable computational cost. So far, the higher-order expansion has been derived under fairly general conditions, mathematically justified, and demonstrated on simple numerical examples involving the identification of a spherical inhomogeneity in an unbounded 3D medium.

6.4.3. Complete transmission invisibility in waveguides

Participant: Anne-Sophie Bonnet-Ben Dhia.

In collaboration with Lucas Chesnel (Inria, Defi) and Sergei Nazarov (Saint-Petersburg University), we consider time harmonic acoustic problems in waveguides. We are interested in finding localized perturbations of a straight waveguide which are not detectable in the far field, as they produce neither reflection nor conversion of propagative modes. In other words, such *invisible* perturbation produces a scattered field which is exponentially decaying at infinity in the two infinite outlets of the waveguide.

In our previous contributions, we found a way to build smooth and small perturbations of the boundary which were almost invisible, in the sense that they were producing no reflexions but maybe a phase shift in transmission.

During the visit of Sergei Nazarov, we found a new approach which allows to build completely invisible perturbations in the mono-mode regime (i.e. when the frequency is chosen below the first cut-off frequency) with no phase shift in transmission. These perturbations include some kinds of thin resonators whose height is adapted to the frequency.

All our results mainly rely on asymptotic theory.

6.4.4. Energy-based cost functional for three-dimensional transient elastodynamic imaging

Participant: Marc Bonnet.

This work is a continuing collaboration with Wilkins Aquino (Duke University, USA). It is concerned with three-dimensional elastodynamic imaging by means of the modified error in constitutive relation (MECR), combining the energy norm of the constitutive residual and a more-classical L^2 norm on the measurement residuals.

We have in particular considered the case of imaging using interior data. The stationarity equations associated with the minimization of a MECR objective function, subject to the conservation of linear momentum, yields a well-posed problem coupling two elastodynamic fields, even in cases where boundary conditions are initially underspecified (making it difficult to define *a priori* a forward problem). Numerical results demonstrate the robust performance of the method in situations where the available measurement data is incomplete and corrupted by noise of varying levels.

In a separate study, elastodynamic imaging using transient data and based on time-domain solvers has been investigated. In this context, each evaluation of a time-domain MECR cost functional entails solving two elastodynamic problems (one forward, one backward), which moreover are coupled (unlike the case of L^2 misfit functionals). This coupling creates a major computational bottleneck, making MECR-based inversion difficult for spatially 2D or 3D configurations. To overcome this obstacle, we propose to (a) set the entire computational procedure in a consistent time-discrete framework that incorporates the chosen time-stepping algorithm, and (b) use an iterative successive over-relaxation-like method for the resulting stationarity equations. The resulting MECR-based inversion algorithm is formulated under quite general conditions, allowing for 3D transient elastodynamics, straightforward use of available parallel solvers, a wide array of time-stepping algorithms commonly used for transient structural dynamics, and flexible boundary conditions and measurement settings. The proposed MECR algorithm is then demonstrated on computational experiments involving 2D and 3D transient elastodynamics and up to over 500 000 unknown elastic moduli.

6.4.5. Linear Sampling Method with realistic data in waveguides

Participants: Laurent Bourgeois, Arnaud Recoquilly.

Our activities in the field of inverse scattering in waveguides with the help of sampling methods has now a quite long history. We now intend to apply these methods in the case of realistic data, that is surface data in the time domain. This is the subject of the PhD of Arnaud Recoquilly. It is motivated by Non Destructive Testing activities for tubular structures and is the object of a partnership with CEA List (Vahan Baronian).

Our strategy consists in transforming the time domain problem into a multi-frequency problem by the Fourier transform. This allows us to take full advantage of the established efficiency of modal frequency-domain sampling methods. We have already proved the feasibility of our approach in the 2D acoustic case. In particular, we have shown how to optimize the number of sources/receivers and the distance between them in order to obtain the best possible identification result. The next steps consist in extending such an approach to the elastic case and trying it experimentally, that is with real data. Experiments will be carried in CEA.

6.5. Integral equations

6.5.1. Fast BEM solvers based on \mathcal{H} -matrices for 3-D frequency-domain elastodynamics

Participants: Stéphanie Chaillat, Patrick Ciarlet, Luca Desiderio.

The main advantage of the Boundary Element Method (BEM) is that only the domain boundaries are discretized leading to a drastic reduction of the total number of degrees of freedom. In traditional BE implementation the dimensional advantage with respect to domain discretization methods is offset by the fully-populated nature of the BEM coefficient matrix. Using the \mathcal{H} -matrix arithmetic and low-rank approximations (performed with Adaptive Cross Approximation) it is possible to derive fast iterative and direct solvers for the BEM system. We extend the method to 3-D frequency-domain elastodynamics. To this end, the Adaptive Cross Approximation is adapted to deal with vectorial problems. To validate the accuracy of the solution of the LU based direct solver, we derive an error estimate. Finally, we check numerically the theoretical estimate of the storage costs. In particular, we study the efficiency of low-rank approximations when the frequency is increased. This is done in partnership with SHELL company in the framework of the PhD of Luca Desiderio.

6.5.2. OSRC preconditioner for 3D elastodynamics

Participant: Stéphanie Chaillat.

This work is done in collaboration with Marion Darbas from University of Picardie and Frédérique Le Louer from Technological University of Compiègne. The fast multipole accelerated boundary element method (FM-BEM) is a possible approach to deal with scattering problems of time-harmonic elastic waves by a three-dimensional rigid obstacle. In 3D elastodynamics, the FM-BEM has been shown to be efficient with solution times of order $O(N \log N)$ per iteration (where N is the number of BE degrees of freedom). However, the number of iterations in GMRES can significantly hinder the overall efficiency of the FM-BEM. To reduce the number of iterations, we propose a clever integral representation of the scattered field which naturally incorporates a regularizing operator. When considering Dirichlet boundary value problems, the regularizing operator is a high-frequency approximation to the Dirichlet-to-Neumann operator. For a spherical obstacle, the approximation of the DtN is a linear combination of the tangential and normal parts. The numerical efficiency of the preconditioned integral equation (i.e. the independence of the number of iterations from the mesh size and frequency) is verified for spherical obstacles, validating the concept of analytical preconditioners for 3D elastodynamics FM-BEM. For more general shapes, this approximation of the DtN is more complex to derive. As a first step, we construct and validate the approximation in the framework of the On-Surface Radiation Condition (OSRC) method.

6.5.3. A wideband Fast Multipole Method for oscillatory kernels

Participant: Stéphanie Chaillat.

This work is done in collaboration with Francis Collino. We derive a new Fast Multipole Method (FMM) based on plane wave expansions (PWFMM), combining the advantages of the low and high frequency formulations. We revisit the method of Greengard et al. (1998) devoted to the low frequency regime and based on the splitting of the Green's function into a propagative and an evanescent part. More precisely, we give an explicit formula of the filtered translation function for the propagative part, we derive a new formula for the evanescent part and we provide a new interpolation algorithm. At all steps, we check the accuracy of the method by providing error estimates. These theoretical developments are used to propose a wideband FMM based entirely on plane wave expansions. The numerical efficiency and accuracy of this broadband PWFMM are illustrated with a numerical example.

6.5.4. *Coupling integral equations and high-frequency methods for ultrasonic NDT modelling*

Participants: Marc Bonnet, Laure Pesudo.

This work, in partnership with CEA LIST and in collaboration with Francis Collino, is undertaken in the context of the PhD thesis of Laure Pesudo. Modelling ultrasonic non destructive testing (NDT) experiments simultaneously involves the scattering of waves by defects of moderate size (for which discretization-based methods such as the BEM are appropriate) and propagation over large distances (requiring high-frequency approximations). Those two types of simulation methods are therefore simultaneously needed in NDT modelling but do not lend themselves easily to coupling. The coupling approach proposed here takes advantage of the fact that the far-field asymptotic approximation of integral representation formulas (which accurately account for the scattering by defects) yields a superposition of rays (satisfying the leading-order equations arising from high-frequency asymptotics). This allows to convert incoming rays into plane waves, compute their scattering by obstacles, and convert the scattered field into rays. A defect of given shape and characteristics becomes (approximately) represented as a point-like scatterer with anisotropic reflection properties that are computed (offline) from BEM solutions of near-field problems. Using a partition of unity on the obstacle boundary allows to approximate the obstacle by a set of point-like reflectors, thereby enlarging the size of obstacles amenable to this approach. Preliminary tests on 2D scalar wave propagation problems show that sufficient far-field accuracy is achieved for wavelength-sized defects.

6.5.5. *Dynamic soil-structure interaction*

Participants: Marc Bonnet, Stéphanie Chaillat, Zouhair Adnani.

This work, undertaken in the context of the PhD thesis of Zouhair Adnani (CIFRE partnership with EDF), concerns the simulation of dynamic soil-structure interaction (SSI) in connection with seismic assessment of civil engineering structures. The main goal is to formulate, implement, and evaluate on realistic test examples, a computational strategy that combines the fast multipole integral equation method for elastic wave propagation in unbounded regions (COFFEE FMM-accelerated BEM solver), and finite elements for modelling civil engineering structures and neighboring soil regions (the EDF in-house code Code_Aster). In a preliminary phase, the evaluation of transient elastodynamic responses via the Fourier synthesis of frequency-domain solutions computed using COFFEE (see Section 5.1) has been studied on several test problems, achieving substantial improvements of computational efficiency for this component of SSI analysis.

6.5.6. *Volume Integral Formulations*

Participant: Marc Bonnet.

Volume integral equations (VIEs), also known as Lippmann-Schwinger integral equations, arise naturally when considering the scattering of waves by penetrable, and possibly heterogeneous, inhomogeneities embedded in a homogeneous background medium. In contrast with the vast existing literature on boundary integral equations, comparatively few studies are available regarding the mathematical properties of VIEs. In this work, we investigate the solvability of VIE formulations arising in elastodynamic scattering by penetrable obstacles. The elasticity tensor and mass density are allowed to be smoothly heterogeneous inside the obstacle and may be discontinuous across the background-obstacle interface, the background elastic material being homogeneous. Both materials may be anisotropic, within certain limitations for the background medium. The VIE associated with this problem is derived, relying on known properties of the background fundamental tensor.

To avoid difficulties associated with existing radiation conditions for anisotropic elastic media, we propose an alternative definition of the radiating character of transmission solutions. The unique solvability of the volume integral equation (and of the scattering problem) is established. For the important special case of isotropic background properties, our definition of a radiating solution becomes equivalent to the classical Sommerfeld-Kupradze radiation conditions.

6.6. Domain decomposition methods

6.6.1. *Transparent boundary conditions with overlap in unbounded anisotropic media*

Participants: Anne-Sophie Bonnet-Ben Dhia, Sonia Fliss, Antoine Tonnoir.

We are interested in acoustic or elastic wave propagation in time harmonic regime in a 2D or 3D medium which is a local perturbation of an infinite anisotropic homogeneous medium. We investigate the question of deriving a formulation which is suitable for numerical computations. This question is difficult due to the anisotropy of the surrounding medium. Our approach consists in coupling several plane-waves representations of the solution in half-spaces surrounding the defect with a FE computation of the solution around the defect. The difficulty is to ensure that all these representations match, in particular in the infinite intersections of the half-spaces. It leads to a formulation which couples, via integral operators, the solution in a bounded domain including the defect and its traces on the edge of the half-planes. We have shown that this formulation has good properties from theoretical and numerical points of view.

6.6.2. *Electromagnetic scattering by objects with multi-layered dielectric coatings*

Participants: Patrick Joly, Matthieu Lecouvez.

The PhD thesis of Matthieu Lecouvez, undertaken in collaboration with the CEA-CESTA and Francis Collino, has been defended in July. It concerned the diffraction of time harmonic electromagnetic waves by perfectly conducting objects covered by multi-layered (possibly thin) dielectric coatings. This idea was to use a domain decomposition method in which each layer would constitute a subdomain. The transmission conditions between the subdomains involve some specific impedance operators in order to achieve a geometric convergence of the method (compared to the slow algebraic convergence obtained with standard Robin conditions). This year, the theoretical aspects of our work have been completed and are the object of an article in preparation.

6.6.3. *Domain Decomposition Methods for the neutron diffusion equation*

Participants: Patrick Ciarlet, Léandre Giret.

Studying numerically the steady state of a nuclear core reactor is expensive, in terms of memory storage and computational time. In its simplest form, one must solve a neutron diffusion equation with low-regularity solutions, discretized by finite element techniques, totaling millions of unknowns or more, within a loop. Iterating in this loop allows to compute the smallest eigenvalue of the system, which determines the critical, or non-critical, state of the 3D core configuration. This problem fits within the framework of high performance computing so, in order both to optimize the memory storage and to reduce the computational time, one can use a domain decomposition method, which is then implemented on a parallel computer. The definition of an efficient DD method has been addressed for conforming meshes prior to the PhD research of Léandre Giret. The development of non-conforming, hence more flexible, DD methods has recently been finalized.

The optimization of the eigenvalue loop is under way. The current research also focuses on the numerical analysis of the full suite of algorithms to prove convergence for the complete multigroup SPN model (which involves coupled diffusion equations). Realistic computations will be carried out with the APOLLO3 neutronics code.

This topic is developed in partnership with CEA-DEN (Erell Jamelot).

6.7. Aeroacoustics

6.7.1. Time-harmonic acoustic scattering in a vortical flow

Participants: Antoine Bensalah, Patrick Joly, Jean-François Mercier.

This activity is done in the framework of the PhD of Antoine Bensalah, in partnership with Airbus Group. We study the time-harmonic acoustic radiation in a fluid in a general flow which is not curl free, but has restricted vortical areas. The objective is to take into account the complicated coupling between acoustics and hydrodynamics. The Galbrun approach developed previously in 2D is too expensive in terms of degrees of freedom for 3D simulations. As an alternative, we propose to consider instead the Goldstein equations, which are vectorial only in the vortical areas and remain scalar elsewhere.

We have proved that the Goldstein equations are well-posed in a domain Ω for a potential flow, or for a vortical flow if the flow is Ω -filling (each point of Ω is reached by a streamline coming from the inflow boundary in a finite time). A non Ω -filling flow corresponds to the presence of recirculation areas and we have shown that, for such flows, some of the closed streamlines can be resonant. To study deeper this phenomenon, we focused on the case of a rotating flow in an annular geometry. We proved that outside the set of resonance frequencies, the radiation problem is well-posed. Work is under progress to determine the solution on a resonant streamline.

6.7.2. Propagation of solitons through Helmholtz resonators

Participant: Jean-François Mercier.

With Bruno Lombard (Laboratoire de Mécanique et Acoustique of Marseille), we studied the propagation of an acoustic solitary wave in a 1D waveguide connected to a periodic array of Helmholtz resonators. Starting from a model of the literature, obtained by approximations, our goal was to provide a numerical modeling, which validates (or not) the underlying model and the assumptions. The model consists of two coupled equations evolution: a nonlinear PDE describing acoustic waves (similar to the Burgers equation), and a linear ODE describing oscillations in the Helmholtz resonators. We have developed a numerical method based on two main ingredients: a diffusive representation of fractional derivatives and a splitting method applied to the evolution equations. The numerical scheme has been validated by comparison with exact solutions. The properties of non-linear solutions have been investigated numerically.

In collaboration with O. Richoux of the LAUM, this work has been extended, comparing to experimental results. Adjustments had to be made, the attenuation of the numerical model being weaker than that observed experimentally. To remedy this, we have incorporated some attenuation mechanisms that we had neglected. One consequence of these additions is that a more sophisticated numerical method had to be developed. A good agreement has been found with experimental results.

SELECT Project-Team

6. New Results

6.1. Model selection in Regression and Classification

Participants: Gilles Celeux, Serge Cohen, Erwan Le Pennec, Pascal Massart, Kevin Bleakley.

The well-documented and consistent variable selection procedure in model-based cluster analysis and classification that Cathy Maugis (INSA Toulouse) designed during her PhD thesis in SELECT, makes use of stepwise algorithms which are painfully slow in high dimension. In order to circumvent this drawback, Gilles Celeux, in collaboration with Mohammed Sedki (Université Paris XI) and Cathy Maugis), proposed to sort the variables using a lasso-like penalization adapted to the Gaussian mixture model context. Using this ranking to select variables, they avoid the combinatory problem of stepwise procedures. After tests on challenging simulated and real data sets, their algorithm has shown encouraging performance. Moreover, the possibility to sort the variables with their marginal likelihoods is under study. The first results are encouraging, and this approach requires no regularization hyperparameters, and is much more rapid.

In collaboration with Jean-Michel Marin (Université de Montpellier) and Olivier Gascuel (LIRMM), Gilles Celeux has continued research aiming to select a short list of models rather a single model. This short list is declared to be compatible with the data using a p -value derived from the Kullback-Leibler distance between the model and the empirical distribution. Furthermore, the Kullback-Leibler distances at hand are estimated through nonparametric and parametric bootstrap procedures. Different strategies are compared through numerical experiments on simulated and real data sets.

Emilie Devijver, Yannig Goude and Jean-Michel Poggi have proposed a new methodology for customer segmentation, in the context of load profiles in energy consumption. The method is based on high-dimensional regression models which perform clustering and model selection at the same time. They have focused on uncovering classes corresponding to different regression models, and compute clustering and model identification in each cluster simultaneously. They have shown the feasibility of the approach on a real data set of Irish customers.

Emilie Devijver has studied a dimension-reduction method for finite mixtures of multivariate response regression models in high-dimension. The size of the response and the number of predictors may exceed the sample size. She considers jointly predictor selection and rank reduction to obtain lower-dimensional approximations of parameter matrices. A penalty, for which the model selected by penalized likelihood satisfies an oracle inequality, is given.

The detection of change-points in a spatially or time-ordered data sequence is an important problem in many fields such as genetics and finance. Kevin Bleakley, with Gérard Biau (LSTA, Paris 6 University) and David Mason (University of Delaware), has found asymptotic distributions of statistics used to detect change-points, and developed methods to provide stopping criteria (model selection) for the number of change-points found.

6.2. Statistical learning methodology and theory

Participants: Gilles Celeux, Christine Keribin, Erwan Le Pennec, Michel Prenat, Solenne Thivin, Kevin Bleakley.

Gilles Celeux has started a collaboration with Jean-Patrick Baudry on strategies to avoid traps in the EM algorithm in mixture analysis. They analyze the effect of spurious local maximizers, and regularized algorithms to avoid such solutions. They show the link that exists between the degree of regularization and slope heuristics. Moreover, their strategy to initiate the EM algorithm, embedding the solution with K components and the starting position with $K + 1$ components to avoid suboptimal solutions, has been proved to be efficient, and is extended to a more complex framework of latent block models.

In the context of algorithms that depend on distributed computing and collaborative inference, Kevin Bleakley, with Gérard Biau (LSTA, Paris 6) and Benoît Cadre (ENS Rennes), have proposed a collaborative framework that aims to estimate the unknown mean θ of a random variable X . In the model they present, a certain number of calculation units, distributed across a communication network represented by a graph, participate in the estimation of θ by sequentially receiving independent data from X while exchanging messages via a stochastic matrix A defined over the graph. They give precise conditions on the matrix A under which the statistical precision of the individual units is comparable to that of a (gold standard) virtual centralized estimate, even though each unit does not have access to all of the data.

6.3. Reliability

Participants: Yves Auffray, Gilles Celeux, Florence Ducros, Patrick Pamphile, Jana Kalawoun.

Since June 2015, in the framework of a CIFRE convention with Nexter, Florence Ducros has commenced a thesis on the modeling of aging of vehicles, supervised by Gilles Celeux and Patrick Pamphile. This thesis should lead to designing an efficient maintenance strategy according to vehicle use profiles. It will involve the estimation of mixtures and competing risk models in a highly censored setting.

Jana Kalawoun has defended her thesis supervised by Gilles Celeux, Patrick Pamphile and Maxime Montaru (CEA) on the estimation of the battery State of Charge (SoC). For vehicles powered by an electric motor, SoC estimation is essential to guarantee vehicle autonomy, as well as safe utilization. The aim is to create a reliable SoC model to closely fit battery dynamics in embedded applications (e.g., electric vehicles). The SoC is modeled by a switching Markov state-space model. Parameters are estimated by combining the EM algorithm and particle filter methods. The model is validated using real-world electric vehicle data. This model has been proved to be highly superior to a simple state space model. The optimal number of battery modes is then identified, using model selection criteria such as AIC and BIC, which has proved to be superior to cross-validation in this particular context.

In the framework of a study on the dispatch availability of Dassault Aviation business jets, Yves Auffray and Gilles Celeux have contributed to methodology aiming to discover the root causes of reliability flaws.

6.4. Statistical analysis of genomic data

Participants: Gilles Celeux, Mélina Gallopin, Christine Keribin, Yann Vasseur.

Mélina Gallopin defended her thesis supervised by Gilles Celeux, Florence Jaffrezic and Andrea Rau (INRA, animal genetics department), This thesis was concerned with modeling and model selection in the analysis of RNA-seq data. Its highlights are the following:

- Presentation of a model selection criterion for model-based clustering of annotated gene expression data. This criterion is an ICL-like criterion taking into account annotation.
- An objective comparison of discrete and continuous modeling after transformations for RNA-seq data based on a comparison of likelihoods (possibly penalized) of the possible models.
- A block diagonal covariance selection method for high dimensional Gaussian graphical models. This non-asymptotic model selection procedure is supported by strong theoretical guarantees, based on an oracle inequality and a minimax lower bound. This work was in collaboration with Emilie Devijver.

The subject of Yann Vasseur's PhD Thesis, supervised by Gilles Celeux and Marie-Laure Martin-Magniette (INRA URGV), is the inference of a regulatory network on Transcriptions Factors (TFs), which are specific genes, of *Arabidopsis thaliana*. To that purpose, a transcriptome dataset with a similar number of TFs and statistical units is available. The first aim consists of reducing the dimension of the network to avoid high-dimensional difficulties. Representing this network with a Gaussian graphical model, the following procedure has been defined:

1. *Selection step:* choose the set of TF regulators (supports) of each TF.
2. *Classification step:* deduce co-factors groups (TFs with similar expression levels) from these supports.

Thus, the reduced network would be built on the co-factors groups. Currently, several selection methods based on Gauss-LASSO and resampling procedures have been applied to the dataset. The study of stability and parameter calibration of these methods is in progress. The TFs are clustered with the Latent Block Model in a number of co-factor groups, selected with BIC or the exact ICL criterion.

In a collaboration with Marie-Laure Martin-Magniette, Cathy Maugis and Andrea Rau, Gilles Celeux has studied gene expression obtained from high-throughput sequencing technology. The focus is on the question of clustering gene expression profiles as a means to discover groups of co-expressed genes. A Poisson mixture model is proposed, using a rigorous framework for parameter estimation as well as for the choice of the appropriate number of clusters. They illustrate co-expression analyses using this approach on two real RNA-seq datasets. A set of simulation studies also compares the performance of the proposed model with that of several related approaches developed to cluster RNA-seq and serial analysis of gene expression data. The proposed method is implemented in the open-source R package `HTSCluster`, available on CRAN. It can now be compared with Gaussian mixtures obtained after relevant data transformations.

6.5. Model based-clustering for pharmacovigilance data

Participants: Gilles Celeux, Christine Keribin, Valérie Robert.

In collaboration with Pascale Tubert-Bitter, Ismael Ahmed and Mohamed Sedki, Gilles Celeux and Christine Keribin have started research concerning the detection of associations between drugs and adverse events in the framework of the PhD of Valerie Robert. At first, this team developed a model-based clustering inspired by latent block models, which consists of co-clustering rows and columns of two binary tables, imposing the same row ranking. This enables it to highlight subgroups of individuals sharing the same drug profile, and subgroups of adverse effects and drugs with strong interactions. Furthermore, some sufficient conditions are provided to obtain the identifiability of the model, and some results are shown for simulated data. This year, the exact ICL criterion has been extended to this double block latent model. Moreover, the possible added value of this model, compared with standard contingency table analysis, is currently under scrutiny.

TAO Project-Team

7. New Results

7.1. Optimal Decision Making under Uncertainty

The Tao-uct-sig is working mainly on mathematical programming tools useful for power systems. In particular, we advocate a data science approach, in order to reduce the model error - which is much more critical than the optimization error, in most cases. Real data are the best way for handling uncertainties. Our main works are as follows:

- **Noisy optimization** In the context of stochastic uncertainties, noisy optimization handles the model error by simulation-based optimization. Our results include:
 - A formalization of noisy optimization in continuous domains, often poorly modeled in the evolutionary computation community [64], [6]. We also proposed heuristic rules for reaching slope $-1/2$ in log-log scale [34]. We also show that in some settings the slope -1 (classical in mathematical programming) can be recovered in evolution strategies (unpublished: <http://www.lri.fr/~teytaud/mca.pdf>), and we provided complexity bounds [20].
 - An extension of portfolio algorithms for noisy optimization. Portfolio methods are already usual in combinatorial optimization, some works exist in the continuous case, this is the first work in the noisy case[8].
 - Pragmatic approaches of noisy optimization, for improving robustness and for taking into account human expertise, including: Applying sieves methods in noisy optimization [27], paired optimization [35], and combining various policies [25].
 - Upper bounds on noisy optimization in discrete domains [5].
- **Quasi-random numbers** Continuous optimization is a key component of our works, hence we improve evolution strategies (which have simplicity and convenience qualities) by quasi-random numbers (showing that even in simple cases it is beneficial[52], and provides great improvements in highly multimodal cases (unpublished, <http://www.lri.fr/~teytaud/qrr.pdf>)). We also developed criteria and testbeds, pointing out some key points not widely studied in the optimization literature[26]. We also extended our earlier results in parallel optimization to additional algorithms[30], and used cutting planes as in the ellipsoid method, hence combining the best of both worlds, i.e. fast rates from cutting planes methodologies and parallel behavior as in evolution strategies[36].
- **Dynamical problems** The dynamical nature of power systems is critical, as transient regimes, ramping constraints are ubiquitous in unit commitment and dispatch. Optimizing policies, with their temporal components, is a challenge when the high dimension and the nonlinearities are taken into account. Games provide a nice testbed for experiments and are used in several of our works. We provided:
 - An original algorithm for learning opening books, by an unexpected use of random seeds[32]. The principle is to randomly sample policies, by modifying the random seed. This can be used for any stochastic policy: we generate thousands of deterministic policies (by setting the random seed to arbitrary values) and select the best ones. This can be applied for games (always the most convincing application for a proof of concept), and any control problem where stochastic policies are available.
 - An extension of the previous work for dynamically adapting the probability distribution for specific positions[51]. This work provides a MCTS without the scalability limitations of MCTS. This work might give birth to many future works.

7.2. Continuous Optimization

- **Markov Chain Analysis of Evolution Strategies** The theory of Markov chains with discrete time and continuous state space turns out to be very useful to analyze the convergence of adaptive evolution strategies (including simplified versions of the state-of-the-art CMA-ES). Exploiting invariance of the algorithms, we can indeed construct homogeneous Markov chains underlying the algorithms whose stability implies the linear convergence of the algorithm [65]. We have also shown how the convergence on constrained problems can be analyzed with Markov chains theory [10].

However the stability can be very difficult to prove; even the irreducibility can be very challenging to prove with current Markov chain theory. We have hence been developing new theoretical tools exploiting deterministic control models to prove more easily the irreducibility and T-chain property of general Markov chains [67]. Those theoretical tools can be applied to the optimization algorithms we are interested in, and trivialize some stability proofs [1], [10].

- **Benchmarking of continuous optimizers** We have been pursuing our effort towards improving the standards in benchmarking of continuous optimizers. We tackled the benchmarking of bi-objective problems and transferred and adapted standard benchmarking techniques from the single-objective optimization and classical derivative free optimization community to the field of EMO [28]. In addition, we have been rewriting part of the COCO platform to improve its modularity and make it less error prone and started its extension to multiobjective optimization.
- **Concentration inequalities for sampling without replacement** We studied the concentration of measure phenomenon in the case of sampling without replacement, which is directly relevant for a recent MCMC technique for large data sets, see [7] accepted to the Bernoulli journal.
- **Random projections for confident MCMC** In the paper [66] accepted at the NIPS "Bayesian Optimization Workshop", we study the benefit of replacing uniform subsampling by random projections in recent MCMC techniques for linear regression of tall datasets.
- **Automatic step size adaptation** We have derived a new, low-cost strategy for online adaptation of the step size in stochastic gradient descent and related algorithms [72]. This problem is of crucial importance in many machine learning algorithms, as current approaches often rely on exploring a grid of step sizes and performing a full optimization for each of them, a lengthy process.

7.3. Data Science

- **High Energy Physics** The success of the 2014 HiggsML challenge has created a willingness for structured collaboration from the High Energy Physics experiments. A working group has been set up and new challenges are currently explored. A yearly workshop has been decided, with a first edition at CERN 9-13 Nov. 2015, *DataScience@LHC*.

The challenge exemplifies a new machine learning task [58][56]: *learning to discover* evaluating the significance of a scientific discovery. It can be formally casted into a two-class classification problem, but with two major departures from a regular setting. 1) Discovery: labeled training examples of the positive class (signals) are not available and must be obtained from simulation. The learning machine can then address the "inverse problem" of predicting which events are signals in real data. 2) Evaluation: because the classes are enormously imbalanced and overlapping, the objective function of the classifier is a metric of a statistical test.

- **Personal Semantics** Our algorithm for inducing a taxonomy from a set of domain terms placed first in the international Taxonomy Induction task, part of the SemEval 2015 conference in Denver. Since then, we have developed a robust technique for discovering the domain vocabulary for a new topic using a directed crawler we created. We are currently creating hundreds of taxonomy for personal themes (hobbies, illnesses) that can be integrated into our Personal Semantics platform PTraces. The challenges for the coming year will be deploying and evaluating the taxonomies, and introducing newer machine learning methods, such as Latent Dirichlet Allocation, for better recognizing domain vocabularies.

- **Distributed system observation** The work on distributed system automated analysis and description [59][60], has been pursued through the continued development of the GAMA multi-agent framework <https://github.com/gama-platform/gama/wiki>. The simulation framework has been applied to the study of a new anytime reverse auctions protocol [53]. Philippe Caillou is associated to the young researcher ANR ACTEUR, coordinated by Patrick Taillandier (IDEES, Rouen university). With this project, a new BDI cognitive agent model, designed to be easy to use for non computer scientist, has been proposed [29] and applied to Rouen traffic simulation [57]. Finally, agent behavior has been extracted from human player logs to study the perception of emotive behaviors in board games [37].
- **Digital humanities** Amiqap and Cartolabe projects both start in 2016. The Cartolabe project applies machine learning techniques to determine comprehensible structures in unstructured data. The goal is to use raw textual data, and underspecified ontologies, to provide intuitive access to pertinent research activities in a large research organisation. Amiqap studies the relation between worker well-being and company performance, in collaboration with Mines ParisTech sociology department and La Fabrique de l'Industrie for research, Secafi and DARES for the data. These activities will benefit from Paola Tubaro's arrival (researcher CNRS in sociology and economy) in 2016.

7.4. Designing criteria

- **Criterion design and optimization methods for computer vision** On the topic of large-scale image segmentation with multiple object detection, targeting as an application the analysis of high-resolution multispectral satellite images covering the Earth, challenges are numerous: scalable complexity, finding good features to distinguish objects, designing shape statistics as well as an optimization method able to incorporate them. We propose a solution [42], [43] based on the construction of binary partition trees and on their optimization, whose cost is alleviated thanks to theoretical results reducing the search space. Concerning video segmentation, we have extended previous work, on the inclusion of shape growth constraints into classical MRF settings (graph cuts with globally optimal segmentation), to the case of multimodal sequences of medical 3D scans [19]. We also studied a new family of metrics in [9], together with a redefinition of the associated gradient and practical ways to compute it. This allows the consideration of new types of priors on planar curve evolution, such as piecewise-rigid motions. Surprisingly, the problem of finding the best piecewise-rigid approximation of a motion turns out to be convex, and to be linked to sparsity approaches.
- **Algorithm selection and configuration** Two PhD theses are still related to the former *Crossing the Chasm* SIG: Nacim Belkhir has worked on inline parameter tuning for the CMA-ES algorithm in the context of a large number of cores [21], and is now using surrogate models to compute the features of expensive continuous optimization (submitted). François Gonard's PhD is dedicated to algorithm selection. The original application domain is that of expensive car industry simulations (within the IRT-ROM project). Initial results concern combinatorial optimization, and François obtained a "Honorable mention from the jury" for his submission to the ICON Challenge (<http://iconchallenge.insight-centre.org/>), for its original approach coupling a pre-scheduler and an algorithm selector. A paper describing the algorithms and analyzing the results has been submitted.
- **A statistical physics perspective** In the topic of MRF design, with motivating applications in large scale inference problems like traffic congestions, we have finalized in [13] an approach based on the disordered Ising model relying on approximate solutions to the Inverse Ising problem. To this specific problem we also propose new approximate solutions, compliant with the generalized belief-propagation algorithm in the static [63] and a new l_0 regularized method based on a maximum likelihood maximization for the dynamical case [11]. In fact in [63] we have developed a method adapted to the generalized belief propagation framework, aiming at addressing directly and systematically the loop corrections without loss of scalability, offering new possibilities in the context of inference by MRF models. In parallel, a better understanding of the so-called mean-field approximation when the phase space is clustered has been derived [68] giving a direct method to solve static inverse problem

in the weak coupling limit. Apart from the method point of view, some consideration over what can be said on the data has been considered, still in the topic of MRF design. In this sense, it is shown in [69] that the reconstruction of the MRF model depends strongly on how the data are gathered, and how to remove redundant data and keep a good reconstruction.

- **Multi-objective AI Planning** This activity had almost stopped since the end of the DESCARWIN ANR project. However, a productive internship resulted in some new benchmarks in the ZenoTravel domain together with an exact solver ensuring the knowledge of the true Pareto front [48], [47].

7.5. Deep Learning and Information Theory

- **Natural Gradients for Deep Learning** Deep learning is now established as a state-of-the-art technology for performing different tasks such as image or sequence processing. Nevertheless, much of the computational burden is spent on tuning the hyper-parameters. On-going work, started during the TIMCO project, is proposing, in the framework of Riemannian gradient descents, invariant algorithms for training neural networks that effectively reduce the number of arbitrary choices, e.g., affine transformations of the activation functions or shuffling of the inputs. Moreover, the Riemannian gradient descent algorithms perform as well as the state-of-the-art optimizers for neural networks, and are even faster for training complex models. The proposed approach is based on Amari's theory of information geometry and consists in practical and well-grounded approximations for computing the Fisher metric. The scope of this framework is larger than Deep Learning and encompasses any class of statistical models.
- **Training dynamical systems online without backtracking** with application to recurrent neural networks [73]. We propose an algorithm to learn the parameters of a dynamical system in an online, memoryless setting, thus requiring no backpropagation through time, and consequently scalable, avoiding the large computational and memory cost of maintaining the full gradient of the current state with respect to the parameters. The algorithm essentially maintains, at each time, a single search direction in parameter space. The evolution of this search direction is partly stochastic and is constructed in such a way to provide, at every time, an unbiased random estimate of the gradient of the loss function with respect to the parameters.
- **Approximating Bayesian predictors thanks to Laplace's rule of succession** Laplace's "add-one" rule of succession modifies the observed frequencies in a sequence of heads and tails by adding one to the observed counts. This improves prediction by avoiding zero probabilities and corresponds to a uniform Bayesian prior on the parameter. We prove that, for any exponential family of distributions, arbitrary Bayesian predictors can be approximated by taking the average of the maximum likelihood predictor and the sequential normalized maximum likelihood predictor from information theory, which generalizes Laplace's rule. The proof heavily involves the geometry provided by the Fisher information matrix. Thus it is possible to approximate Bayesian predictors without the cost of integrating or sampling in parameter space [46].

AMIB Project-Team

6. New Results

6.1. RNA Design

In collaboration with J. Hales, J. Manuch and L. Stacho (Simon Fraser University/Univ. British Columbia, Canada), we have investigated the combinatorial RNA design problem, a minimal instance of the RNA design problem which aims at finding a sequence that admits a given target as its unique base pair maximizing structure. We obtained provide complete characterizations for the structures that can be designed using restricted alphabets. We provided a complete characterization of designable structures without unpaired bases. When unpaired bases are allowed, we provides partial characterizations for classes of designable/undesignable structures, and showed that the class of designable structures is closed under the stutter operation. Membership of a given structure to any of the classes can be tested in linear time and, for positive instances, a solution could be found in linear time. Finally, we considered a structure-approximating version of the problem that allows to extend helices and, assuming that the input structure avoids two motifs, we provided a linear-time algorithm that produces a designable structure with at most twice more base pairs than the input structure, as illustrated by Fig. 3 .

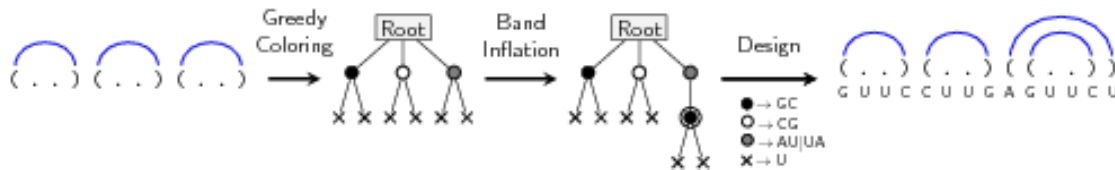


Figure 3. Principle of our structure-approximating version of RNA design: Starting from a potentially undesignable structure, a greedy coloring can be performed and corrected such that the final structure is provably designable in linear time.

Theses results were presented at the CPM 2015 conference in Italy [17], and open new avenues of research, both towards practical, tractable versions of design, and constitute a first step towards long-awaited theoretical foundations for the problem.

6.2. Combinatorics of motifs and algorithms

We developed an $O(n)$ -time and $O(n)$ -space algorithm to compute minimal absent words. Their computation is used in sequence comparison [32] or to detect biologically significant events. For instance, in [52], it was shown that there exist three minimal words in *Ebola* virus genomes which are absent from human genome. The identification of such species-specific sequences may prove to be useful for the development of both diagnosis and therapeutics. In our new contribution [21] we provided an implementation that can be executed in parallel. Experimental results show that excluding the indexing data structure construction time, it achieves near-optimal speed-ups. The computation on the human genome is accelerated by a factor of 10 when using 16 processors, but it consumes a huge amount of RAM. Thus we are currently working on an external memory implementation, that will provide a trade-off between space and time consumption.

Combinatorial tools have been developed to predict the length of repetitions in a random sequence. This allows to distinguish biologically significant repetitions or tune some parameters in assembly or re-sequencing algorithms. For instance, unique mappability is strongly related to the length of the repetitions. A *trie profile* was defined in [45] to address this issue for binary alphabets, by the means of analytic combinatorics. General alphabets, where no closed formula exist, were addressed in [24]. An alternative, and simpler, approach is derived, that exhibits a Large deviation Principle and makes use of Lagrange multipliers. Different domains and transition phases are exhibited. It is expected that this approach extends to a Markov model and to approximate repetitions.

6.3. Structural variants

D. Iakovishina defended in 2015 a PhD thesis co-advised by M. Régnier and V. Boeva (Curie Institute). She proposed a new computational method to detect structural variants using whole genome sequencing data. It combines two techniques that are based either on the detection of paired-end mapping abnormalities or on the detection of the depth of coverage. SV-BAY relies on a probabilistic Bayesian approach and includes a modelization of possible sequencing errors, read mappability profile along the genome and changes in the GC-content. Keeping only somatic SVs is an additional option when matched normal control data are provided. SV-BAY compares favorably with existing tools on simulated and experimental data sets [12] Software SV-BAY is freely available <https://github.com/InstitutCurie/SV-Bay>.

As a side product, a novel exhaustive catalogue of SV types -to date the most comprehensive SV classification- was built. On the grounds of previous publications and experimental data, seven new SV types, ignored by the existing SV calling algorithms, were exhibited.

Structural variations can also be observed and analyzed at larger time scales, and computational methods can be used to predict the structure of ancestral genomes. Within two collaborations with C. Chauve, A. Rajaraman (Simon Fraser University, Canada) and J. Zanetti (SFU, Canada & UniCAMP, Brazil), we revisited the problem of predicting a parsimonious set of adjacencies between ancestral genes, i.e. the most likely structure of an ancestral genome. More specifically, we modified the dynamic programming scheme underlying the DeCo algorithm [28] to compute indicators of robustness for predicting adjacencies. Our reimplementation, which relies on interesting meta-programming strategies, is available at <https://github.com/yannponty/DeClone>.

In a first study, we postulated a Boltzmann-Gibbs distribution over the set of evolutionary scenarii [9]. Our initial experiments relied on Boltzmann sampling to estimate the probabilities of ancestral adjacencies, but our extended version describes an exact polynomial-time computation of such probabilities, through an adaptation of the inside-outside algorithm. We interpreted such probabilities as supports for predicted adjacencies, and found that discarding adjacencies associated with low supports provided a good strategy for resolving syntenic conflicts.

However, the costs associated with the main operations (gaining/breaking adjacencies) in the underlying evolutionary models must be set beforehand in a somewhat arbitrary fashion. This has led us to investigate the influence of those costs on the characteristics of parsimonious predictions, i.e. the robustness of predictions with respect to perturbations of the scoring scheme [18]. To that purpose, we have performed an exact parametric analysis of the DeCo dynamic programming scheme (see Fig. 4 for details). This analysis revealed a quasi-independence, for a large subset of gene trees, of predicted adjacencies to the actual numerical values involved in the scoring scheme.

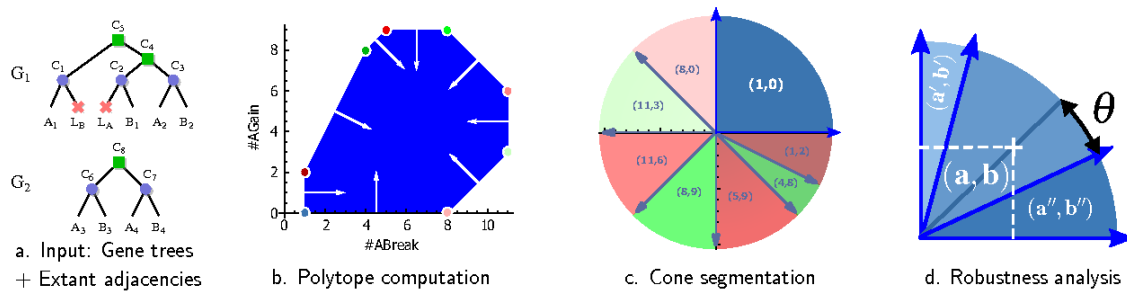


Figure 4. Main steps involved in the parametric prediction of ancestral adjacencies. Starting from two reconciled gene trees and a list of contemporary adjacencies (a.), the polytope of admissible Adjacency Gains/Breaks (+Presence/Absence of a given adjacency) is computed (b.) and projected onto a dual space which partitions the space of cost schemes into (infinite) regions leading to equivalent predictions (c.). The angular distance of the reference cost scheme $(1, 1)$ to a region representing an alternative prediction (d.) is used as a measure of robustness for the prediction.

GALEN Project-Team

7. New Results

7.1. Optimizing Average Precision

Participants: Pawan Kumar

Average precision (AP) is one of the most commonly used measures for ranking. However, due to the inefficiency of optimizing it during learning, a common approach is to use surrogate loss functions such as 0-1 loss. We have developed a novel latent AP-SVM classifier [1], that minimizes a carefully designed upper bound on the AP-based loss function over weakly supervised samples. Using publicly available datasets, we demonstrate the advantage of our approach over standard loss-based learning frameworks on three challenging problems: action classification, character recognition and object detection.

7.2. Region-based Semantic Segmentation

Participants: Pawan Kumar

In [9] we consider the problem of parameter estimation and energy minimization for a region-based semantic segmentation model. The main problem we face in the context of energy minimization, is the large number of putative pixel-to-region assignments. We address this problem by designing an accurate linear programming based approach for selecting the best set of regions from a large dictionary, which is constructed by merging and intersecting segments obtained from multiple bottom-up over-segmentations. The lack of fully supervised data is tackled by using a latent structural SVM formulation, where the latent variables model any missing information in the human annotation. Using large, publicly available datasets we show that our methods are able to significantly improve the accuracy of the region-based model.

7.3. Parsimonious Labeling

Participants: Puneet Dokania, Pawan Kumar

In [22], we propose a new family of discrete energy minimization problems, which we call parsimonious labeling, that is to use as few labels as possible. This allows us to capture useful cues for important computer vision applications such as stereo correspondence and image denoising. Furthermore, we propose an efficient graph-cuts based algorithm for the parsimonious labeling problem that provides strong theoretical guarantees on the quality of the solution. Using both synthetic and standard real datasets, we show that our algorithm significantly outperforms other graph-cuts based approaches.

7.4. Structured Learning and Inference

Participants: Jiaqian Yu, Matthew Blaschko

We have developed computationally efficient structured output prediction methods for learning with non-modular losses [19], [29], [40]. We both demonstrate the feasibility of learning with submodular losses, as well as show that learning with multiple correct outputs can lead to NP-hard optimization problems even when learning with a single correct output is feasible.

7.5. Novel graph kernels

Participants: Katerina Gkirtzou, Matthew Blaschko

We have developed a novel family of graph kernels that are capable of incorporating local curvature properties of 3D meshes [6]. We have additionally demonstrated their application to the modelling of interdependencies between different brain regions in an fMRI based classification task for predicting cocaine addiction.

7.6. Structured Sparsity Regularization & Statistical Hypothesis Testing

Participants: Eugene Belilovsky, Wacha Bounliphone, Katerina Gkirtzou, Andreas Argyriou, Matthew Blaschko

We have developed novel methods for structured sparsity regularization & hypothesis testing. We have applied these methods to fMRI [3], [2], [36] and the analysis of large medical databases [10]. We have also developed novel statistical hypothesis tests for relative dependency [21], [37] and similarity [14]. We have applied these methods to the problem of identifying relative dependencies between languages using a multi-lingual corpus, and for discovering the relative relationships between gliomas and genetic information. Additionally, we have shown the application of relative tests to the problem of model selection in deep generative models, and currently an important question in machine learning.

7.7. High-Order MRF models

Participants: Nikos Komodakis, Nikos Paragios

We developed a very general algorithm for structured prediction learning [7] that is able to efficiently handle discrete MRFs/CRFs (including both pairwise and higher-order models) so long as they can admit a decomposition into tractable subproblems. By properly combining dual decomposition with a max-margin learning method, the framework manages to reduce the training of a complex high-order MRF to the parallel training of a series of simple slave MRFs that are much easier to handle.

7.8. Graph-based registration and segmentation

Participants: Enzo Ferrante, Vivien Fecamp, Aimilia Gastouniotti, Bharat Singh, Stavros Alchatzidis, Nikos Paragios

Deformable image registration plays a fundamental role in many clinical applications. We investigated the use of graphical models in the context of a particular type of image registration problem, known as slice-to-volume registration. We introduced a scalable, modular and flexible formulation that can accommodate low-rank [5] and high order [16] terms, that simultaneously selects the plane and estimates the in-plane deformation through a single shot optimization approach. We applied our models on simulated and real-data in the context of ultrasound and magnetic resonance registration, demonstrating the potential of our methods.

We also developed a novel methodology for graph-based motion-driven segmentation [24] and applied it for carotid plaque segmentation in ultrasound images. We identified the plaque region by exploiting kinematic dependencies between the atherosclerotic and the normal arterial wall. The methodology exploits group-wise image registration towards recovering the deformation field, on which information theory criteria are used to determine dominant motion classes and a map reflecting kinematic dependencies, which is then segmented using Markov random fields.

Moreover, in order to address the problem of general purpose multi-modal deformable registration/fusion we developed a novel and robust method using a metric defined in an appropriate sub-space which is adaptive to the image-content/image-modality [18]. We adopted a graph-based formulation that assumes that intensities of corresponding pixels in the two image domains are related through an unknown piece-wise constant linear function. This relation is propagated to an appropriate sub-space (wavelets coefficients) where a criterion that couples the estimation of the deformation field with optimal transport function on the subspace and the smoothness of the deformation is considered.

7.9. Object Detection from RGB-Depth images

Participant: Siddhartha Chandra, Iasonas Kokkinos

In [11] we explore RGB-Depth representations for the training of Deformable Models, and describe strategies to improve an object detection pipeline by introducing viewpoint based mixture components. Our contributions are threefold. First, we use surface-based object representations (3D mesh models) from available 3D object model repositories to learn strongly supervised viewpoint classifiers. Second, we develop a geometric dataset augmentation scheme that uses scene geometry to ‘take another look’ at the training data, simulating the effect of camera viewpoint changes. Third, to better exploit depth information, we develop a novel depth-based dense feature extraction method that provides a robust statistical description of scene geometry. We evaluate our learned detectors on the common NYU dataset, and demonstrate that each of our advances results in systematic performance improvements over the traditional detection pipeline.

7.10. Deep CNN for Modelling Deformations and Semantic Segmentation

Participant: Iasonas Kokkinos

Invariance to deformations in Deep Convolutional Neural Networks (DCNN) is commonly achieved by using multiple ‘max-pooling’ (MP) layers. In [26] we show that alternative methods of modeling deformations can improve the accuracy and efficiency of DCNNs. For this, (i) we introduce epitomic convolution as an alternative to the common convolution-MP cascade of DCNNs, (ii) we introduce a Multiple Instance Learning algorithm to accommodate global translation and scaling in image classification and (iii) we develop a DCNN sliding window detector that explicitly, but efficiently, searches over the object’s position, scale, and aspect ratio. We provide competitive image classification and localization results on the ImageNet dataset and object detection results on Pascal VOC2007.

In [25] we bring together methods from DCNNs and probabilistic graphical models for addressing the task of pixel-level classification (“semantic image segmentation”). We overcome the poor localization property of deep networks by combining the responses at the final DCNN layer with a fully connected Conditional Random Field (CRF). Qualitatively, our “DeepLab” system is able to localize segment boundaries at a level of accuracy which is beyond previous methods.

7.11. Learning Low-level Image Representations with Deep CNN

Participant: Iasonas Kokkinos

In [27] we propose a novel framework for learning local image descriptors in a discriminative manner. For this purpose we explore a siamese architecture of DCNNs, with a Hinge embedding loss on the L2 distance between descriptors. Since a siamese architecture uses pairs rather than single image patches to train, there exist a large number of positive samples and an exponential number of negative samples. We propose to explore this space with a stochastic sampling of the training set, in combination with an aggressive mining strategy over both the positive and negative samples which we denote as “fracking”. We perform a thorough evaluation of the architecture hyper-parameters, and demonstrate large performance gains compared to both standard CNN learning strategies, hand-crafted image descriptors like SIFT, and the state-of-the-art on learned descriptors: up to 2.5x vs SIFT and 1.5x vs the state-of-the-art in terms of the area under the curve (AUC) of the Precision-Recall curve.

In [4] we explore connections between DCNNs and texture understanding. First, instead of focusing on texture instance and material category recognition, we propose a human-interpretable vocabulary of texture attributes to describe common texture patterns, complemented by a new describable texture dataset for benchmarking. Second, we look at the problem of recognizing materials and texture attributes in realistic imaging conditions, including when textures appear in clutter, developing corresponding benchmarks on top of the recently proposed OpenSurfaces dataset. Third, we revisit classic texture representations, including bag-of-visual-words and the Fisher vectors, in the context of deep learning and show that these have excellent efficiency and generalization properties if the convolutional layers of a deep model are used as filter banks. We obtain in this manner state-of-the-art performance in numerous datasets well beyond textures, an efficient method to apply deep features to image regions, as well as benefit in transferring features from one domain to another.

In [35] we propose a new DCNN architecture that learns pixel embeddings, such that pairwise distances between the embeddings can be used to infer whether or not the pixels lie on the same region. That is, for any two pixels on the same object, the embeddings are trained to be similar; for any pair that straddles an object boundary, the embeddings are trained to be dissimilar. Experimental results show that when this embedding network is used in conjunction with a DCNN trained on semantic segmentation, there is a systematic improvement in per-pixel classification accuracy. Our contributions are integrated in the popular Caffe deep learning framework, and consist in straightforward modifications to convolution routines. As such, they can be exploited for any task involving convolution layers.

7.12. Human-Limb Segmentation for Intelligent Mobility Assistance Robots

Participants: Siddhartha Chandra, Stavros Tsogkas, Iasonas Kokkinos

We developed a computer vision component [12] to be used as part of an intelligent robotic assistant. This component exploits RGB and depth information extracted from Kinect sensors mounted on the robot, to accurately segment human limbs, using fully convolutional neural networks (FCNNs). We trained our network using an in-house Human-Limb dataset composed of video frames, and described a scheme for dynamically exploiting RGB and depth data in a single framework for training and testing. Our method demonstrated promising performance, being very efficient at the same time, with a run-time of 8 frames per second on a single GPU.

M3DISIM Team

7. New Results

7.1. Modeling

7.1.1. *Model-based analysis of continually measured signals of aortic pressure and flow*

Participants: Radomir Chabiniok, Dominique Chapelle [correspondant], Arthur Le Gall, Philippe Moireau, Fabrice Vallée.

We have started an application of reduced-order cardiac modeling in identifying relevant functional properties and state of heart from clinical records obtained during long-term (minutes-hours) monitoring of patients. Those are obtained either from anesthetized or intensive care patients by Fabrice Vallée, medical doctor in the department of anesthesia and intensive care at Lariboisière Hospital, Paris, who has joined the M3DISIM team in November 2015. The collaboration was initiated already in February 2015, and together with Fabrice we supervised the master's internship of Arthur Le Gall (medical doctor in his last year of specialization residency training). The internship took place at Lariboisière Hospital and in our lab at Inria Saclay (50:50%, period of April-September 2015). First published results are expected in 2016, when also a master's internship of a second student of Fabrice Vallée is scheduled. In addition, we intend to start a PhD on this topic in late 2016.

7.1.2. *Thermodynamical framework for modeling chemical-mechanical coupling in muscle contraction – Formulation and validation*

Participants: Matthieu Caruel, Dominique Chapelle [correspondant], Philippe Moireau.

Muscle contraction occurs at the nanoscale of a hierarchical multi-scale structure with the attachment of so-called cross-bridges within sarcomeres, namely, the creation of chemical bonds between myosin heads and specific sites on actin filaments. A cross-bridge in itself can be seen as a special chemical entity having internal mechanical variables – or degrees of freedom – pertaining to the actual geometric configuration, which implies that the free energy of the cross-bridge – whether in an attached or unattached state – must be made dependent on these internal variables (T.L. Hill, *Free Energy Transduction And Biochemical Cycle Kinetics*, Dover, 2004). This provides a thermodynamical basis for modeling the complex interplay of chemical and mechanical phenomena at the sarcomere level. Within this framework we propose a muscle model with two mechanical variables associated with a cross-bridge. For the action of individual cross-bridges occurring at the nanometer scale, the energy provided by the Langevin thermostat cannot be neglected, and we therefore propose to endow the internal mechanical variables with stochastic dynamics. Important motivations for this modeling choice include the ability to represent (i) the so-called power-stroke phenomenon and (ii) short-time responses of a muscle, e.g. to load steps. Our approach allows for systematic treatment of the model energetics, and in particular one goal of the proposed description is to investigate the potential benefit in mechanical efficiency with systems including – in addition to chemically-induced transformations – thermally-induced conformational changes such as the power-stroke.

7.1.3. *Biophysical modeling of seismocardiograms measurements*

Participants: Alexandre Laurin, Sébastien Imperiale [correspondant], Philippe Moireau, Dominique Chapelle.

We are developing models of various levels of complexity to represent seismocardiograms (SCG) that record mechanical thoracic vibrations induced by the beating heart. Our model combines a complete heartbeat model with a mechanical model of the thorax. The coupling is ensured by a unilateral contact modeling the non-penetration between the beating heart and the thoracic chest. In parallel, we are fine-tuning signal processing algorithms to identify the relevant characteristics of SCG and creating an iPhone application that is capable of acquiring the signal with its standard sensors. The application is also developed to integrate a simplified version of the cardio-thoracic model.

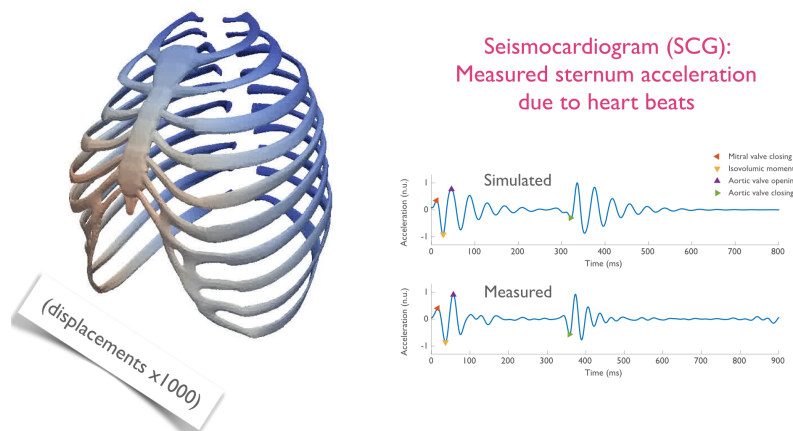


Figure 1. Model-based synthetic SCG compared to a measured SCG.

7.2. Numerical Analysis

7.2.1. Dirichlet-to-Neumann operator for diffraction problems in stratified anisotropic acoustic waveguides

Participants: Antoine Tonnoir [correspondant], Sonia Fliss [Poems team], Anne-Sophie Bonnet-Ben Dhia [Poems team].

In this work, we are interested in the construction of a Dirichlet-to-Neumann operator for the diffraction problem in stratified anisotropic acoustic waveguides. The key idea consists in using an adapted change of coordinates that allows to recover the completeness and the orthogonality of the modes on “deformed” cross-sections of the waveguide. Thus, we can properly define the diffraction problem and construct transparent boundary conditions to reformulate this problem in a bounded domain. Using classical arguments we easily prove the well-posedness. The method has also been implemented in a C++ code and has been validated.

7.2.2. Fourth-order energy-preserving locally implicit time discretization for linear wave equations

Participants: Sébastien Imperiale [correspondant], Juliette Chabassier [MAGIQUE-3D team].

In collaboration with Juliette Chabassier, we have constructed a family of fourth-order implicit-explicit time schemes for linear wave equations. Our application is the simulation of elastic waves propagation in a locally stiff medium. The domain of propagation is decomposed into several regions where different fourth-order time discretization are used, chosen among a family of implicit (for the stiff regions) or explicit fourth-order schemes. The coupling is based on a Lagrangian formulation on the boundaries between several non-conforming meshes of the regions. A global discrete energy is shown to be preserved and leads to global fourth-order consistency in time. Numerical results in 1D and 2D illustrate the good behavior of the schemes and their potential for the efficient simulation of realistic highly heterogeneous media for which using an explicit scheme everywhere can be extremely penalizing. Accuracy up to fourth-order reduces the numerical dispersion inherent to implicit methods used with a large time step, and makes this family of schemes attractive compared to second-order accurate methods.

7.2.3. Numerical methods for poromechanics: Applications to cardiac perfusion

Participants: Bruno Burtschell, Dominique Chapelle [correspondant], Philippe Moireau.

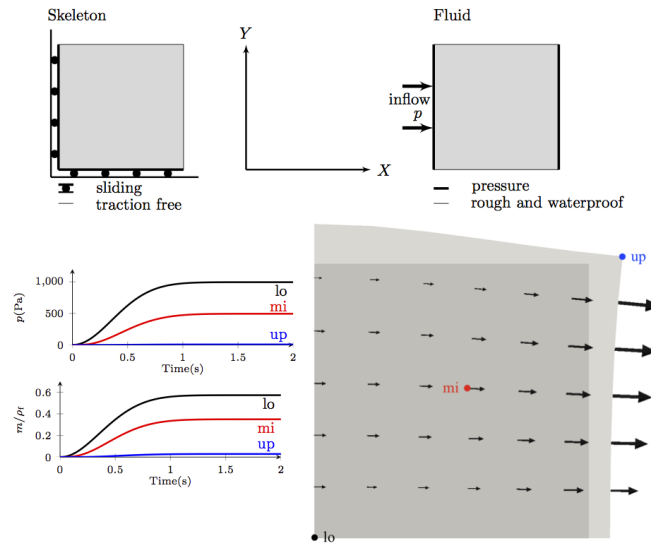


Figure 2. Swelling of a porous sample. On the left, fluid pressure and mass at three points in time during the swelling test. On the right, the fluid velocity field vector at the steady state in the deformed configuration.

We have previously formulated a rather general modeling framework of poromechanics – formulations that combine solid and fluid components to represent the behavior of a porous medium – to take into account large deformations and rapid fluid flows, see [6]. This allows to consider, in particular, the application of blood perfusion within the cardiac tissue, which features these specific complex phenomena, out of the scope of classical poromechanical models. One of our major objectives now, within the PhD of Bruno Burtschell, is to propose and analyse some associated relevant numerical schemes.

Some existing algorithms of fluid-structure interaction, with which our poromechanics formulations feature deep similarities, have been implemented – in FreeFEM++, both in axisymmetric configuration and in 3D – and compared. Their numerical and theoretical analysis – consistency, convergence – has been performed. Then, the adaptation of these algorithms to our poromechanics formulations enabled us to propose a time discretisation well-fitted to our framework, and to present its energy stability analysis. Spatial discretizations issues have also been specifically addressed, based on a complete analysis performed on a linearized problem, in order to guarantee pressure stability – via the selection of adequate inf-sup-compatible discretization spaces – including when the solid constituent is nearly or fully incompressible. Implementation and detailed numerical validations of these schemes have been performed. Integration into FELISCE (“HappyHeart” module) in 3D, and into a reduced model of cardiac cycle to take into account myocardium perfusion, are ongoing work.

7.3. Model-Data Interaction

7.3.1. Displacement Reconstructions in Ultrasound Elastography

Participant: Sébastien Imperiale.

In collaboration with Guillaume Bal (Columbia University, New York, USA), we have considered the reconstruction of internal elastic displacements from ultrasound measurements, which finds applications in the medical imaging modality called elastography. By appropriate interferometry and windowed Fourier transforms of the ultrasound measurements, we have proposed a reconstruction procedure of the vectorial

structure of spatially varying elastic displacements in biological tissues. This provides a modeling and generalization of scalar reconstruction procedures routinely used in elastography. The proposed algorithm has been justified using a single scattering approximation and local asymptotic analysis. Its validity has been assessed by numerical simulations.

7.3.2. *Recursive joint state and parameter estimation*

Participants: Atte Aalto, Philippe Moireau [correspondant].

We propose a method for estimating the parameters of a linear dynamical system from noisy measurements over a given, finite time, interval. For this purpose we develop a recursive modification of the joint state and parameter estimation method proposed in [7]. As the time interval is fixed, any errors in the initial state of the system may cause a significant error in the parameter estimate. Therefore, the parameter estimator is complemented by the so called back and forth nudging (BFN) method for estimating the system's initial state. The proposed strategy can also be regarded as a hybrid least squares optimization method for minimizing the quadratic discrepancy between the measured and simulated outputs over the set of all possible initial states and system parameters.

The optimality of the BFN method with colocated feedback has been considered as well. We have shown that in the case when the system's dynamics are governed by a skew-adjoint generator, the initial state estimate given by the BFN method converges to the minimizer of the quadratic output discrepancy – provided that the observer gains are chosen suitably. If the system's generator is essentially skew-adjoint and dissipative, a certain modification of the feedback operator is required in order to obtain such convergence.

7.3.3. *Convergence of discrete-time Kalman filter estimate to continuous-time estimate*

Participant: Atte Aalto [correspondant].

The Kalman(-Bucy) filter gives the optimal (minimum variance) solution to the state estimation problem for linear systems with Gaussian initial state, and white input and output noise processes. The implementation of the discrete-time Kalman filter is straightforward as it is readily formulated in an algorithmic manner. Thus, it may be tempting to use the discrete-time filter on the time-sampled continuous-time system. We study the convergence of the state estimate obtained from the discrete-time Kalman filter to the continuous-time estimate as the temporal discretization is refined. The convergence follows from the martingale convergence theorem, but surprisingly, no results exist on the rate of convergence. We derive convergence rate estimates for the discrete-time estimate under a number of different sets of assumptions starting from finite-dimensional systems and infinite-dimensional systems with bounded output operators and then proceeding to systems with unbounded output operators and systems with analytic semigroups. The proofs are based on applying the discrete-time Kalman filter on a dense, numerable subset of the time interval of interest, and bounding the change in the state estimate as the new data points are being added. These bounds, in turn, are based on smoothness estimates of the noise-free output.

7.3.4. *Observers for the wave equation in unbounded domains*

Participants: Sébastien Imperiale, Philippe Moireau [correspondant], Antoine Tonnoir, Sonia Fliss [Poems team], Karim Ramdani [Sphinx team].

We are interested in the reconstruction of initial data for the wave equation problem in unbounded domains using an observer strategy. A major advantage of this method for problems set in bounded domains is the exponential convergence of the algorithm of reconstruction. In our case, the specificity is the unboundedness of the domain which requires to bound it with artificial boundaries for numerical computations. To avoid spurious reflections due to these artificial boundaries, we consider transparent boundary conditions. The difficulty then is to adapt the classical observers technique to this case. Indeed, after enough time, the outgoing waves have left the computational domain and the related information is in some sense “lost”.

First results have been obtained for the 1D case: the theoretical proof of the (exponential) convergence of the algorithm has been done, and the method has been numerically validated. We are currently working on the extension to the 2D case, which raises new difficulties. In particular, the construction of the transparent boundary condition is not obvious and implies a non-local operator in both time and space. Due to this non-local operator, the theoretical analysis of the convergence of the method is then much more difficult.

7.3.5. A Luenberger observer for reaction-diffusion models with front position data

Participants: Dominique Chapelle, Annabelle Collin, Philippe Moireau [correspondant].

We propose a Luenberger observer for reaction-diffusion models with propagating front features, and for data associated with the location of the front over time. Such models are considered in various application fields, such as electrophysiology, wild-land fire propagation and tumor growth modeling. Drawing our inspiration from image processing methods by considering a data similarity measure of Mumford-Shah type, we start by proposing an observer for the eikonal-curvature equation that can be derived from the reaction-diffusion model by an asymptotic expansion. We then carry over this observer to the underlying reaction-diffusion equation by an “inverse asymptotic analysis”, and we show that the associated correction in the dynamics has a stabilizing effect for the linearized estimation error. We also discuss the extension to joint state-parameter estimation by using the earlier-proposed ROUKF strategy. We published a first work [17] where the observer feedback is derived from the shape-derivative of the data similarity measure. Then, in [21], in order to improve the observer formulation, we followed a strategy of increasing importance in shape optimization or “level-set”-based image segmentation by complementing the required shape derivatives, used to modify the shape contours, by a topological derivative that represents the sensitivity of the similarity measure when removing a small part of the domain. Both results are illustrated with test problems pertaining to electrophysiology modeling, including with a realistic model of cardiac atria. Our numerical trials show that state estimation is directly very effective with the proposed Luenberger observer.

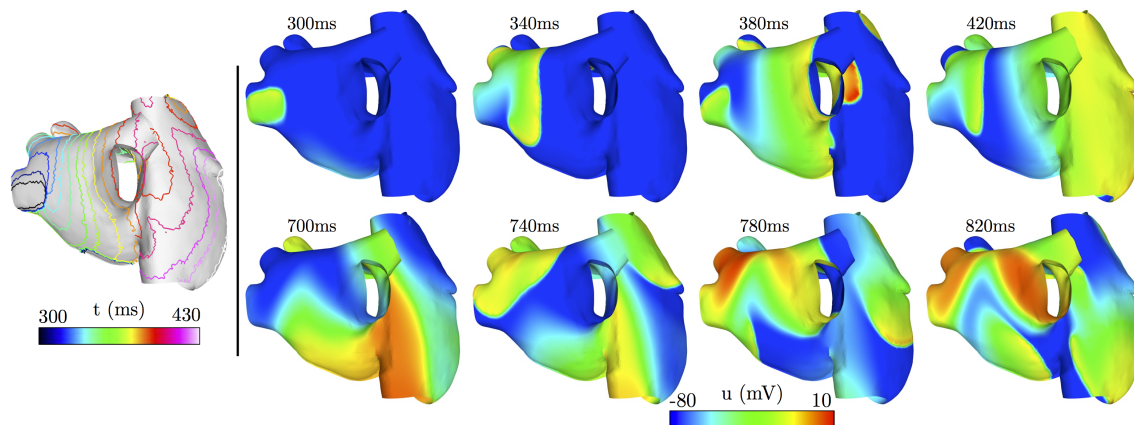


Figure 3. Atrial fibrillation: synthetic front data with noise (left, note that only the first times of passage after the onset of fibrillation are displayed, for the sake of clarity), and corresponding observer solutions (right)

7.3.6. Identification of weakly coupled multiphysics problems. Application to the inverse problem of electrocardiography

Participants: Cesare Corrado [Reo team], Jean-Frédéric Gerbeau [Reo team], Philippe Moireau [correspondant].

This work addresses the inverse problem of electrocardiography from a new perspective, by combining electrical and mechanical measurements. Our strategy relies on the definition of a model of the electromechanical contraction which is registered on ECG data, but also on measured mechanical displacements of the heart tissue typically extracted from medical images. In this respect, we establish in this work the convergence of a sequential estimator which combines for such coupled problems various state-of-the-art sequential data assimilation methods in a unified consistent and efficient framework. Indeed, we aggregate a Luenberger observer for the mechanical state and a Reduced-Order Unscented Kalman Filter applied on the parameters to be identified and a POD projection of the electrical state. Then, using synthetic data we show the benefits of our approach for the estimation of the electrical state of the ventricles along the heart beat, compared with more classical strategies that only consider an electrophysiological model with ECG measurements. Our numerical results actually show that the mechanical measurements improve the identifiability of the electrical problem, allowing to reconstruct the electrical state of the coupled system more precisely. Therefore, this work is intended to be a first proof of concept, with theoretical justifications and numerical investigations, of the advantage of using available multi-modal observations for the estimation and identification of an electromechanical model of the heart.

7.3.7. Data assimilation of cine-MR images by a biophysical model

Participants: Radomir Chabiniok, Dominique Chapelle [correspondant], Alexandra Groth, Philippe Moireau, Juergen Weese.

Within the European project VP2HF, we participated in extending the image segmentation tool developed by Philips Hamburg (Alexandra Groth, Jürgen Weese) to process clinically routine cine-MR images for creating anatomical models of heart. Secondly, together with A. Groth and J. Weese we defined a discrepancy operator – between a biomechanical heart model and cine-MR images – that does not require segmenting MR images prior to data assimilation. Initial results of the state estimation using this discrepancy operator were presented at the 2nd VP2HF evaluation meeting (December 2015), and extending these results into a journal paper is a joint objective of the M3DISIM team and of Philips Hamburg.

PARIETAL Project-Team

7. New Results

7.1. Semi-Supervised Factored Logistic Regression for High-Dimensional Neuroimaging Data

Imaging neuroscience links human behavior to aspects of brain biology in ever-increasing datasets. Existing neuroimaging methods typically perform either discovery of unknown neural structure or testing of neural structure associated with mental tasks. However, testing hypotheses on the neural correlates underlying larger sets of mental tasks necessitates adequate representations for the observations. We therefore propose to blend representation modelling and task classification into a unified statistical learning problem. A multinomial logistic regression is introduced that is constrained by factored coefficients and coupled with an auto-encoder. We show that this approach yields more accurate and interpretable neural models of psychological tasks in a reference dataset, as well as better generalization to other datasets.

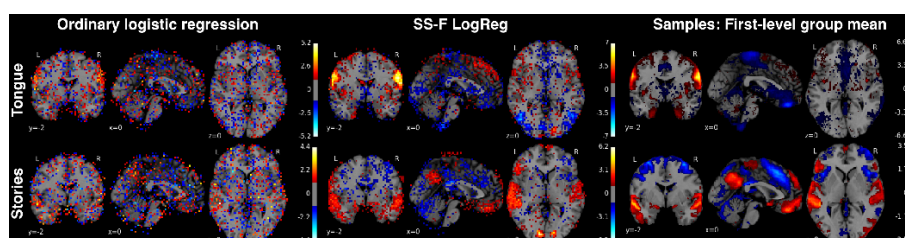


Figure 3. Classification weight maps. The voxel predictors corresponding to 2 exemplary (of 18 total) psychological tasks (rows) from the Human Connectome Project dataset. Left column: multinomial logistic regression (same implementation but without bottleneck or autoencoder), middle column: Semi-Supervised Factored Logistic Regression (SSFLogReg), right column: voxel-wise average across all samples of whole-brain activity maps from each task. SSFLogReg puts higher absolute weights on relevant structure, lowers ones on irrelevant structure, and yields BOLD-typical local contiguity (without enforcing an explicit spatial prior). More information can be found in [50].

More information can be found in [50].

7.2. NeuroVault.org: a web-based repository for collecting and sharing unthresholded statistical maps of the human brain

Here we present NeuroVault — a web based repository that allows researchers to store, share, visualize, and decode statistical maps of the human brain. NeuroVault is easy to use and employs modern web technologies to provide informative visualization of data without the need to install additional software. In addition, it leverages the power of the Neurosynth database to provide cognitive decoding of deposited maps. The data are exposed through a public REST API enabling other services and tools to take advantage of it. NeuroVault is a new resource for researchers interested in conducting meta- and coactivation analyses.

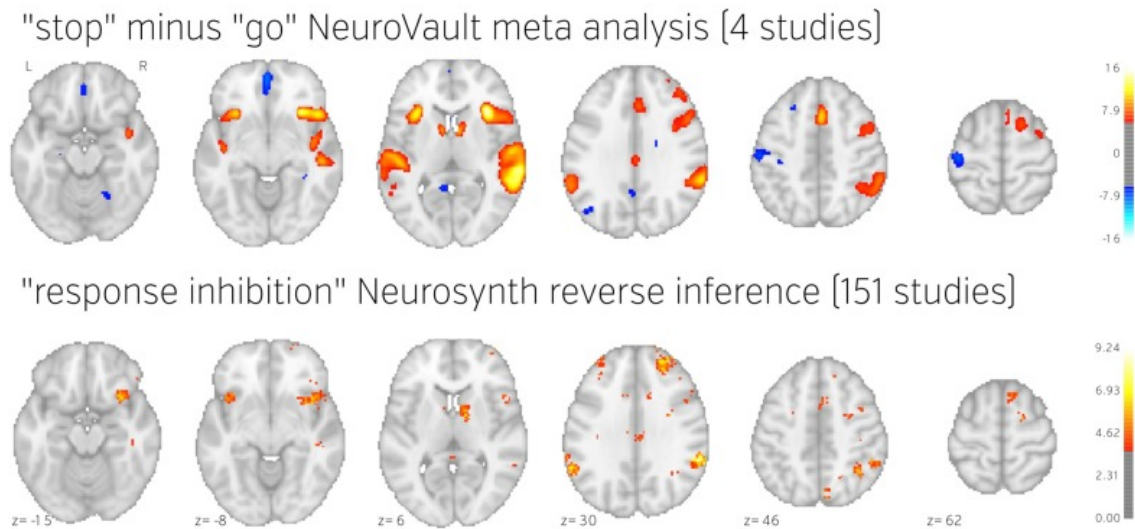


Figure 4.

Comparison of image based and coordinate based meta analysis of response inhibition. Meta analysis based on unthresholded statistical maps obtained from NeuroVault (top row) managed to recover the pattern of activation obtained using traditional methods despite including much fewer studies. NeuroVault map has been thresholded at $z = 6$, response inhibition map has been thresholded at $z = 1.77$ (the threshold values were chosen for visualization purposes only, but both are statistically significant at $p < 0.05$). Unthresholded versions of these maps are available at <http://neurovault.org/collections/439/>

More information can be found in [18] and [17].

7.3. FAASTA: A fast solver for total-variation regularization of ill-conditioned problems with application to brain imaging

The total variation (TV) penalty, as many other analysis-sparsity problems, does not lead to separable factors or a proximal operator with a closed-form expression, such as soft thresholding for the ℓ_1 penalty. As a result, in a variational formulation of an inverse problem or statistical learning estimation, it leads to challenging non-smooth optimization problems that are often solved with elaborate single-step first-order methods. When the data-fit term arises from empirical measurements, as in brain imaging, it is often very ill-conditioned and without simple structure. In this situation, in proximal splitting methods, the computation cost of the gradient step can easily dominate each iteration. Thus it is beneficial to minimize the number of gradient steps. We present fAASTA, a variant of FISTA, that relies on an internal solver for the TV proximal operator, and refines its tolerance to balance computational cost of the gradient and the proximal steps. We give benchmarks and illustrations on “brain decoding”: recovering brain maps from noisy measurements to predict observed behavior. The algorithm as well as the empirical study of convergence speed are valuable for any non-exact proximal operator, in particular analysis-sparsity problems.

Convergence of currently available optimization algorithms, for 3 scenarios, with weak, medium and strong regularization, where medium regularization corresponds to the value chosen by cross-validation. These are log-log plots with the 0 defined as the lowest energy value reached across all algorithms.

More information can be found in [47].

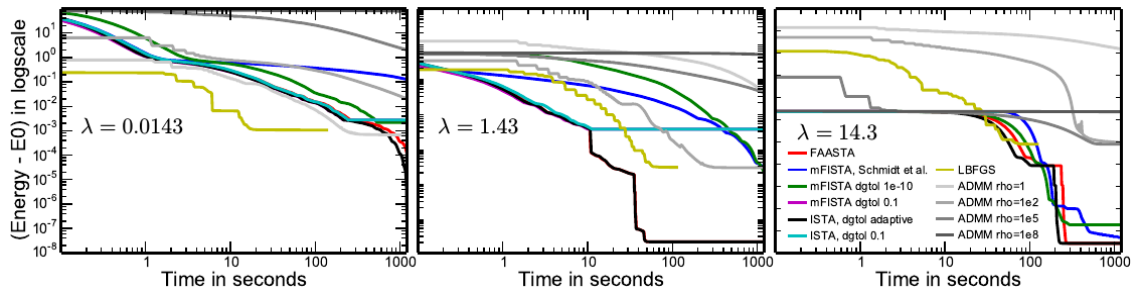


Figure 5.

7.4. Bootstrapped Permutation Test for Multiresponse Inference on Brain Behavior Associations.

Despite that diagnosis of neurological disorders commonly involves a collection of behavioral assessments, most neuroimaging studies investigating the associations between brain and behavior largely analyze each behavioral measure in isolation. To jointly model multiple behavioral scores, sparse multi-response regression (SMR) is often used. However, directly applying SMR without statistically controlling for false positives could result in many spurious findings. For models, such as SMR, where the distribution of the model parameters is unknown, permutation test and stability selection are typically used to control for false positives. In this paper, we present another technique for inferring statistically significant features from models with unknown parameter distribution. We refer to this technique as bootstrapped permutation test (BPT), which uses Studentized statistics to exploit the intuition that the variability in parameter estimates associated with relevant features would likely be higher with responses permuted. On synthetic data, we show that BPT provides higher sensitivity in identifying relevant features from the SMR model than permutation test and stability selection, while retaining strong control on the false positive rate. We further apply BPT to study the associations between brain connectivity estimated from pseudo-rest fMRI data of 1139 fourteen year olds and behavioral measures related to ADHD. Significant connections are found between brain networks known to be implicated in the behavioral tasks involved. Moreover, we validate the identified connections by fitting a regression model on pseudo-rest data with only those connections and applying this model on resting state fMRI data of 337 left out subjects to predict their behavioral scores. The predicted scores are shown to significantly correlate with the actual scores of the subjects, hence verifying the behavioral relevance of the found connections.

Real data results: Statistically significant connectivity differences between populations (a) Significant network connections found on pseudo-rest fMRI data. (b) Pearson's correlation between predicted and actual scores with p-values noted. Each set of three bars (top to bottom) correspond to spatial working memory strategy, spatial working memory between errors, and rapid visual information processing accuracy scores. Significance is declared at $p < 0.05$.

More information can be found in [43].

7.5. Total Variation meets Sparsity: statistical learning with segmenting penalties

Prediction from medical images is a valuable aid to diagnosis. For instance, anatomical MR images can reveal certain disease conditions, while their functional counterparts can predict neuropsychiatric phenotypes. However, a physician will not rely on predictions by black-box models: understanding the anatomical or functional features that underpin decision is critical. Generally, the weight vectors of classifiers are not

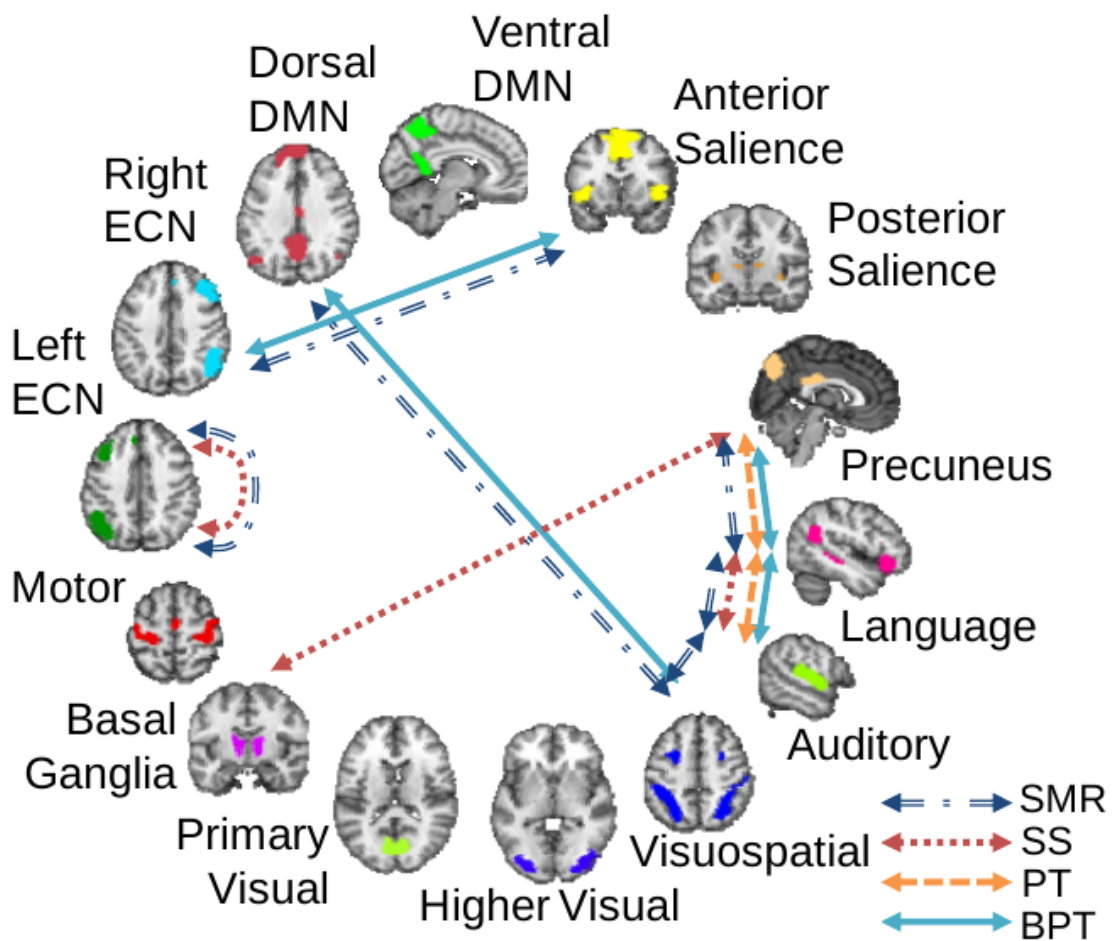


Figure 6.

easily amenable to such an examination: Often there is no apparent structure. Indeed, this is not only a prediction task, but also an inverse problem that calls for adequate regularization. We address this challenge by introducing a convex region-selecting penalty. Our penalty combines total-variation regularization, enforcing spatial contiguity, and l_1 regularization, enforcing sparsity, into one group: Voxels are either active with non-zero spatial derivative or zero with inactive spatial derivative. This leads to segmenting contiguous spatial regions (inside which the signal can vary freely) against a background of zeros. Such segmentation of medical images in a target-informed manner is an important analysis tool. On several prediction problems from brain MRI, the penalty shows good segmentation. Given the size of medical images, computational efficiency is key. Keeping this in mind, we contribute an efficient optimization scheme that brings significant computational gains.

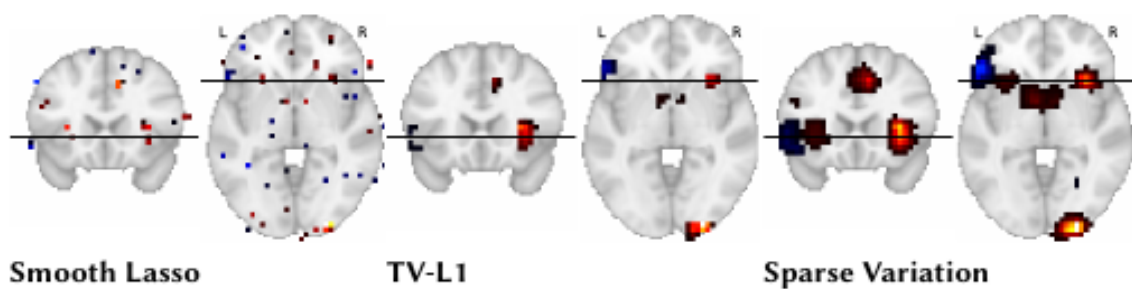


Figure 7.

Weight vectors from estimating gain on the mixed gambles task for three sparse methods: Graphnet, TV-11 and Sparse Variation. This inter- subject analysis shows broader regions of activation. Mean correlation scores on held out data: Graphnet: 0.128, TV-11 : 0.147, Sparse Variation: 0.149. One can see that both TV-11 and Sparse Variation regularizations yield more interpretable patterns than Graphnet.

More information can be found in [40].

7.6. Improving sparse recovery on structured images with bagged clustering

The identification of image regions associated with external variables through discriminative approaches yields ill-posed estimation problems. This estimation challenge can be tackled by imposing sparse solutions. However, the sensitivity of sparse estimators to correlated variables leads to non-reproducible results, and only a subset of the important variables are selected. In this paper, we explore an approach based on bagging clustering-based data compression in order to alleviate the instability of sparse models. Specifically, we design a new framework in which the estimator is built by averaging multiple models estimated after feature clustering, to improve the conditioning of the model. We show that this combination of model averaging with spatially consistent compression can have the virtuous effect of increasing the stability of the weight maps, allowing a better interpretation of the results. Finally, we demonstrate the benefit of our approach on several predictive modeling problems.

Z-score obtained across bootstraps for two discriminative tasks, using the candidate approaches. Higher values hint at lower variability across bootstrap replications. SCLR decreases the variability and yields larger standardized effects.

More information can be found in [42].

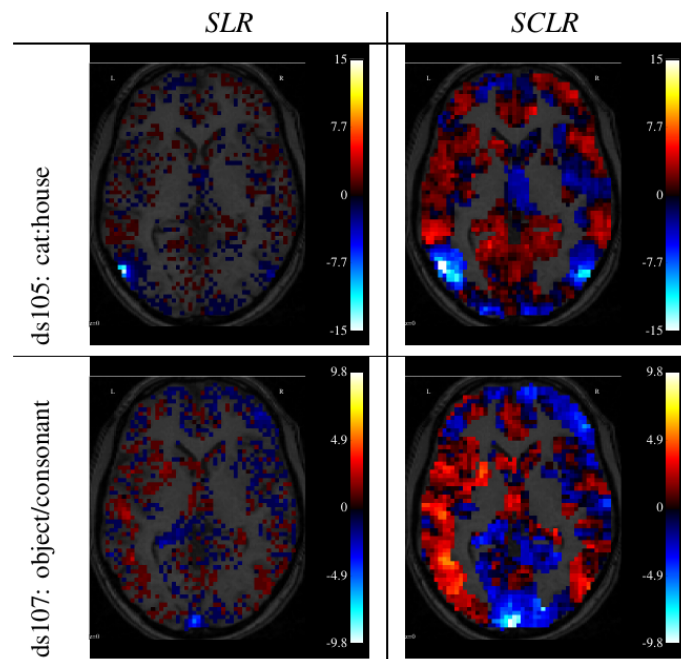


Figure 8.

7.7. Integrating Multimodal Priors in Predictive Models for the Functional Characterization of Alzheimer’s Disease

Functional brain imaging provides key information to characterize neurodegenerative diseases, such as Alzheimer’s disease (AD). Specifically, the metabolic activity measured through fluorodeoxyglucose positron emission tomography (FDG-PET) and the connectivity extracted from resting-state functional magnetic resonance imaging (fMRI), are promising biomarkers that can be used for early assessment and prognosis of the disease and to understand its mechanisms. FDG-PET is the best suited functional marker so far, as it gives a reliable quantitative measure, but is invasive. On the other hand, non-invasive fMRI acquisitions do not provide a straightforward quantification of brain functional activity. To analyze populations solely based on resting-state fMRI, we propose an approach that leverages a metabolic prior learned from FDG-PET. More formally, our classification framework embeds population priors learned from another modality at the voxel-level, which can be seen as a regularization term in the analysis. Experimental results show that our PET-informed approach increases classification accuracy compared to pure fMRI approaches and highlights regions known to be impacted by the disease.

Overview of the proposed classification pipeline: The inputs are ROI-to-voxel connectivities computed from the rs-fMRI time-series. FDG-PET model weights are integrated as prior for the classification. Then, predictions of all ROIs are the inputs of a stacking model to predict the clinical group.

More information can be found in [44].

7.8. Inverse problems with time-frequency dictionaries and Gaussian non-white noise

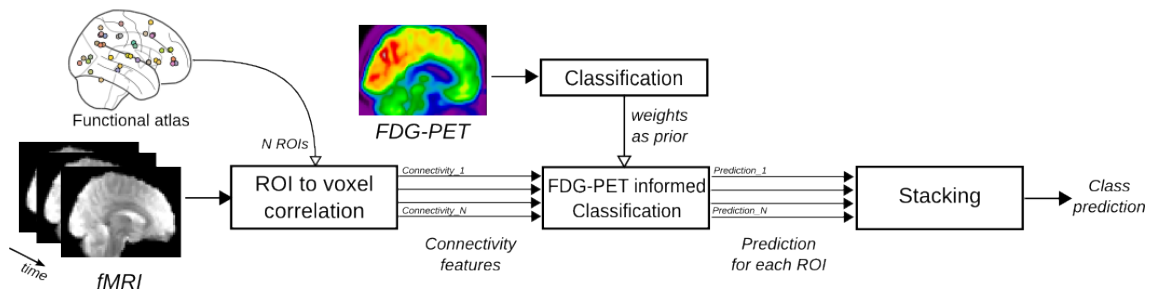


Figure 9.

Sparse regressions to solve ill-posed inverse problems have been massively investigated over the last decade. Yet, when noise is present in the model, it is almost exclusively considered as Gaussian and white. While this assumption can hold in practice it rarely holds when observations are time series as they are corrupted by auto-correlated and colored noise. In this work we study sparse regression under the assumption of non white Gaussian noise and explain how to run the inference using proximal gradient methods. We investigate an application in brain imaging: the problem of source localization using magneto- and electroencephalography (M/EEG) which allow functional brain imaging with high temporal resolution. We use a time-frequency representation of the source waveforms and a sparse regularization which promotes focal sources with smooth and transient activations. Our approach is evaluated using simulations comparing it to strategies that assume the noise is white or to simple prewhitening.

More information can be found in [30].

7.9. Variable density sampling based on physically plausible gradient waveform. Application to 3D MRI angiography

Performing k-space variable density sampling is a popular way of reducing scanning time in Magnetic Resonance Imaging (MRI). Unfortunately, given a sampling trajectory, it is not clear how to traverse it using gradient waveforms. In this paper, we actually show that existing methods can yield large traversal time if the trajectory contains high curvature areas. Therefore, we consider here a new method for gradient waveform design which is based on the projection of unrealistic initial trajectory onto the set of hardware constraints. Next, we show on realistic simulations that this algorithm allows implementing variable density trajectories resulting from the piecewise linear solution of the Travelling Salesman Problem in a reasonable time. Finally, we demonstrate the application of this approach to 2D MRI reconstruction and 3D angiography in the mouse brain.

Full k-space acquisition with an EPI sequence (a) and corresponding reference image (f). Comparison between an exact parameterization of the TSP trajectory (b) and projection from Travelling Salesman Problem trajectory onto the set of constraints (c),(d). In experiments (b,c), the number of measured locations is fixed to 9% ($r = 11.2$), whereas in (b,d), the time to traverse the curve is fixed to 62 ms. (e): Spiral trajectory with acquisition of the k-space center. (g-j): Reconstructed images corresponding to sampling strategies (b-e).

More information can be found in [38].

7.10. A projection method on measures sets.

We consider the problem of projecting a probability measure π on a set MN of Radon measures. The projection is defined as a solution of the following variational problem:

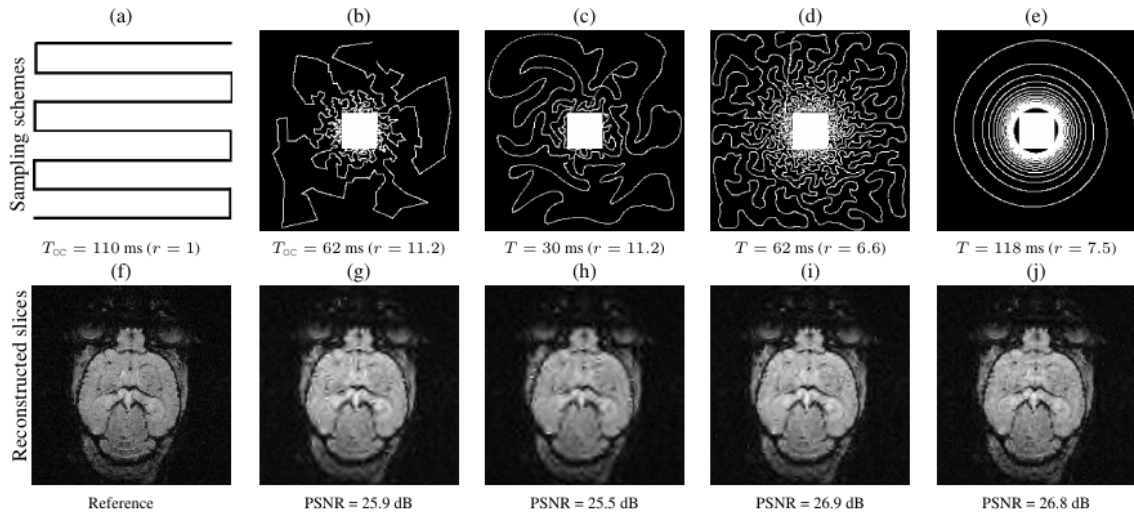


Figure 10.

$$\inf_{\mu \in \mathcal{M}_N} \|h(\mu - \pi)\|_2^2,$$

where $h \in L^2(\Omega)$ is a kernel, $\Omega \subset \mathbb{R}^d$ and denotes the convolution operator. To motivate and illustrate our study, we show that this problem arises naturally in various practical image rendering problems such as stippling (representing an image with N dots) or continuous line drawing (representing an image with a continuous line). We provide a necessary and sufficient condition on the sequence $(\mathcal{M}_N)_{N \in \mathbb{N}}$ that ensures weak convergence of the projections $(\mu_N^*)_{N \in \mathbb{N}}$ to π . We then provide a numerical algorithm to solve a discretized version of the problem and show several illustrations related to computer-assisted synthesis of artistic paintings/drawings.

Projection of a lion image onto $P_N^{1,\infty}$ with $N = 8,000$. The figure depicts the resulting line with several values of the iterates of our Algorithm.

More information can be found in [55].

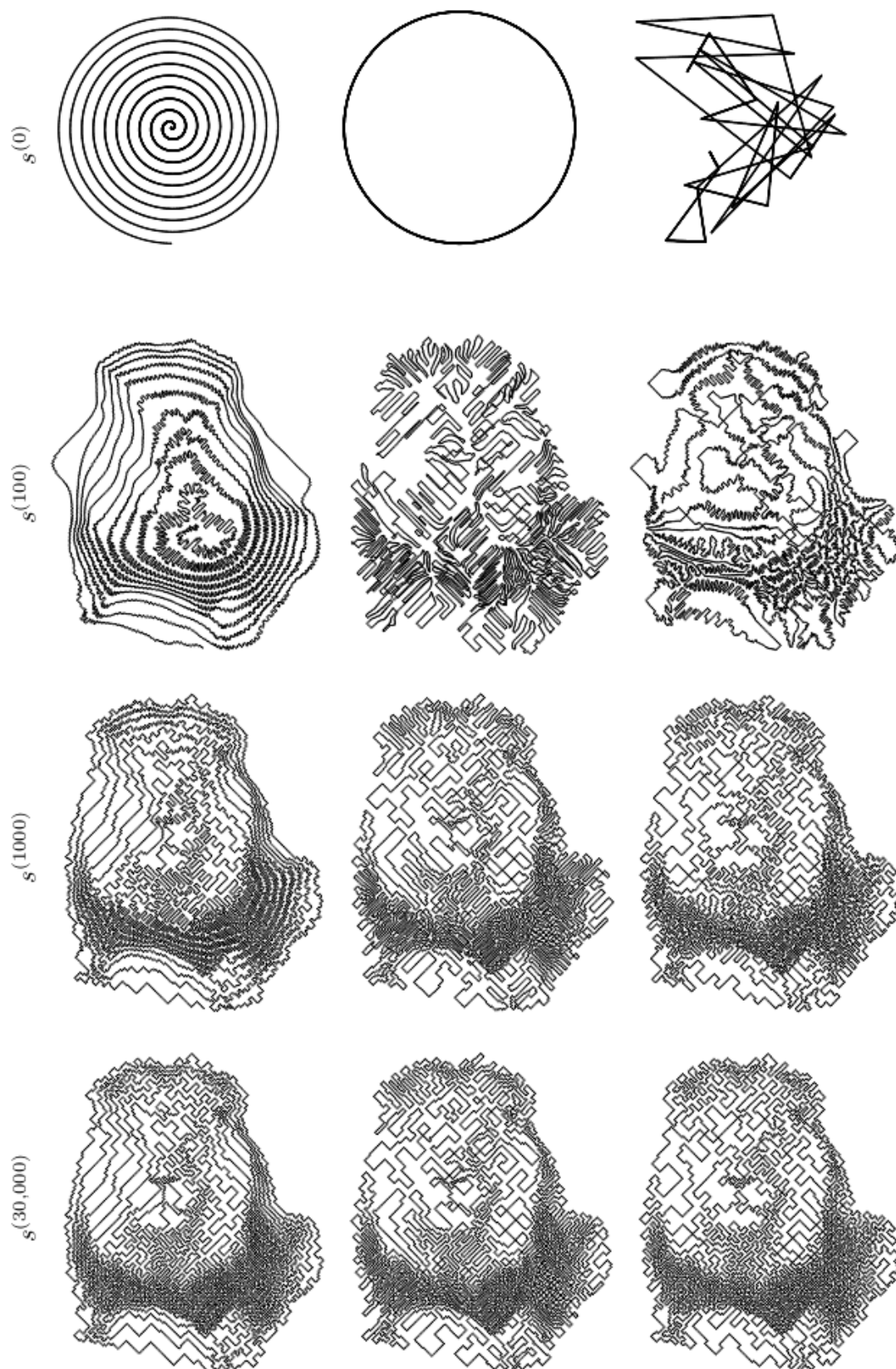


Figure 11.

POPIX Team

7. New Results

7.1. Model identifiability

We have discussed the question of model identifiability within the context of nonlinear mixed effects models. Although there has been extensive research in the area of fixed effects models, much less attention has been paid to random effects models. In this context we distinguish between theoretical identifiability, in which different parameter values lead to non-identical probability distributions, structural identifiability which concerns the algebraic properties of the structural model, and practical identifiability, whereby the model may be theoretically identifiable but the design of the experiment may make parameter estimation difficult and imprecise. We have explored a number of pharmacokinetic models which are known to be non-identifiable at an individual level but can become identifiable at the population level if a number of specific assumptions on the probabilistic model hold. Essentially if the probabilistic models are different, even though the structural models are non-identifiable, then they will lead to different likelihoods.

7.2. Model of tumor growth

Both molecular profiling of tumors and longitudinal tumor size data modeling are relevant strategies to predict cancer patients' response to treatment. Herein we have proposed a model of tumor growth inhibition integrating a tumor's genetic characteristics that successfully describes the time course of tumor size in patients with low-grade gliomas treated with first-line temozolomide chemotherapy. The model captures potential tumor progression under chemotherapy by accounting for the emergence of tissue resistance to treatment following prolonged exposure to temozolomide. Using information on individual tumors' genetic characteristics, in addition to early tumor size measurements, the model was able to predict the duration and magnitude of response, especially in those patients in whom repeated assessment of tumor response was obtained during the first 3 months of treatment. Combining longitudinal tumor size quantitative modeling with a tumor's genetic characterization appears as a promising strategy to personalize treatments in patients with low-grade gliomas

7.3. Methods for PDEs based model

We have extended the methodologies previously developed for ordinary differential equations (ODE) to partial differential equations (PDE). A finite element method solver for a given family of PDEs has been developed. This solver can now be used with a prototype version of Monolix, a platform for population modeling of longitudinal data. We have implemented the well-known Lagrange finite element method in one, two and three dimensions of the space.

INFINE Team

6. New Results

6.1. Online Social Networks (OSN)

Community detection; bandit algorithms; privacy preservation; reward mechanisms

6.1.1. Community detection

Participants: Laurent Massoulié, Marc Lelarge, Charles Bordenave.

We have progressed in the design of spectral methods for community detection and in the corresponding analysis, in particular by proving the so-called spectral redemption conjecture. This has been published in IEEE FOCS'15. The abstract of the paper is as follows. A non-backtracking walk on a graph is a directed path such that no edge is the inverse of its preceding edge. The nonbacktracking matrix of a graph is indexed by its directed edges and can be used to count non-backtracking walks of a given length. It has been used recently in the context of community detection and has appeared previously in connection with the Ihara zeta function and in some generalizations of Ramanujan graphs. In this work, we study the largest eigenvalues of the non-backtracking matrix of the Erdős-Rényi random graph and of the Stochastic Block Model in the regime where the number ℓ of edges is proportional to the number of vertices. Our results confirm the “spectral redemption conjecture” that community detection can be made on the basis of the leading eigenvectors above the feasibility threshold.

6.1.2. Bandit algorithms for active learning of content type at low spam cost

Participants: Laurent Massoulié, Mesrob Ohanessian, Alexandre Proutière.

Progress on “bandit algorithms” for targeted news dissemination. We developed a framework in which to cast the problem, and the so-called “greedy Bayes” algorithm to determine which user to expose to a given content. We proved corresponding optimality properties, and observed that “greedy Bayes” beats the so-called Thompson sampling approach, that is the state-of-the-art method in bandit problems. This work was published at ACM Sigmetrics'15.

6.1.3. Clustering and Inference From Pairwise Comparisons

Participants: Rui Wu, Jiaming Xu, Srikant Rayadurgam, Marc Lelarge, Laurent Massoulié, Bruce Hajek.

In a short publication at ACM Sigmetrics'15, we do the following. Given a set of pairwise comparisons, the classical ranking problem computes a single ranking that best represents the preferences of all users. In this paper, we study the problem of inferring individual preferences, arising in the context of making personalized recommendations. In particular, we assume users form clusters; users of the same cluster provide similar pairwise comparisons for the items according to the Bradley-Terry model. We propose an efficient algorithm to estimate the preference for each user: first, compute the net-win vector for each user using the comparisons; second, cluster the users based on the net-win vectors; third, estimate a single preference for each cluster separately. We show that the net-win vectors are much less noisy than the high dimensional vectors of pairwise comparisons, therefore our algorithm can cluster the users reliably. Moreover, we show that, when a cluster is only approximately correct, the maximum likelihood estimation for the Bradley-Terry model is still close to the true preference.

6.2. Spontaneous Wireless Networks and Internet of Things

internet of things; wireless sensor networks; dissemination; resource management

6.2.1. Platform Design for the Internet of Things

Participants: Emmanuel Baccelli, Cedric Adjih, Oliver Hahm, Matthias Waehlich, Thomas Schmidt, Hauke Petersen.

Within this activity, we have further developed the platforms we champion for the Internet of Things: the open source operating system RIOT and open-access IoT-lab testbeds. RIOT now aggregates open source contributions from 120+ people (and counting) from all over the world, coming both from academia and from industry, and received financial backing from top companies including Cisco and Google in 2015. Revisiting concepts from the early Internet, we have designed and introduced a new software architecture that fits the (memory, CPU, energy) constraints of low-end IoT devices, while being full-featured and easily extensible, thus more future-proof than state of the art. This work was published in ACM MobiSys'15 (IoT-Sys workshop), and released as open source code, integrated in the latest version of RIOT 2015-12. We have also designed a distributed test framework which supports advanced continuous integration techniques, allows for the integration of project contributors to volunteer hardware and software resources to the test system, and can function as a permanent distributed plugtest for network interoperability testing. This work was published in ACM MobiSys'15 (IoT-Sys workshop). Concerning IoT-lab, we have contributed to the completion of the design and the roll-out of IoT-lab testbeds in multiple sites in France and started deploying an additional one in Berlin. Description of completed work and design was published in IEEE IoT-WF'15.

6.2.2. Standards for Spontaneous Wireless Networks

Participant: Emmanuel Baccelli.

Within this activity, we have contributed to new network protocol standards for spontaneous wireless networking, applied to ad hoc networks and the Internet of Things. In particular, collaborating with Fraunhofer, we have published Directional Airtime Metric (DAT), a new wireless metric standard targeting wireless mesh networks. The standard is in the RFC editor's queue (which means the corresponding IETF standard, an RFC, will be published within weeks). Furthermore, collaborating with ARM and Sigma Designs, we published RFC 7733, which provides guidance in the configuration and use of protocols from the RPL protocol suite to implement the features required for control in building and home environments. In collaboration with various industrial partners, we have also published a number of other Internet drafts, including an analysis of the characteristics of multi-hop ad hoc wireless communication between interfaces in the context of IP networks, and an analysis of the challenges of information-centric networking in the Internet of Things.

6.3. Resource and Traffic Management

Traffic offloading; infrastructure deployment; opportunistic routing; traffic modeling; intermittently connected networks.

6.3.1. On the Interaction between Content Caching and Routing

Participants: Kolar Purushothama Naveen, Laurent Massoulié, Emmanuel Baccelli, Aline Carneiro Viana, Don Towsley.

Nowadays Internet users are mobile over 60% of their time online, and mobile data traffic is expected to increase by more than 60% annually to reach 15.9 exabytes per month by 2018. This evolution will likely incur durably congested wireless access at the edge despite progress in radio technologies. To alleviate congestion at the Internet edge, one promising approach is to target denser deployments of wireless access points. As a result, mobile users are potentially within radio reach of several access points (AP) from which content may be directly downloaded. In this context, distinct AP's can have very different bandwidth and memory capacities. Such differences raise the following question: When requests can be sent to several such access points, how to optimize performance through both load balancing and content replication?

In this work, we introduce formal optimization models to address this question, where bandwidth availability is represented via a cost function, and content availability is represented either by a cost function or a sharp constraint. For both formulations we propose dynamic caching and request assignment algorithms. Crucially our request assignment scheme is based on a server price signal jointly reflecting content and bandwidth availability. Using mean field approximation and Lyapunov functions techniques, we prove that our algorithms are optimal and stable in a limiting fluid regime with large arrival rates and content chunking. Through simulations we exhibit the efficacy of our request assignment strategy in comparison to the common practices

of assigning requests purely based on either bandwidth or content availability. Finally, using the popular LRU (Least Recently Used) strategy instead for cache replacements, we again demonstrate the superior performance of our request assignment strategies. This work was published in the ACM SIGCOMM'15 workshop on All Things Cellular.

6.3.2. *From Routine to Network Deployment for Data Offloading in Metropolitan Areas*

Participants: Eduardo Mucceli, Aline Carneiro Viana.

Smartphone sales are booming, nearly half billion were sold in 2011; more smartphones, more mobile data traffic, and Currently, 3G cellular networks in metropolitan areas are struggling to attend the recent boost up of mobile data consumption. Carefully deploying WiFi hotspots allow to maximize WiFi offloading and can both be cheaper than upgrade the cellular network structure and concede substantial improvement in the network capacity. In this context, in this work, we first propose a new way to map into a graph the *people behavior* (i.e., mobility context) in an urban scenario. Our proposed behavior-to-graph solution is simple, take into consideration the restrictions imposed by transportation modes to traffic demand, the space-time interaction between people and urban locations, and finally, is powerful to be used as input to any popular area identification problem (key points for an efficient network planning). Secondly, we propose a metric to identify locations more capable of providing coverage for people and consequently, more suitable for receiving hotspots. Deploying a small percentage of hotspots ranked by the herein proposed metric provides high percentages of coverage time for people moving around in the city. Using a real-life metropolitan trace, we show our routine-based strategy guarantees higher offload ratio than current approaches in the literature while using a realistic traffic model. This work, including new characterization results of the used trace and new analysis of space-traffic correlation, is under submission in a transaction.

6.3.3. *Mobile Data Traffic Modeling: Revealing Hidden Facets*

Participants: Eduardo Mucceli, Aline Carneiro Viana, Kolar Purushothama Naveen, Carlos Sarraute.

Smartphone devices provide today the best means of gathering users information about content consumption behavior on a large scale. In this context, the literature is rich in work studying and modeling users mobility, but little is publicly known about users content consumption patterns. The *understanding of users' mobile data traffic demands* is of fundamental importance when looking for solutions to manage the recent boost up of mobile data usage and to improve the quality of communication service provided. Hence, the definition of a *usage pattern* can allow telecommunication operators to better foresee future demanded traffic and consequently, to better (1) deploy data offloading hotspots or (2) timely plan network resources allocation and then, set subscription plans.

Using a large-scale dataset collected from a major 3G network in a big metropolitan area, in this work, we present the first detailed measurement-driven modeling of mobile data traffic usage of smartphone subscribers. Our main outcome is a synthetic measurement-based mobile data traffic generator, capable of imitating traffic-related activity patterns of different categories of subscribers and time periods of a routinary normal day in their lives. For this, we first characterize individual subscribers routinary behaviour, followed by the detailed investigation of subscribers' usage pattern (i.e., "when" and "how much" traffic is generated). Broadly, our observations bring important insights into network resource usage. We then classify the subscribers into six distinct profiles according to their usage pattern and model these profiles according to two different journey periods: peak and non-peak hours. We show that the synthetic trace generated by our data traffic model consistently imitates different subscriber profiles in two journey periods, when compared to the original dataset. We discuss relevant issues in traffic demands and describe implications in network planning and privacy. This work, including a new characterization results of the used trace, including analysis correlating age and gender to traffic demands, as well as new profiling results, is under submission in a transaction.

6.3.4. *Data Delivery in Opportunistic and Intermittently Connected Networks*

Participants: Ana Cristina Vendramin, Anelise Munaretto, Myriam Delgado, Aline Carneiro Viana, Mauro Fonseca.

The pervasiveness of computing devices and the emergence of new applications and cloud services are factors emphasizing the increasing need for adaptive networking solutions. In most cases, this adaptation requires the design of interdisciplinary approaches as those inspired by nature, social structures, games, and control systems. The approach presented in this work brings together solutions from different, yet complementary domains, i.e., networking, artificial intelligence, and complex networks, and is aimed at addressing the problem of efficient data delivery in intermittently connected networks.

As mobile devices become increasingly powerful in terms of communication capabilities, the appearance of opportunistic and intermittently connected networks referred to as Delay Tolerant Networks (DTNs) is becoming a reality. In such networks, contacts occur opportunistically in corporate environments such as conferences sites, urban areas, or university campuses. Understanding node mobility is of fundamental importance in DTNs when designing new communication protocols that consider opportunistic encounters among nodes. This work proposes the Cultural Greedy Ant (CGrAnt) protocol to solve the problem of data delivery in opportunistic and intermittently connected networks. CGrAnt is a hybrid Swarm Intelligence-based forwarding protocol designed to address the dynamic and complex environment of DTNs. CGrAnt is based on: (1) Cultural Algorithms (CA) and Ant Colony Optimization (ACO) and (2) operational metrics that characterize the opportunistic social connectivity between wireless users. The most promising message forwarders are selected via a greedy transition rule based on local and global information captured from the DTN environment. Using simulations, we first analyze the influence of the ACO operators and CA knowledge on the CGrAnt performance. We then compare the performance of CGrAnt with the PROPHET and Epidemic protocols (two well known related protocols in the literature) under varying networking parameters. The results show that CGrAnt achieves the highest delivery ratio (gains of 99.12% compared with PROPHET and 40.21% compared with Epidemic) and the lowest message replication (63.60% lower than PROPHET and 60.84% lower than Epidemic). This work is under submission to an international journal.

6.3.5. Designing Adaptive Replication Schemes in Distributed Content Delivery Networks

Participants: Mathieu Leconte, Marc Lelarge, Laurent Massoulié.

In a paper published at the ITC'15 conference, we address the problem of content replication in large distributed content delivery networks, composed of a data center assisted by many small servers with limited capabilities and located at the edge of the network. We aim at optimizing the placement of contents on the servers to offload the data center as much as possible. We model the sub-system constituted by the small servers as a loss network, each loss corresponding to a request to the data center. Based on large system / storage behavior, we obtain an asymptotic formula for the optimal replication of contents and propose adaptive schemes to attain it by reacting to losses, as well as faster algorithms which can react before losses occur. We show through simulations that our adaptive schemes outperform significantly standard replication strategies both in terms of loss rates and adaptation speed.

6.3.6. Vehicular Network under a Social Perception

Participants: Felipe D. Cunha, Aline Carneiro Viana, Raquel A. F. Mini, Antonio A.f. Loureiro.

Vehicular Mobility is strongly influenced by the speed limits, destinations, traffic conditions, period of the day, and direction of the public roads. At the same time, the driver's behavior produces great influences in vehicular mobility. People tend to go to the same places, at the same day period, through the same trajectories, which lead them to the appearance of driver's daily routines. These routines lead us to the study of mobility in VANETs under a social perspective and to investigate how effective is to explore social interactions in this kind of network. In this work, we thus characterize and evaluate social properties of a realistic vehicular trace found in literature. Our aim is to study the vehicles' mobility in accordance to social behaviors. Social metrics are computed and the obtained results are compared to random graphs. With our analysis, we could verify the existence of regularity and common interests among the drivers in vehicular networks.

After having identified routine in vehicles mobility patterns and their correlation with the period of the day, we then leverage the identified social aspects to design a *Socially Inspired Broadcast Data Dissemination* for VANETs. We claim that protocols and applications designed for Vehicular Ad Hoc Networks need to adapt to

vehicles routines in order to provide better services. With this issue in mind, we designed a data dissemination solution for these networks that considers the daily road traffic variation of large cities and the relationship among vehicles. The focus of our approach is to select the best vehicles to rebroadcast data messages according to social metrics, in particular, the clustering coefficient and the node degree. Moreover, our solution is designed in such a way that it is completely independent of the perceived road traffic density. Simulation results show that, when compared to related protocols, our proposal provides better delivery guarantees, reduces the network overhead and possesses an acceptable delay.

6.3.7. Design and Analysis of an Efficient Friend-to-Friend Content Dissemination System

Participants: Kanchana Thilakarathna, Aline Carneiro Viana, Aruna Seneviratne, Henrik Petander.

In this work, we focus on dissemination of content for delay tolerant applications/services, (i.e. content sharing, advertisement propagation, etc.) where users are geographically clustered into communities. Due to emerging security and privacy concerns, majority of users are becoming more reluctant to interact with strangers and are only willing to share information/content with the users who are previously identified as friends. As a result, despite its promise, opportunistic communications systems have not been widely adopted. In addition, in this environment, opportunistic communication will not be effective due to the lack of known friends within the communication range. We thus propose a novel architecture which combines the advantages of distributed decentralized storage and opportunistic communications. The proposed system addresses the trust and privacy concerns of opportunistic communications systems, and enables the provision of efficient distributed mobile social networking services. We exploit the fact that users will trust their friends, and the friends will help in disseminating content by temporarily storing and forwarding content. This can be done by replicating content on friends' devices who are likely to consume that content and provide the content to other friends when the device has access to low cost networks. The fundamental challenge then is to minimize the number of replicas, to ensure high and timely availability. We provide a formal definition of this content replication problem, and show that it is NP hard. Then, we propose a community based greedy heuristic algorithm with novel dynamic centrality metrics that replicates the content on a minimum number of friends' devices, and maximizes the availability of content. Using both real world and synthetic traces, we validate effectiveness of the proposed scheme. In addition, we demonstrate the practicality of the the proposed system, through an implementation on Android smartphones. This work is under submission in an international transaction.

6.3.8. Telling Apart Social and Random Relationships in Dynamic Networks

Participants: Pedro Olmo Vaz de Melo, Aline Carneiro Viana, Marco Fiore, Katia Jaffrès-Runser, Frédéric Le Mouël, Antonio A. F. Loureiro, Lavanya Addepalli, Guangshuo Chen.

Recent studies have analyzed data generated from mobile individuals in urban regions, such as cab drivers or students in large campuses. Particular attention has been paid to the dynamics of user movement, whose real-world complexity cannot be fully captured through synthetic models. Indeed, understanding user mobility is of fundamental importance when designing new communication protocols that exploit opportunistic encounters among users. In this case, the problem mainly lies in correctly forecasting future contacts. To that end, the regularity of daily activities comes in handy, as it enforces periodic (and thus predictable) space-time patterns in human mobility. Although human behavior is characterized by an elevated rate of regularity, random events are always possible in the routines of individuals. Those are hardly predictable situations that deviate from the regular pattern and are unlikely to repeat in the future.

We argue that the ability to accurately spot random and social relationships in dynamic networks is essential to network applications that rely on a precise description of human routines, such as recommendation systems, forwarding strategies and opportunistic dissemination protocols. We thus propose a strategy to analyze users' interactions in mobile networks where users act according to their interests and activity dynamics. Our strategy, named *Random rElationship ClAssifier sTrategy (RECAST)*, allows classifying users' wireless interactions, separating random interactions from different kinds of social ties. To that end, RECAST observes how the real system differs from an equivalent one where entities' decisions are completely random. We evaluate the effectiveness of the RECAST classification on five real-world user contact datasets collected in diverse networking contexts. Our analysis unveils significant differences among the dynamics of users' wireless

interactions in the datasets, which we leverage to unveil the impact of social ties on opportunistic routing. We show that, for such specific purpose, the relationships inferred by classifier are more relevant than, e.g., self-declared friendships on Facebook. This work was published in 2015 at the Performance Evaluation Elsevier Journal [9].

AVIZ Project-Team

7. New Results

7.1. Design Considerations for Composite Physical Visualizations

Participants: Mathieu Le Goc [correspondant], Pierre Dragicevic, Samuel Huron, Jean-Daniel Fekete.

Physical visualization has existed for thousands of years, yet the Information Visualization community is just starting to study it. Many current physical visualizations are monolithic, static, and not interactive. Some of them are made of multiple individual objects that can be rearranged in order to represent a variety of informative configurations. We call them composite physical visualizations. A major benefit of such visualizations is that they support modularity and updatability, but their design space is not well understood.

We show [28] that composite physical visualizations can be classified according to two orthogonal dimensions: i) their level of actuation and ii) their manipulability. Among existing systems, some have a high manipulability but no support for actuation, while others are fully actuated but not manipulable. Only a few systems are combining both qualities and none supports both full manipulability and full actuation. We discuss the tradeoffs between these two dimensions, and identify the opportunities and challenges for future research and design.

7.2. Design Considerations for Enhancing Word-Scale Visualizations with Interaction

Participants: Pascal Goffin, Wesley Willett, Jean-Daniel Fekete, Petra Isenberg.

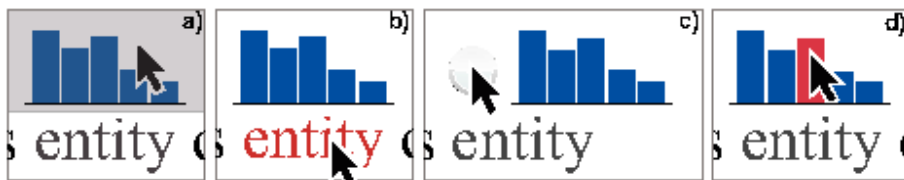


Figure 8. Illustration of where interaction can take place in the context of word-scale visualizations.

This paper presents a design space for interaction with word-scale visualizations. Most sparklines and word-scale visualizations are static and do not support any interaction. However, when word-scale visualizations are used in digital environments, interaction can enhance their use by allowing various data manipulation and management operations. Our design space covers where (Figure 8), when, and how interaction can be triggered for word-scale visualizations embedded in a text document. It also includes how and when to transition from a view where the text with word-scale visualizations is the focus (document-centric view) to a view in which the visualizations becomes the reading focus (visualization-centric view).

7.3. Drawing Characteristics for Reproducing Traditional Hand-Made Stippling

Participants: Domingo Martín, Vicente Del Sol, Celia Romo, Tobias Isenberg [correspondant].

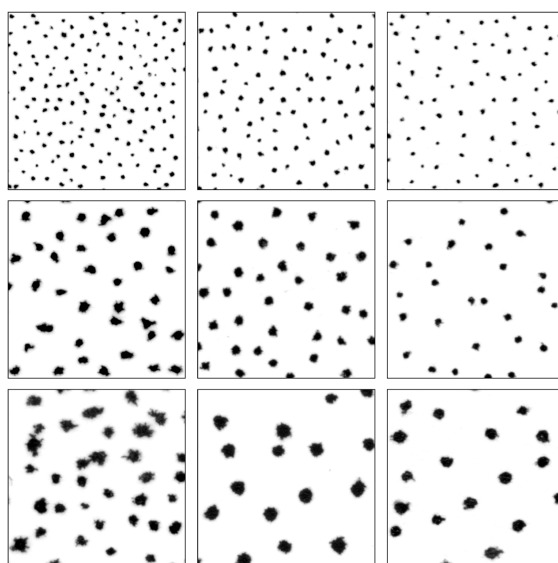


Figure 9. Samples from the stipple dot experiments.

We contribute an in-depth analysis of the characteristics of traditional stippling and relate these to common practices in NPAR stippling techniques as well as to the abilities and limitations of existing printing and display technology. We focus specifically on the properties of stipple dots and consider the dimensions and attributes of pens and paper types used in artistic practice (see Figure 9). With our analysis we work toward an understanding of the requirements for digital stippling, with the ultimate goal to provide tools to artists and illustrators that can replicate the stippling process faithfully in the digital domain. From the results of our study we provide a dataset for use in new example-based stippling techniques, derive a taxonomy of characteristics and conditions for the reproduction of stippling, and define future directions of work.

More on the project Web page: <http://tobias.isenberg.cc/VideosAndDemos/Martin2015DCR>.

7.4. Evaluation of an IEC Framework for Guided Visual Search

Participants: Nadia Boukhelifa [correspondant], Anastasia Bezerianos, Waldo Cancino, Evelynne Lutton.

We evaluated and analysed a framework for Evolutionary Visual Exploration (EVE) [13] (Figure 10) that guides users in exploring large search spaces. EVE uses an interactive evolutionary algorithm to steer the exploration of multidimensional datasets towards two dimensional projections that are interesting to the analyst. This method smoothly combines automatically calculated metrics and user input in order to propose pertinent views to the user. We revisited this framework and a prototype application that was developed as a demonstrator, and summarized our previous study with domain experts and its main findings. We then reported on results from a new user study with a clear predefined task that examined how users leverage the system and how the system evolved to match their needs.

While previously we showed that using EVE, domain experts were able to formulate interesting hypotheses and reach new insights when exploring freely, the new findings indicated that users, guided by the interactive evolutionary algorithm, were able to converge quickly to an interesting view of their data when a clear task was specified. We provided a detailed analysis of how users interact with an evolutionary algorithm and how the system responded to their exploration strategies and evaluation patterns. This line of work aims at building a bridge between the domains of visual analytics and interactive evolution. The benefits are

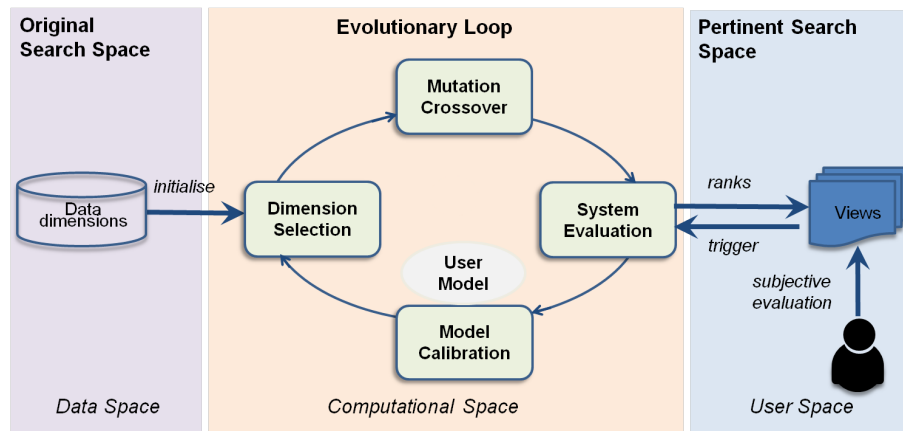


Figure 10. Illustration of the evolutionary visual exploration framework.

numerous, in particular for evaluating Interactive Evolutionary Computation (IEC) techniques based on user study methodologies.

Next, we summarized and reflected upon our experience in evaluating our guided exploratory visualization system [34]. This system guided users in their exploration of multidimensional datasets to pertinent views of their data, where the notion of pertinence is defined by automatic indicators, such as the amount of visual patterns in the view, and subjective user feedback obtained during their interaction with the tool. To evaluate this type of system, we argued for deploying a collection of validation methods that are: user-centered, observing the utility and effectiveness of the system for the end-user; and algorithm-centered, analysing the computational behaviour of the system. We reported on observations and lessons learnt from working with expert users both for the design and the evaluation of our system.

More on the project Web page: <http://www.aviz.fr/EVE><http://www.aviz.fr/EVE>.

7.5. Exploring the Effect of Word-Scale Visualizations on Reading Behavior

Participants: Pascal Goffin, Wesley Willett, Anastasia Bezerianos, Petra Isenberg.

We studied how the integration of small visualizations (word-scale visualizations) into a sentence affects reading speed and memorization during a brief reading task. In particular, we were interested in how different placement types—with their inherent text appearance and layout changes—affect readers. We designed a quantitative study in which we gave sentences with or without small visualizations for participants to read (study conditions are shown in Figure 11). Then, we invited them to answer questions on the sentences. We found that the information encoded in the visualizations is more prominent and easily remembered than information in the written text, but that different placement options had little to no effect on reading performance, even if participants had different preferences.

7.6. Exploration of the Brain's White Matter Structure through Visual Abstraction and Multi-Scale Local Fiber Tract Contraction

Participants: Maarten H. Everts, Eric Begue, Henk Bekker, Jos B. T. M. Roerdink, Tobias Isenberg [correspondant].

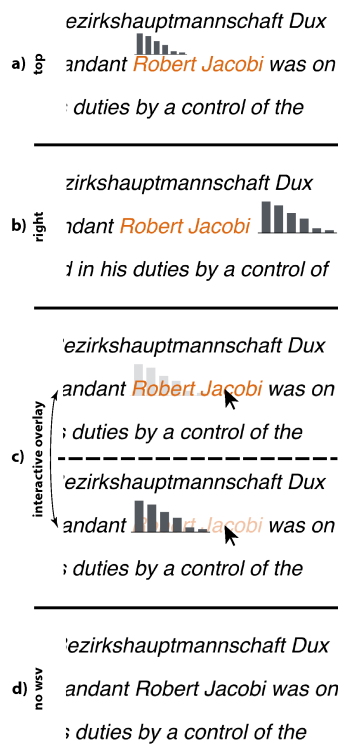


Figure 11. Illustration of the study conditions.

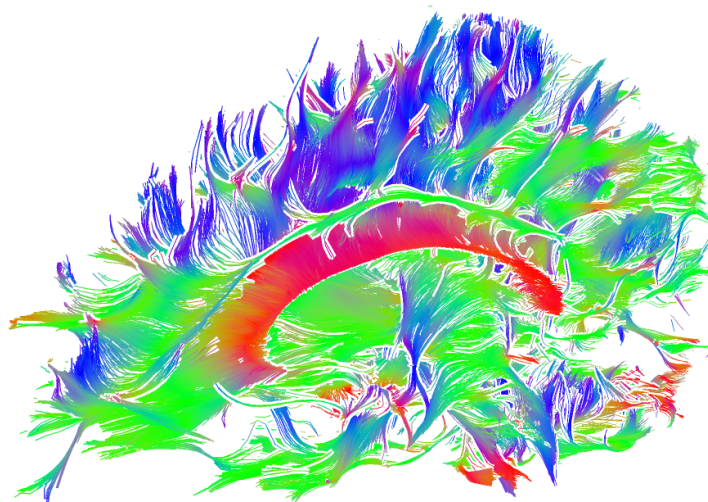


Figure 12. Contraction of the white matter fiber tracts in the brain.

We developed a visualization technique for brain fiber tracts from DTI data that provides insight into the structure of white matter through visual abstraction. We achieve this abstraction by analyzing the local similarity of tract segment directions at different scales using a stepwise increase of the search range. Next, locally similar tract segments are moved toward each other in an iterative process, resulting in a local contraction of tracts perpendicular to the local tract direction at a given scale. This not only leads to the abstraction of the global structure of the white matter as represented by the tracts, but also creates volumetric voids (see Figure 12). This increase of empty space decreases the mutual occlusion of tracts and, consequently, results in a better understanding of the brain's three-dimensional fiber tract structure. Our implementation supports an interactive and continuous transition between the original and the abstracted representations via various scale levels of similarity. We also support the selection of groups of tracts, which are highlighted and rendered with the abstracted visualization as context.

More on the project Web page: <http://tobias.isenberg.cc/VideosAndDemos/Everts2015EBW>.

7.7. Interactive Illustrative Line Styles and Line Style Transfer Functions for Flow Visualization

Participants: Maarten H. Everts, Henk Bekker, Jos B. T. M. Roerdink, Tobias Isenberg [correspondant].

We present a flexible illustrative line style model for the visualization of streamline data. Our model partitions view-oriented line strips into parallel bands whose basic visual properties can be controlled independently. We thus extend previous line stylization techniques specifically for visualization purposes by allowing the parametrization of these bands based on the local line data attributes. Moreover, our approach supports emphasis and abstraction by introducing line style transfer functions that map local line attribute values to complete line styles. With a flexible GPU implementation of this line style model we enable the interactive exploration of visual representations of streamlines. We demonstrate the effectiveness of our model by applying it to 3D flow field datasets (see Figure 13).

More on the project Web page: <http://tobias.isenberg.cc/VideosAndDemos/Everts2015IIL>.

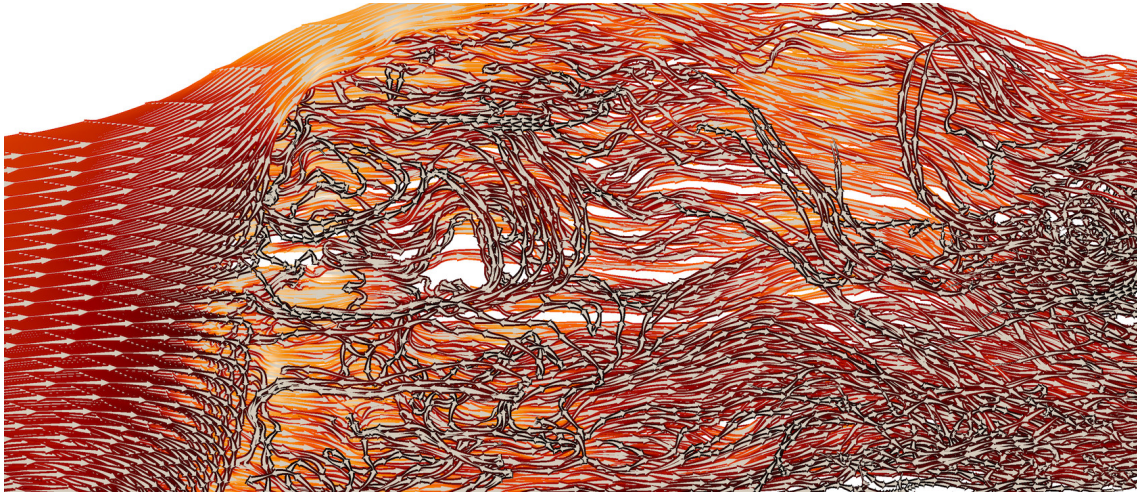


Figure 13. Illustrative line styles applied to streamline visualization of a complex 3D flow.

7.8. Research Agenda for Data Physicalization

Participants: Yvonne Jansen, Pierre Dragicevic [correspondant], Petra Isenberg, Jason Alexander, Abhijit Karnik, Johan Kildal, Sriram Subramanian, Kasper Hornbæk.

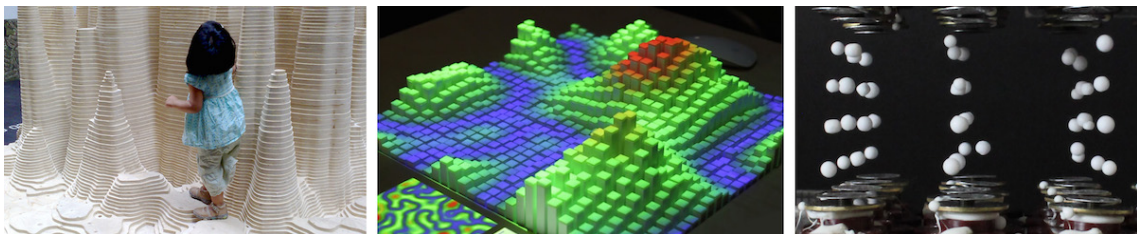


Figure 14. Three examples of data physicalizations.

Physical representations of data have existed for thousands of years. Yet it is now that advances in digital fabrication, actuated tangible interfaces, and shape-changing displays are spurring an emerging area of research that we call Data Physicalization. It aims to help people explore, understand, and communicate data using computer-supported physical data representations. We call these representations physicalizations, analogously to visualizations – their purely visual counterpart. We joined our efforts with research teams from Europe and published a research agenda where we go beyond the focused research questions addressed so far by delineating the research area, synthesizing its open challenges and laying out opportunities for future work. Examples can be seen in Figure 14 .

More on the Data Physicalization Wiki: dataphys.org/.

7.9. Storytelling and Engagement

Participants: Jeremy Boy, Jean-Daniel Fekete, Françoise Detienne.

We conducted three web-based field experiments, in which we evaluated the impact of using initial narrative visualization techniques and storytelling on user-engagement with exploratory information visualizations. We conducted these experiments on a popular news and opinion outlet, and on a popular visualization gallery website. While data journalism exposes visualizations to a large public, we do not know how effectively this public makes sense of interactive graphics, and in particular if people explore them to gain additional insight to that provided by the journalists. In contrast to our hypotheses, our results indicated that augmenting exploratory visualizations with introductory ‘stories’ does not seem to increase user-engagement in exploration.

Many online data graphics use narrative design elements to explain a given dataset in a straightforward and compelling way. According to New York Times graphic editors Mike Bostock and Shan Carter, these explanatory graphics are preferable for data-journalism, as they have the advantage of exposing up-front what the main insights from the data are, without making people ‘have to work for them.’ However, most only provide limited interactivity, which reduces the potential for personal extraction of insight. In essence and by definition, Information visualization (Infovis) is interactive and exploratory. Thus, finding ways to make exploratory graphics more accessible and engaging to people is important, because if open/public/civic data is to truly empower people, then these people should be able to use appropriate tools to gain their own insights and knowledge—not only that provided by journalists in articles written or designed from a specific perspective. We explored the potential of narrative visualization techniques and storytelling to trigger this desired user-engagement. By engagement, we specifically mean a user’s investment in the exploration of a visualization.

DAHU Project-Team

6. New Results

6.1. Specification and verification of data-driven systems

Process-centric views of data-driven workflows. Declarative, data-aware workflow models are becoming increasingly pervasive. While these have numerous benefits, views describing valid sequences of tasks are also useful to provide stake-holders with high-level descriptions of the workflow. In [23], we study the problem of recovering process-centric views from declarative, data-aware workflow specifications. The views are most naturally specified by finite-state transition systems describing regular languages. The results characterize when process-centric views of artifact systems are regular, with both linear and branching-time semantics.

Complexity in counter systems and in proof systems. The static analysis of queries on XML trees and data streams relies in a majority of cases on decision procedures expressed in terms of formal systems like counter systems or proof systems. For instance, two-variables first-order data queries on words can be related to reachability in vector addition systems (VAS), and the same queries on trees to reachability in a branching extension of VAS. We have fundamental results on the computational complexity of these problems, including the first explicit upper bound for reachability in VAS [24] and the best known lower bound for reachability in branching VAS [17] (where it is currently unknown whether reachability is decidable at all). We have furthermore defined a first sequent calculus for a modal data logic [29] as preliminary groundwork for the ANR PRODAQ project on proof systems for data queries.

6.2. Query processing for the Web

Query languages for graph databases. Graph-structured data on the Web can be found in emerging applications such as *RDF* and *linked data*, or *social networks*. Classical database languages are not suitable to query such data, essentially because they do not allow to (easily) express simple connectivity queries, which are the basic building block in graph navigation. We use Regular Path Queries, computing pairs of nodes reachable via a path satisfying a regular expression. We have tackled the problem of answering queries over graph databases which are available only through a given set of views. We have shown that in the “asymptotic case”, i.e. when the query is large enough relative to the view definition, it is decidable whether the view determines the query [22].

6.3. Distributed knowledge base.

Webdamlog The Webdamlog language is an extension of datalog to the distributed context, with *delegation* as the main novelty. A summary of the project was presented in [20].

We introduced an access control mechanism based on provenance in [26]. This access control is designed for a distributed setting. Peers getting data are also willing to enforce access control on that data, so that the owner of the data keeps some control over it when the data is passed around in the network. A second version of Webdamlog was developed in 2015 at Drexel, primarily by Vera Moffit also as part of her thesis (in collaboration with S. Abiteboul). It includes access control mechanism.

EX-SITU Team

7. New Results

7.1. Fundamentals of Interaction

Participants: Sarah Fdili Alaoui, Michel Beaudouin-Lafon, Cédric Fleury, Wendy Mackay, Theophanis Tsandilas.

In order to better understand fundamental aspects of interaction, ExSitu studies interaction in extreme situations. We conduct indepth observational studies and controlled experiments which contribute to theories and frameworks that unify our findings and help us generate new, advanced interaction techniques. Although we continue to explore the theory of Instrumental Interaction in the context of multi-surface environments [23], we are also extending it into a wider framework we call *information substrates*. This has resulted in several prototypes, such as Webstrates [18]. We also continue to study elementary interaction tasks in large-scale environments, such as pointing [11] and object manipulation [15].

Information substrates – “Instrumental interaction” argues that, since our interaction with the physical world is often mediated by tools, or instruments, we should do the same in the digital world. Our work on multisurface environments has demonstrated the value of this model, for example, to support distributed interfaces in which the user controls the content of a wall-sized display using handheld devices [23]. Instrumental interaction does not, however, describe the “objects of interest” that instruments interact with, nor does it explain how an object becomes an instrument, nor how users appropriate them in unexpected ways (the principle of “co-adaptation”).

“Information substrates” embrace a wider scope than instrumental interaction: both objects and instruments are “substrates” that hold information and behavior, and can be combined in arbitrary ways. What makes an object an instrument is defined not by what it is but by how the user uses it. We started to explore this concept with Webstrates [23], a web-based environment in which content and tools are embedded in the same information substrate—in this case the Document Object Model (DOM) (Figure 7).

Our work on information substrates has influenced other projects in the group. For example, our work on tools to help programmers parallelize and optimize their code [22] uses coordinated views of the code: a traditional text view and a graphical polyhedral visualization (Figure 2). These two substrates afford different types of manipulation by the user, but share the same underlying information, i.e. the algorithm being designed. The SketchSliders technique [20], described in the following section, provides users with an easily customizable approach to control complex visual displays. SketchSliders act as a substrate for creating slider instruments, which are independent from but tightly coupled with the visual display they control. By letting users define their own sliders, we solve the long-standing problem of combining power and simplicity. Finally, the ColorLab prototypes [17], described in the following section, provide artists and graphic designers with substrates that offer novel ways to interact with and display color relationships.

Interaction in the large – ExSitu and its predecessor InSitu have a long history of studying the most fundamental action in visual environments: pointing. We recently published an extensive 64-page journal article [11] on our studies of pointing on large, wall-sized displays. In such environments, users must be able to point from a distance, typically up to a few meters from the screen, with great accuracy. Existing techniques are ill-suited for this task, due to the combination of the high index of difficulty and the constraint that users must be able to move around in the room while pointing.

We have designed and tested a number of techniques, including dual-mode techniques that combine coarse pointing with direct techniques, such as ray-casting or using the orientation of the head, and fine pointing with relative techniques, such as using a hand-held touchpad 3. Rather than proposing the “ultimate” pointing technique for such environments, we provide a set of criteria, a set of techniques derived from those criteria, and a calibration technique for optimizing the transfer functions used by relative pointing techniques under extreme conditions.

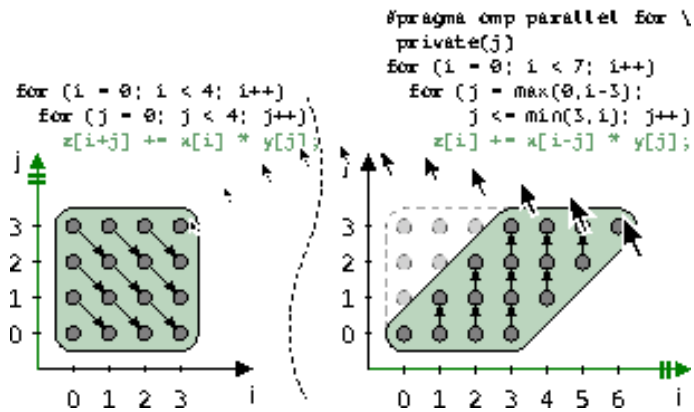


Figure 2. Performing a skew transformation to parallelize polynomial multiplication. The code is automatically transformed from its original form (left) to the skewed one (right).

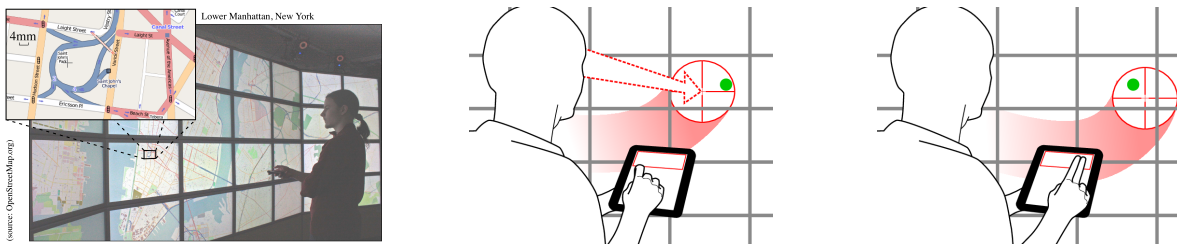


Figure 3. The challenge of pointing on a wall from a distance on a ultra-high resolution wall-sized display (left). Two of the pointing techniques that we evaluated: coarse pointing using the orientation of the head (center) vs. a two-finger swipe (right). In both cases, a one-finger swipe controls precise pointing.

In collaboration with the Inria REVES group in Sophia Antipolis, we proposed a framework for analyzing 3D object manipulation in immersive environments [15]. We decomposed 3D object manipulation into the component movements, taking into account both physical constraints and mechanics. We then fabricated five physical devices that simulate these movements in a measurable way under experimental conditions. We implemented the devices in an immersive environment and conducted an experiment to evaluate direct finger-based against ray-based object manipulation. We identified the compromises required when designing devices that (i) are reproducible in both real and virtual settings, and (ii) can be used in experiments to measure user performance.

7.2. Creativity

Participants: Sarah Fdili Alaoui, Michel Beaudouin-Lafon, Ghita Jalal, Wendy Mackay, Joseph Malloch, Nolwenn Maudet, Theophanis Tsandilas.

ExSitu is interested in understanding the work practices of creative professionals, particularly artists, designers, and scientists, who push the limits of interactive technology. This year, we conducted studies and created tools for a variety of such users. Based on contextual interviews with artists, designers and scientists, we created the *Color Portraits* design space [17] to characterize color manipulation activities, which influenced the design of a set of color manipulation tools (*Color Lab*). We designed *BricoSketch* [21] to enable professional illustrators to work at different levels of detail on paper. We studied how makers *remix* each others' designs by analyzing metadata from over 175,000 digital designs from Thingiverse [19]. We created *SketchSliders* [20] to help scientists explore their data by sketching and manipulating free-form interactive controllers. Finally, we studied the meaning and use of the term *evaluation* within the NIME (New Interfaces for Musical Expression) community [14].

Our studies of these “extreme users” allows us to obtain empirical grounding for the theoretical concepts of instrumental interaction, information substrates and co-adaptive systems. We expect to transfer what we learn to the design of creative tools, first for expert users, then for non-specialists and non-professional users.

Color Portraits – We conducted contextual interviews with 16 participants, who provided detailed examples of how they used color to create 69 different artistic or technical artifacts [17]. Based on results from these interviews, we created the Color Portraits design space to help identify color manipulation requirements that are poorly addressed by today's color manipulation tools. We then developed a set of novel color-manipulation tools that test the generative power of the design space. We presented these to users as probes. Our observations of how users interacted with the color probes provide implications for the design of more advanced tools.

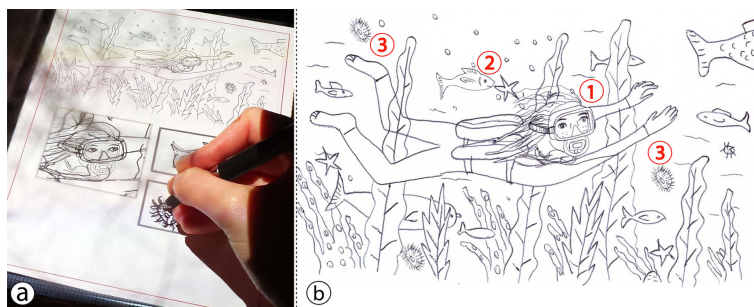


Figure 4. An artist drawing on paper with *BricoSketch*. (a) The artist has created three views on paper to draw parts of the illustration with higher detail: (1) the head of the diver, (2) a fish, and (3) an urchin. (b) The final composition after blending the partial views together.

BricoSketch – We conducted interviews with four professional illustrators and investigated how they use technology and paper in their creative process [21]. We also studied the evolution of the work of one of these illustrators for a period of two years. In interaction with this artist, we designed BricoSketch. BricoSketch enables illustrators to interactively create partial views of their drawings. Such views can be transposed and rescaled. Artists can then use them to create variations of their illustrations or add details with higher drawing precision. Our implementation is based on interactive paper technology that allows for above-the-surface interaction and supports traditional drawing tools such as common pens and pencils.

Remixing Designs – We investigated [19] how makers remix digital designs for physical objects on “Thingiverse”, a well-established online 3D-printing maker community. We collected metadata from over 175,000 digital designs and analyzed the *remixing graph* – links between sources and remixes that primarily exhibit a tree-like inheritance structure. We also used this data to identify particularly influential and surprising “Things”, which we further examined via qualitative case studies. We concluded with specific suggestions for online design repositories and design software so as to provide better support for remixing, and thus build stronger online maker communities.

SketchSliders – We developed SketchSliders [20], range sliders that users can freely sketch on a mobile device to parametrize and customize their data exploration on a wall display. With a small combination of sketches and gestures, users can create complex interactive controllers, such as slider branches and data transformation sliders 5. In addition to their natural custom shape, the sketched sliders can also be enhanced by interaction aids such as slider cursors, markers and distribution visualizations. We evaluated the sketching interface with six visualization experts and found that SketchSliders accommodate a wide range of exploration strategies, as well as help users focus and customize their visual explorations.

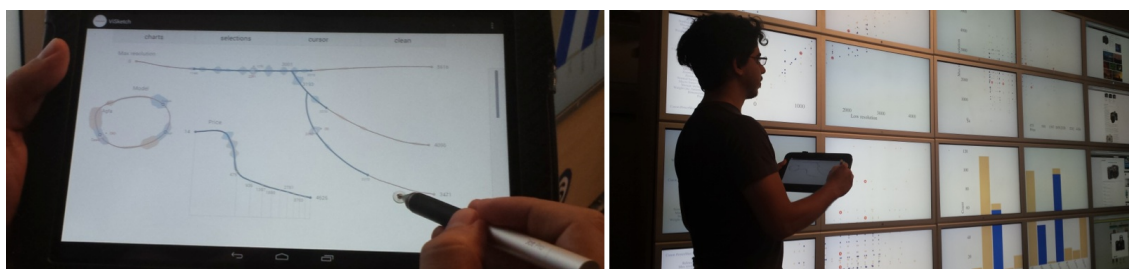


Figure 5. SketchSliders let users directly sketch viusalization controllers to explore multidimensional datasets.

Evaluation for NIME – We explored the use of evaluation techniques and terminology within the past three years of the New Interfaces for Music Expression (NIME) conference [14]. We categorized each paper that mentioned evaluation according to five criteria: a) targets and stakeholders considered, b) goals set, c) criteria used, d) methods used, and e) duration of evaluation. Results suggest that the NIME community does not share a common culture with respect to evaluation, with little consistency regarding use of the term. This paper raises the issue of evaluation within NIME community, with the goal of using it more consistently and effectively in the future.

7.3. Collaboration

Participants: Michel Beaudouin-Lafon, Cédric Fleury, Wendy Mackay, Can Liu, Ignacio Avellino Martinez.

ExSitu is interested in exploring new ways to support collaborative interaction, especially within and across large interactive spaces such as those of the Digiscope network (<http://digiscope.fr/>). We started to investigate how to support telepresence among large, heterogeneous interactive spaces [24], [25]. In particular, we studied how accurately a user can interpret deictic gestures in a video feed of a remote user [12]. These deictic

gestures are important for conveying non-verbal cues for communication between remote users. We also created *Webstrates* [18], an environment for exploring shareable dynamic media and the concept of *information substrate*.

Telepresence among large, heterogeneous interactive spaces – Large interactive spaces are powerful tools that can help scientific, industrial and business users to collaborate on large and complex data sets. In order to reach their full potential, these spaces must not only support local collaboration, but also collaboration with remote users, who may have significantly different display and interaction capabilities, such as a wall-display connected to an immersive CAVE.

We explain why supporting telepresence across large interactive spaces is critical for remote collaboration [24]. We have also started to explore how such asymmetric interaction capabilities provide interesting opportunities for new collaboration strategies in large interactive spaces [25].

Accuracy of deictic gestures for telepresence – In the context of telepresence on large wall-sized displays, we investigated how accurately a user can interpret the video feed of a remote user showing a shared object on the display, by looking at it or by looking and pointing at it (Figure 6) [12]. We also analyzed how sensitive distance and angle errors are to the relative position between the remote viewer and the video feed. We showed that users can accurately determine the target, that eye gaze alone is more accurate than when combined with the hand, and that the relative position between the viewer and the video feed has little effect on accuracy. These findings can inform the design of future telepresence systems for wall-sized displays.

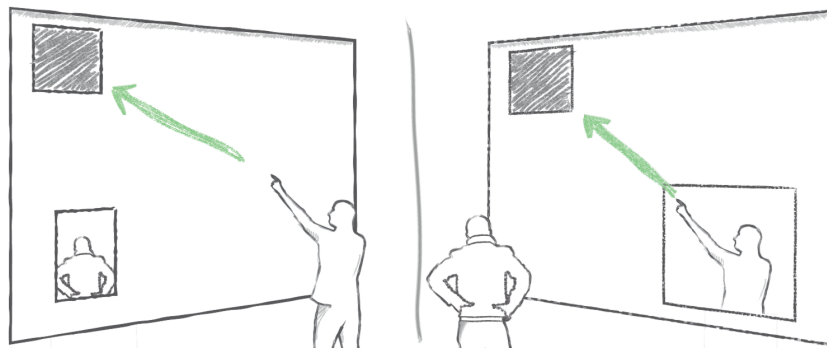


Figure 6. Users working on shared objects using two remote wall-sized displays : a user (left) shows a shared object by pointing at it and the remote user (right) can see which object is being shown through the video feed.

Webstrates – In collaboration with Université of Aarhus (Denmark) and Institut Mines Telecom, we created *Webstrates* [18], a system inspired by Alan Kay's early vision of interactive dynamic media. *Webstrates* is based on web technology: web pages served by the *Webstrates* server can be shared in real time among multiple users, on any web-enabled device. By using transclusion, a *webstrate* page can include other *Webstrates*. *Webstrates* can also include code, making them dynamic and interactive. A *Webstrate* that can act on another, transcluded *Webstrate*, is similar to an editor on a classical desktop environment. However the distinction between content and tools, documents and applications is blurred, e.g. content can be used as a tool, and tools can be shared like regular content. We implemented two case studies to illustrate *Webstrates* (Figure 7). We authored the article collaboratively, using functionally and visually different editors that we could personalize and extend at run-time. We also used *Webstrates* to orchestrate a presentation, using multiple devices to control the presentation, to let the audience participate and the session chair organize the session. We demonstrated the simplicity and generative power of *Webstrates* with three additional prototypes and evaluated them from a systems perspective. *Webstrates* runs in our WildOS middleware on the WILD and WILDER rooms, and is used for some of our projects on telepresence.



Figure 7. Sample uses of Webstrates: (a) Collaborative document authoring with different editors personalized at run-time; (b) Multiple devices used to sketch a figure (tablet 1), see it in a print preview (tablet 2), and adjust it in a graphics editor (laptop). (c) Distributed talk controlled remotely by a speaker with a separate interface for audience participation.

ILDA Team

7. New Results

7.1. An Evaluation of Interactive Map Comparison Techniques

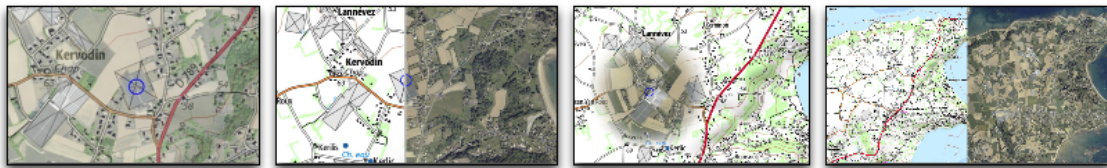


Figure 4. Empirical evaluation of multiplexing strategies using juxtaposition or overlaying for spatially-registered map comparison tasks [4]. Research conducted in the context of ANR project MapMuxing with IGN (Institut National de l'Information Géographique et Forestière).

Geovisualization applications typically organize data into layers. These layers hold different types of geographical features, describe different characteristics of the same features, or represent those features at different points in time. Layers can be composited in various ways, most often employing a juxtaposition or superimposition strategy, to produce maps that users can explore interactively. From an HCI perspective, one of the main challenges is to design interactive compositions that optimize the legibility of the resulting map and that ease layer comparison. We characterized five representative techniques, and empirically evaluated them using a set of real-world maps in which we purposefully introduced six types of differences amenable to inter-layer visual comparison. We discussed the merits of these techniques in terms of visual interference, user attention and scanning strategy. Those results can help inform the design of map-based visualizations for supporting geo-analysis tasks in many application areas.

This work was published at ACM CHI 2015 [4], and received an honorable mention (top 5% of all submissions).

7.2. Reciprocal Drag and Drop

Drag-and-drop has become ubiquitous, both on desktop computers and touch-sensitive surfaces. It is used to move and edit the geometry of elements in graphics editors, to adjust parameters using controllers such as sliders, or to manage views (e.g., moving and resizing windows, panning maps). Reverting changes made via a drag-and-drop usually entails performing the reciprocal drag-and-drop action. This can be costly as users have to remember the previous position of the object and put it back precisely. We introduced the DnD^{-1} model that handles all past locations of graphical objects. We redesigned the Dwell-and-Spring widget to interact with this history. Applications can implement DnD^{-1} to enable users to perform reciprocal drag-and-drop to any past location for both individual objects and groups of objects. We performed two user studies, whose results show that users understand DnD^{-1} , and that Dwell-and-Spring enables them to interact with this model effectively.

This work was published in ACM ToCHI [1].

7.3. SketchSliders: Sketching Widgets for Visual Exploration on Wall Displays

Given our interest in how to effectively interact with wall displays, we have started investigating ways to empower end users, by allowing them to easily create themselves their interfaces. We introduced a sketching interface that runs on mobile devices, and allows users to explore multi-dimensional datasets on wall displays by sketching on the fly the interactive controllers they require. We demonstrated this concept with *SketchSliders*, range sliders that users can freely sketch on the mobile surface to customize their exploration. A small combination of sketches and gestures allows the creation of complex interactive sliders, such as circular sliders for periodic data, slider branches for detailed interaction, and fisheye transformation sliders. We augmented sliders with a suite of tools, such as markers, slider cursors, and approximate views of data distributions. These designs were inspired by a design study with three visualization experts, and validated through a user study with six experts using our system.

This work was published at ACM CHI 2015 [9], and received an honorable mention (top 5% of all submissions).

7.4. Ultra-high-resolution Wall-sized Displays

We have worked on the following other projects, also related to the interactive visualization of large datasets on ultra-high-resolution wall displays:

- Mid-air Pointing on Ultra-Walls [5]. The size and resolution of ultra-high resolution wall-sized displays (“ultra-walls”) make traditional pointing techniques inadequate for precision pointing. We studied mid-air pointing techniques that can be combined with other, domain-specific interactions. We explored the limits of existing single-mode remote pointing techniques and demonstrated theoretically that they do not support high-precision pointing on ultra-walls. We then explored solutions to improve mid-air pointing efficiency: a tunable acceleration function and a framework for dual-precision (DP) techniques, both with precise tuning guidelines.
- WallTweet: A Knowledge Ecosystem for Supporting Situation Awareness [20]. Tweets are an important source of information during large-scale events, like tornados or terrorist attacks. Yet, tweets are hard to visualize and put in a geographical context: large quantities of tweets get sent in a short period, that vary greatly in content and relevance with respect to the crisis at hand. WallTweet is a tweet visualization designed for wall displays and aimed at improving the situation awareness of users monitoring a crisis event utilizing tweets.
- The monitoring of road traffic data on wall-sized displays [15]. Road traffic is a complex system that can be very unstable. A little perturbation can lead to a traffic-crippling congestion. To avoid such situations, researchers attempt to model traffic in order to prevent congestions and optimize traffic flow. Traffic is also continually monitored by operators in traffic control rooms. We designed an interactive system to monitor traffic on a wall display, that is coupled to traffic modeling algorithms. The system enables users to interactively adjust traffic parameter settings and visualize the impact of these adjustments at both a local and global scale.

OAK Project-Team

7. New Results

7.1. Scalable and Expressive Techniques for the Semantic Web

On the topic of efficient query answering methods for semantic-rich RDF data, we have obtained new fundamental results for the RDF Schema ontology language [25] and for a simple DL-Lite dialect [23], [34]; we presented our results in a tutorial at IEEE ICDE [10] and in an invited keynote at SEBD, the Italian Database conference [4]. A demonstration issued from this work was presented at VLDB [26] and at BDA, the French database conference [27].

To help users get acquainted with large and complex RDF graphs, we have started to work on an approach for RDF graph summarization: a graph summary is a smaller RDF graph, often by several orders of magnitude, which preserves the core structural information of the original graph and thus allows to reason about several important graph property on a much more manageable structure. Our first results were presented in [17] and demonstrated at [29] and [30]. These results were also presented in the keynote of the Data Engineering and the Semantic Web workshop [5].

On the related topic of analytical RDF schemas, we have published novel techniques for incrementally computing the result of an RDF analytical query (also known as “RDF cube”) out of the result of a previously computed RDF cube [31]. Such computations, commonly known as roll-up, drill-down etc. in the classical relational database setting, require novel solutions for RDF due to the heterogeneity of the graph structure.

7.2. Massively Distributed Data Management Systems

One of the main results of the year is the publication of the full paper [15] and demonstration [14] on CliqueSquare in the highly prestigious IEEE Conference on Data Engineering (ICDE). CliqueSquare has also been released in open source in 2014 (see the Software section). Its main advantage is a novel technique for optimizing conjunctive queries in a massively parallel setting, using n-ary join operators; this allow the optimization algorithm to build plans which are as flat as possible. These results apply beyond the RDF conjunctive query evaluation to the general setting of relational conjunctive query processing in a massively parallel context.

Another crucial result of the year is the publication of the PAXQuery framework for massively processing XML queries based on the Stratosphere (now Apache Flink) platform [3]. We show that our algebra-based approach allows to capture the expressive processing performed by an XQuery query and to compile it efficiently into massively distributed plans which are then evaluated by the Flink platform; this outperforms a set of state-of-the-art approaches for evaluating XQuery queries in a parallel environment. The system was also demonstrated at SIGMOD [11].

7.3. Advanced Algorithms for Data Querying and Transformation

We focused on explaining why some data, so-called missing answers, are not part of the result of a query, even though a developer expects them to be there.

The query-based explanations we return during query analysis serve as the starting point for our query rewriting process. Indeed, knowing the condition combinations pruning data relevant to the missing answers significantly narrows the search space for eligible query rewritings as we can first focus on finding solutions that only affect these query conditions. To further prune the search space, our current solution applies a cost model for rewritings based on several criteria, including edit distance to the original query, or the number of side-effects (tuples additionally appearing in the result of the rewritten query that are not our original missing answers). To select the best solutions w.r.t. the different dimensions of our cost model, we compute and return the skyline over these. We have demonstrated a preliminary version of the proposed algorithm in [8]. This work is reported [7] and in the PhD thesis of K. Tzompanaki [1].

7.4. Social Data Management and Crowdsourcing

Some particular tasks such as annotating data or matching entities have traditionally been outsourced to human workers for many years. But the last few years have seen the rise of a new research field called crowdsourcing that aims at delegating a wide range of tasks to human workers. Crowd workers tend to make mistakes, so that redundant tasks are typically submitted to mitigate errors. As the crowd is a relatively expansive resource, we have worked on building formal frameworks to improve the efficiency of these processes.

Our research has been focused on two kinds of queries: boolean queries (asking the crowd to identify relevant items in a list, e.g., meals containing a specific ingredient), and ranking queries (asking the crowd to retrieve one or a few preferred items; e.g., ski resorts). We proposed new algorithms and heuristics improving the state of the art for boolean queries, and claimed the first algorithms for ranking queries (more specifically, for top-k and skyline queries) in the comparison framework [16].

We considered top-k query answering in social tagging systems, also known as folksonomies, a problem that requires a significant departure from existing, socially agnostic techniques. In a network-aware context, one can and should exploit the social links, which can indicate how users relate to the seeker and how much weight their tagging actions should have in the result build-up. Beyond explicit social links, we also focus on uncovering implicit, potentially richer relationships from user interactions and exploiting them to improve core functionality such as search. Specifically we considered as-you-type search in a social network, where results socially close to the user asking the query are more relevant, and proposed an efficient algorithm presenting, for any (increasingly longer) prefix of the query as the user types it, the k most relevant results [28].